CHAPTER 139

STABILITY ANALYSIS OF OCEAN PIPELINES : A PROBABILISTIC APPROACH

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ABSTRACT

A probabilistic approach to the lateral stability of a pipeline on the sea-bed is considered. The uncertainties involved in the deterministic approach, due to the stochastic nature of variables, inaccuracy of models and statistical error are discussed. Variables representing these uncertainties are identified and their statistical properties investigated. Procedures for computing the probability of stability failure are developed using both analytical and Monte Carlo simulation techniques. Example calculations are presented indicating typical applications. It is concluded that the probabilistic procedure, as presented here, can serve as a valuable tool in the stability analysis of ocean pipelines.

1. INTRODUCTION

An important consideration in the design of pipelines resting on the ocean bottom involves their protection against possible structural instability. Included in the stability design must be a provision for the capacity of the structure to resist forces caused by wave-induced water motion and currents. This is normally achieved through an application of some form of anchorage or weighting down of the pipeline and taking into account the resistance against sliding of the bottom foundation.

Determination of the structural loads and resistances entering the stability design problem is based on theoretical and/or empirical models that consider a variety of influencing factors, such as hydrodynamic forces and soil frictional resistance. Due to uncertainties involved in various aspects of these models (e.g., random variable input parameters, experimentally determined coefficients, etc.), the final design is heavily influenced by the experience and judgement of the designer.

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This has often resulted in conservative attitudes in code directives and design decisions. The latter are commonly based on the concept of a global factor of safety. Recognition of the factor of safety's nonlinear relationship to certain input parameters and inability to distinguish between contributions of parameters separately, have caused designers to propose improvements in the method of expressing safety (Ellinas et al, 1984).

The objectives of the present study are to: (1) identify random variables that are present in the stability analysis; (2) determine their statistical properties and (3) develop a procedure to determine the probability of stability failure. A probabilistic description of the relevant random functions is used. The random variables in the analysis are the wave height and period, force and soil coefficients and variables representing uncertainty in the prediction of wave kinematics. Distribution functions for the variables are either determined from available data or using their commonly accepted representations. Evaluation of the random functions is done through analytical transformation of variables and Monte Carlo simulation.

2. PROBLEM FORMULATION

The case under consideration is that of a straight pipeline situated directly on a flat, rigid, impermeable bottom in the ocean. In terms of structural, lateral stability, the pipeline is considered to be safe if no movement thereof occurs.

For the purpose of this study the demand on the pipeline is considered to be the hydrodynamic forces exerted on it due to it being exposed to wave-induced oscillatory flow and a steady uniform current. The resistance to movement is considered to be provided by either anchorage or frictional resistance of the bottom soil.

The wave and current directions relative to the pipeline, θ_w and θ_c respectively, are assumed to be sufficiently small for the forces along the section of the pipeline which is of interest, to be essentially constant. A unit length of the pipeline is therefore considered to be representative of the section of interest in the design. For the unit length of pipeline the stability criterion requires that the lateral component of the force, F_H, be less than the resisting force, F_R. The stability condition to be investigated can therefore be expressed in terms of a stability parameter, G as

$$G = \frac{F_H}{F_R} < 1 \qquad ---- \quad (1)$$

3. DETERMINISTIC ANALYSIS AND IDENTIFICATION OF UNCERTAINTIES

The deterministic (conventional) methods used to evaluate lateral stability of a pipeline can be generally summarised under the following steps:

- Definition of the environmental conditions, with regard to waves and currents, based on specific statistical extreme states.

- Definition of the kinematics at the seabed using appropriate wave theories.

- Definition of the hydrodynamic forces through application of semi-empirical force models.

- Evaluation of the lateral stability using models describing the resistance to movement provided by either anchorage or the bottom soil, for chosen anchorage specifics or pipeline weight.

The specific models normally applied, and uncertainties involved in the procedure, are discussed in the next sections.

