CHAPTER 97

NUMERICAL MODEL OF BREAKING WAVE AROUND A RIVER MOUTH

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ABSTRACT

Equations for wave kinematics and wave dynamics based on small amplitude wave theory have been used in the prediction of wave deformations and wave-indused currents. However, the applicability of the linear wave theory is questionable in a river mouth where forced wave breaking and strong wave-current interaction take place.

A numerical model based on the non-linear dispersive wave theory has been developed, the results by this model was compared with the values of the experiments and the linear theory. Wave transformations including shoaling, wave-current interaction and wave breaking by the model showed a good agreement with the experimental result. In the prediction of wave-induced currents, the excess momentum flux (Pxx) computed by the model has more reasonable value than the radiation stress (Sxx) calculated by the small amplitude wave theory.

1. INTRODUCTION

To predict wave deformation and wave-induced currents in a shallow water region is an important problem not only for a design of coastal structures but also for a prediction of beach deformations in near shore regions. In general, equations for wave kinematics and dynamics based on linear wave theory have been used to solve these problems. However, the applicability of the linear wave theory is questionable in a river mouth where forced breaking and strong wave-current interaction take place. Recentry, Boussinesq type equations have been often used in numerical modelling of short waves in shallow water regions (Abbott et al. (1978), Haugel(1980) and Haugel et al.(1982)) and the accuracy of these equations was discussed by Abbott et al.(1978) and McCowan (1982).

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In this paper, the applicability of Boussinesq type equations in surf zone dynamics was studied in the problems of wave transformations and wave-induced flows around a river mouth. Results by this model were compared with those of the experiments as well as the results calculated by linear model developed by Sawaragi et al.(1984).

2. BASIC EQUATIONS

Boussinesq equation derived by Peregrine (1967) in a variable water depth are written as follows, where we introduce the viscous term. Fig.1 shows the definition sketch of the coordinate system.

$$\frac{\partial \eta}{\partial t} + \frac{\partial M_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial M_j}{\partial t} + \frac{\partial}{\partial x_i}\left(\frac{M_i M_j}{h+\eta}\right) + g(h+\eta)\frac{\partial}{\partial x_j}\eta\delta_{ij} - \frac{\partial}{\partial x_i}\left(\nu\frac{\partial M_j}{\partial x_i}\right) + \tau_{bj}$$
$$= \frac{h^2}{3}\left[\frac{\partial}{\partial x_j}\left(\frac{\partial^2 M_i}{\partial x_i \partial t}\right) - \frac{M_i}{h+\eta}\frac{\partial}{\partial x_i}\left(\frac{\partial^2 \eta}{\partial x_j \partial t}\right)\right] \tag{2}$$

where i, j = 1 or 2
δ_{ij} = Kronecker's δ function
g = gravitational acceleration
ν = kinematic eddy viscosity
τ_{bj} = bottom friction
$M_i = U_i(h+\eta)$: depth integrated velocity,ie, mass flux per unit width

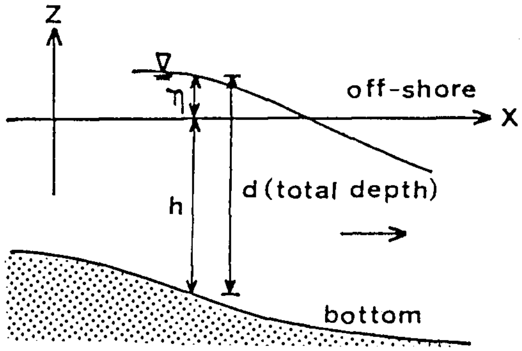


Fig.1 Definition sketch of coordinate system

Now, we define the mass flux per unit width M as a sum of staedy component M_o , phase averaged component of waves M' and turbulent component M''.

$$M_i = M_{oi} + M_i' + M_i'' \tag{3}$$

By using the above expression in eq.(1) and (2), the following equations for mean currents and waves are derived by the time averaging minipulation.

1) For mean currents

$$\frac{\partial \bar{\eta}}{\partial t} + \frac{\partial M_{\circ i}}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial M_{\circ j}}{\partial t} + \frac{\partial}{\partial x_i}\left(\frac{M_{\circ i} M_{\circ j}}{h+\bar{\eta}}\right) + g(h+\bar{\eta})\frac{\partial}{\partial x_j}\bar{\eta}\delta_{ij} + \overline{\tau_{bj}} + \frac{\partial P_{ij}}{\partial x_i}$$
$$- L_i = \frac{h^2}{3}\left[\frac{\partial}{\partial x_j}\left(\frac{\partial^2 M_{\circ i}}{\partial x_i \partial t}\right) - \frac{M_{\circ i}}{d}\frac{\partial}{\partial x_i}\left(\frac{\partial^2 \bar{\eta}}{\partial x_j \partial t}\right)\right] \tag{5}$$

where "$\overline{}$" denotes the time averaged value, d is a total depth $h+\eta$, P_{ij} is a excess momentum flux by the wave motion, L_i is a lateral mixing term of momentum.

$$P_{ij} = \overline{M'_i M'_j}/(h+\eta) \tag{6}$$

$$L_i = \frac{\partial}{\partial x_i}\left(\frac{\partial M_{\circ i}}{\partial x_i}\nu - \frac{1}{h+\bar{\eta}}\overline{M''_i M''_j}\right) \simeq