3.1 Definition of environmental conditions

The parameters which normally are used to define the environmental conditions are wave height, H, and period, T, and uniform current, U_c. These parameters are, due to their stochastic nature, most often determined in terms of an expected frequency of occurrence, or more practically, in terms of a return period, such as 50 or 100 years. The implication is that for decisions in this respect there has to be relied on extrapolation from statistical models which are based on measured data. The accuracy of results derived through this process depends heavily on the quantity and quality of the basic data. Mol et al (1984) describes inaccuracy in prediction of H due to (1) scatter and inaccuracies of the original data; (2) selection of a probability function and (3) extrapolation outside the observation period, resulting in a coefficient of variation typically in the order or 10% to 15%.

Further uncertainty is also present in the selection of the right design wave height and period combination. Chen et al (1979) e.g., determined distributions for T conditional to H from measurements within a stationary sea state. According to their study the conditional distribution of T is normal with a coefficient of variation which decreases with increasing H from about 18% at H_S to about 8% at H_{max}.

3.2 Definition of the kinematics at the sea-bed

Kinematics at the sea-bed such as horizontal velocity and acceleration are normally determined through use of a wave theory. Although verification of certain wave theories such as the linear, vocoidal and stream function theories, in relation to specific conditions such as water depth, has been done (Swart and Loubser, 1979; Dean and Perlin, 1986), it is only in the average sense.

Results of experimental work, especially such as those obtained by Grace (1976) in real ocean conditions, show large scatter and possible errors in the prediction of the maximum horizontal velocity, u_m, by the linear or stream function theories of up to 60% and 50% respectively. Even larger scatter and possible errors exist in the prediction of the maximum horizontal acceleration, a_m. Substantial uncertainty therefore exists in the prediction of kinematics at the sea-bed by wave theories.

3.3 Definition of the hydrodynamic forces

Application of Morison-type equations to determine the magnitude of hydrodynamic forces on a pipeline, is widely accepted to provide good average values (Karal, 1985; Grace and Zee, 1981).

The maximum hydrodynamic forces per unit length can be expressed as follows (Grace and Zee, 1981):

$$F_{Hm} = 0.5 \rho D C_{Hm} u_m^2 \qquad \text{- - - -} \quad (2)$$

and

$$F_{Lm} = 0.5 \rho D C_{Lm} u_m^2 \qquad \text{- - - -} \quad (3)$$

where

F_{Lm}, F_{Lm} = maximum horizontal and lift forces in a wave cycle, normal to the pipeline
ρ = water density
D = pipe outside diameter
C_{Hm}, C_{Lm} = maximum force coefficients (horizontal, lift)
u_m, a_m = maximum horizontal particle velocity and acceleration in a wave cycle, normal to the pipeline.

The accuracy of predictions by the above equations relies heavily on the choice of values for the force coefficients used. There is however a substantial degree of uncertainty related to appropriate choices in this respect, due to large scatter in the results of experimental work performed in the laboratory such as reported by Jacobsen and Bryndum (1984), and even more so for results from ocean experiments such as performed by Grace and Zee (1981) and the Hydraulics Research Station (1982).

The type of uncertainty involved in the prediction of the hydrodynamic forces is therefore mainly related to model inaccuracies. A certain degree of measurement error, in the determination of force coefficient values, would however also be present.

3.4 Uncertainties in the evaluation of the lateral stability

If the resistance to movement is provided by the soil frictional resistance the Coulomb friction force model is normally considered appropriate. The resistance force is thus determined as

$$F_R = S(W_S - F_L) \qquad ---- \quad (4)$$

Therefore the stability condition is described by

$$G = \frac{F_H}{S(W_S - F_L)} \qquad ---- \quad (5)$$

where

S = lateral soil resistance coefficient
W_S = submerged weight of the pipeline.

In the application of this model the main source of uncertainty lies in the value of S. Experimental results show considerable scatter in values of S (Anand and Agarwal, 1981) and choices normally has to be made from a range of possible values. Predictions of S, based on experience of soil conditions similar to those occurring in the case under consideration, or theoretical predictions from measurements of soil characteristics, will be subjected to a large degree of uncertainty. Even if this procedure was totally reliable, the possible variation of soil conditions along the pipeline and random results from measurements of values of soil parameters would introduce uncertainty in the prediction of S for a specific pipeline section.

3.5 The effects of uncertainties on the stability design

The effect of having to cope with uncertainties in the stability design of a pipeline, is that of introducing methods which are aimed at producing an overall conservative result. Upper bound values are used for random parameters such as coefficients and environmental parameters, and procedures, such as linear vectorial addition of wave and current kinematics are implemented. In addition to this a safety factor is included in the final analysis.

Although such practices would normally ensure safety, no quantifiable estimate can be made of the real degree of safety since the procedure does not account for the uncertainty related to true parameter values and model predictions. With conservatism being introduced at each step of the design procedure, the cumulative effect of the conservatism is difficult to quantify. Apart from this situation being unsatisfactory from a scientific viewpoint, the risk for unduly conservative design, or even failure in some cases, is present. The effects of over-conservative design could be drastic due to the sensitivity of installation methods to pipeline weight.

4. PROBABILISTIC APPROACH

The objective of the probabilistic approach is to account for the uncertainties involved in an analysis and to provide an improved measure for the reliability of the structure. The procedures followed in this approach are aimed at determining the probability of failure, P_f, for a structure. These procedures can generaly be summarized as:

- Description of the failure function $F = f(x_1, x_2, x_3 \ldots x_n)$ (where $x_1, x_2, x_3 \ldots x_n$ are the n variables in the analysis);

- Identification of the variables representing the uncertainties in the analysis and description of their statistical distributions or properties;

- Determination of the statistical distribution or properties of F and consequently P_f = Probability that $F < 0$, by approximate methods (Level II) or exact methods (Level III).

In the present study the above listed procedures are followed to develop a method through which the probability of stability failure of a pipeline can be determined. The probabilistic modelling is done according to the exact method (Level III) which implies full description of the random variables in terms of their probability density functions. Calculation of P_f is performed by both analytical development of the failure function and Monte Carlo simulation.

4.1 Description of the failure function

Since the stability criterion determines that $G < 1$, as defined in equation (1), it follows that to satisfy this criterion

$$F = F_R - F_H > 0 \qquad ---- \quad (6)$$

which is the basic expression for the failure function F.

If maximum forces are considered and the resistance is assumed to be provided by the bottom friction force, equation (6) can be expanded to

$$F = S(W_S - BF_{Lm}) - F_{Hm} \qquad ---- \quad (7)$$

where B is a reduction factor on F_{Lm} at the instant when F_{Hm} occurs.

Introducing equations (2) and (3) into (7) gives:

$$F = S(W_S - 0.5\rho DBC_{Lm}u^2{}_m) - 0,5\rho DC_{Hm}u^2{}_m \qquad --- \quad (8)$$

where u_m is e.g. described by the linear wave theory as

$$u_m = \frac{HgT\cosh[2\pi(z+d)/L]}{2L\cosh(2\pi d/L)} + U_c \qquad ---- \quad (9)$$

with
- L = wavelength
- d = waterdepth
- z = - depth below still water level
- U_c = uniform current velocity

4.2 Description of variable statistics

In the description of probability distributions to be used for the variables in the probabilistic analysis, distinction is made between two types of variables. The first type concerns those variables whose distributions will be dependant on the specific conditions of the case under consideration. The variables H, T and S are of this type as their statistical properties will be determined from measured data, only applicable to the relevant conditions. The second type of variable concerns those variables for which distributions can be derived which are generally applicable. It is assumed here that those variables which represent model uncertainty, such as C_{Hm} and C_{Lm} are of this type.

The variables which are treated as random variables in this study are:

Wave parameters, H and T
Soil frictional resistance, S
Wave kinematics model uncertainty parameters,

$$E_u = \frac{u_{wm}\ (\text{true})}{u_{wm}\ (\text{theory})} \qquad E_a = \frac{a_{wm}\ (\text{true})}{a_{wm}\ (\text{theory})}$$

with u_{wm}, a_{wm} = maximum horizontal velocity, acceleration due to wave action in a wave cycle.

Wave force coefficients, C_{Hm} and C_{Lm}.

4.2.1 Wave parameters, H and T

As explained above there is no general distribution function which applies for the description of the distribution of e.g. H. Typical distributions used are the Log-Normal and Extremal Type I, II and III. Similarly there is no general distribution for T which applies. Typically it can be taken to be Normal (Chen et al, 1979). The general models developed in this study allow the use of any distribution function for H and T but for specific examples the Log-Normal and Normal distribution are used for H and T respectively. H and T are taken to be uncorrelated.

4.2.2 Soil frictional resistance parameter, S

Although the distribution of S will in practice ideally be obtained from site specific measurements, it could, due to the lack of such measurements, be necessary to assign to it a distribution from previous knowledge pertaining to similar conditions. For this kind of subjective probabilistic treatment a distribution is chosen and parameters for it determined from existing knowledge concerning the value of S, such as its mean or maximum and minimum. Typical distribution functions which can be used in this fashion are the Rectangular, Triangular and Normal distributions. For example calculations in this study the Normal distribution is used for S with chosen mean value, S, and coefficient of variation, CV_S.

4.2.3 Wave kinematics model uncertainty parameters, E_u and E_a

It is assumed that the true value of e.g. u_{wm} is that of u_{wm} predicted by theory multiplied by a random factor, E_u.

The procedure followed to describe the statistical properties of E_u and E_a was as follows:

Using experimental data from which frequency histograms for E_u and E_a could be obtained, best fit general theoretical distribution functions were determined. It was found that the Log-Normal distribution provides the best fit for both E_u and E_a (see Figure 1). Parameters (mean and coefficient of variation) to be used in this distribution, applicable to both the linear and stream function wave theories, were obtained from appropriate available data sets. A summary of the results obtained is given in Table 1.

TABLE 1: Summary of statistical properties of E_u and E_a.

Variable	Mean	CV	5% value = 95% exceedance according to Log-Normal distribution
E_u (linear theory)	1.10	0.16	1.42
E_u (stream function)	0.96	0.18	1.26
E_a (linear theory)	1.22	0.39	2.10
E_a (stream theory)	1.07	0.36	1.82

Results based on data from: Grace (1976); Swart and Loubser (1979); Dean and Perlin (1986); Grace and Zee (1981).

4.2.4 Force coefficients, C_{Hm} and C_{Lm}

The force coefficients are assumed to represent the model uncertainties in predicting hydrodynamic forces. In the statistical treatment of C_{Hm} and C_{Lm} their distributions conditional to the dimensionless parameter, $P = u^2_m/Da_m$, were obtained. This was done by fitting distributions to available data for P-interval widths equal to 1. The Log-Normal distribution function was found to provide the best fit in general (an example fit is presented in Figure 2). The parameters required in this distribution are the mean and standard deviation. Least squares regression analysis on the means for each interval of P, provided continuous functions of P for $\bar{C}_{Hm}$ and $\bar{C}_{Lm}$. The values of the standard deviations of C_{Hm} and C_{Lm} can be obtained using the "average" coefficient of variation defined from those calculated for each P-interval. Table 2 contains values for the coefficients A, B, C and D obtained from a least squares fit of the equation $C_{Hm}, C_{Lm} = A + B\exp(-CP) + D/P^2$ to the interval means. The latter expression is similar to that used by Grace and Zee (1981).

Values for A, B, C and D resulting from fitting the same equation to the 95% exceedance values obtained for each P-interval, are also shown.

TABLE 2: Statistical properties of C_{Hm} and C_{Lm}.

Variable		$C_{Hm}, C_{Lm} = A + B\exp(-CP) + D/P^2$ for $P > 1$				CV
		A	B	C	D	
C_{Hm}	Mean	1.37	2.45	0.26	4.24	0.21
	5% value	1.89	3.36	0.26	4.47	
C_{Lm}	Mean	1.24	7.66	0.49	-1.47	0.23
	5% value	1.75	10.83	0.49	-2.07	

5% value = 95% exceedance using Log-Normal distribution.
Results based on data from: Grace and Zee (1981); Jacobsen and Bryndum (1984).

4.3 Development of models to calculate P_f

4.3.1 Analytical model

In order to reduce mathematical complexities in the analytical development of P_f, simplification of the failure function is desirable. This proves to be possible as follows:

For the relative narrow range of P values covered in a specific case the ratio between the mean values of C_{Lm} and C_{Hm} can be taken as constant. Also, the analysis is relatively insensitive to variation in the value of B, the reduction factor on the lift force. By assuming B to be constant, the failure function as expressed in equation (7) can now be rewritten as:

$$F = SW_S - F_{Hm}(cS + 1) \qquad \text{- - - - (10)}$$

where $c = BC_{Lm}/C_{Hm}$

If the kinematics model error factor E_u is introduced into equation (10), the final form of the failure function can be written as:

$$F = SW_S - KC_{Hm}(E_u u_{wm} + U_c)^2(cS + 1) \quad -- (11)$$

where $K = 0.5\rho D$

Following the criterion that $P_f = P_r(F < 0)$, the development of (11) in terms of the probability density function of the random variables S, C_{Hm}, E_u and u_{wm} results in:

$$P_f = \iiiint_{F_{min}}^{o\ F_{max}} \frac{f_S(q)\, f_{C_{Hm}}(t)\, f_{E_u}(w)\ f_{u_{wm}}\left[\dfrac{\sqrt{\dfrac{qW_S-r}{Kt(cq+1)}} - U_c}{w}\right]}{2Kt(cq+1)\ \sqrt{\dfrac{qW_S-r}{Kt\ (cq+1)}}}\, dw\ dt\ dq\ dr \quad (12)$$

where $f_x(X)$ is the probability density function of x evaluated at X,

F_{min} and F_{max} are the realistic lower and upper limits respectively of F,

and q, t, w and r are dummy variables in the integration.

The analytical development of $f_{uwm}(u_{wm})$ in terms of $f_H(H)$ and $f_T(T)$ is not possible with existing wave theories. However, if T is assumed constant the linear wave theory allows the development to be performed.

Closed form solutions of the expression in equation (12) is possible only in the most simple cases. A computer program was developed in this study which performs numerical integration to calculate P_f according to equation (12).

4.3.2 Monte Carlo simulation model

Whereas the analytical model prevents development of $f_{uwm}(u_{wm})$ in terms of $f_H(H)$ and $f_T(T)$, the Monte Carlo simulation technique provides a way to include both these variables. A computer program was developed in this study to perform this task using equation (8) and linear wave theory. This entails random generation of a large number of values for each variable according to its specified distribution function, calculation of the failure function for each set of variables and counting of the number of failures. P_f is then calculated as the ratio of the number of failures to the total number of variable sets evaluated.

In both the analytical model and Monte Carlo procedures the distribution function applicable to each variable can be selected by including the appropriate subroutine in the computer programs. The distribution parameters for C_{Hm} and C_{Lm} are determined by establishing the mean value and using the appropriate coefficient of variation in the Log-Normal distribution function. The mean value is calculated in each case through application of the point estimate method, developed by Rosenblueth (1975), to determine moments of the random functions C_{Hm}, C_{Lm} = f(P) = f(u_m, a_m) = f(H,T,E_u,E_a).

5. APPLICATIONS

Calculations for two different cases were performed to serve as examples of the application of the methods described above.

The first example (Case I) comprises a 0.4 m diameter pipeline of which the lateral stability is to be provided by its own weight. The statistical properties assumed for the wave height,period and bottom frictional resistance are given in Table 3.

TABLE 3: Statistical properties of variables in Case I.

Variable	Distribution	Mean	CV	1% Value
H	Log-Normal	6 m	0.13	8 m
T	Normal	12 s	0.08	14 s
S	Normal	0.6	0.08	0.5

1% value = 99% exceedance according to distribution function chosen.

For the statistical properties of E_u, E_a, C_{Hm} and C_{Lm} the values in Tables 1 and 2 are accepted and the value of B was taken as 0.95. The probability of failure, P_f, was calculated at three waterdepths, d = 80 m, 90, m and 100 m for a range of values of pipeline weight, W_S. The results are presented in Figure 3. Also shown are the W_S values obtained in a deterministic analysis with global factor of safety, FS = 1.5, 2.0 and 2.5, for d = 80 m. The analysis was performed using the 1% values in Table 3 for H, T and S, and the 5% values for E_u, E_a, C_{Hm} and C_{Lm} given in Tables 1 and 2.

The second case (Case II) comprises a 0.6 m diameter pipeline of which the stability is to be provided by anchorage. Exactly the same properties for the variables as used in Case I were applied. The calculations of P_f,

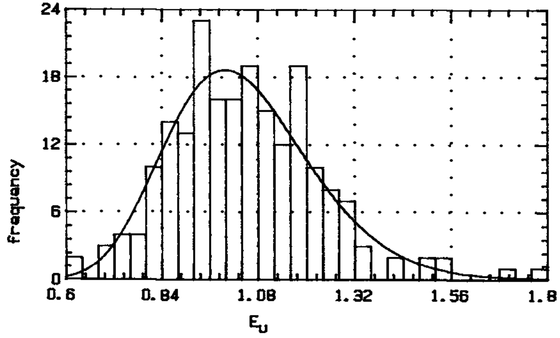

Figure 1: Frequency histogram of E_u with Log-Normal distribution function

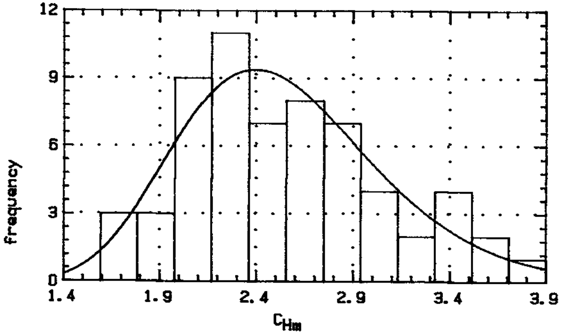

Figure 2: Frequency histogram of C_{Hm} for $3 < P < 4$, with Log Normal distribution function

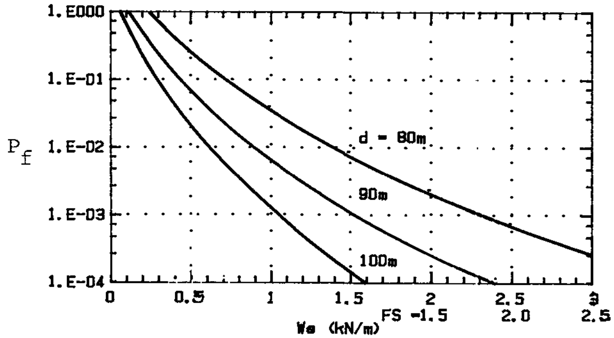

Figure 3: P_f vs W_S for Case I (FS shown for d = 80 m)

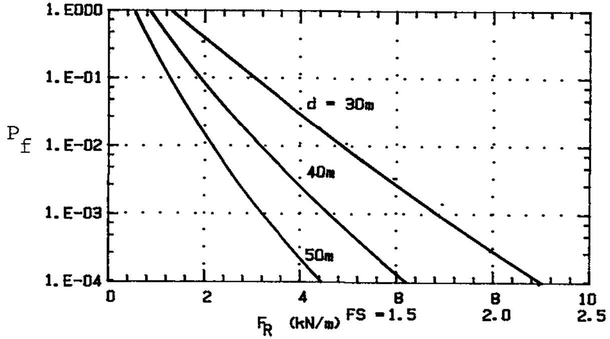

Figure 4: P_f vs F_R for Case II (FS shown for d = 30 m)

for a range of F_R values (F_R = capacity of anchorage), were performed at waterdepth, d = 30 m, 40 m and 50 m. The results are shown in Figure 4. Values for F_R calculated in a deterministic analysis, using the same variable values as in Case I, are also shown for FS = 1.5, 2.0 and 2.5, at d = 30 m.

The almost logarithmic increase in safety, with linear increase in pipeline weight or anchorage capacity, is clearly indicated by the results of the above calculations. In order to utilize these results in the stability design, the important question is that of which P_f value to accept. Meaning can, however, only be attached to such a value in terms of the concept "consequence of failure" incorporated in a risk analysis. Of more immediate practical importance is the evaluation of relative safety between different design options or along a pipeline route, using results such as provided in the examples presented above.

6. CONCLUSIONS

Based on the analyses performed and results obtained in this study the following conclusions can be drawn:

i) The traditional deterministic analysis of ocean pipeline stability involves several uncertainties which are difficult to represent realistically with safety factors alone. It is possible to represent these uncertainties with random variables for which statistical properties can be obtained.

ii) The incorporation of all the random variables representing uncertainties, in terms of their probability density functions, in the probabilistic procedure presented here, provides a more realistic measure of safety than is possible with the application of a safety factor alone.

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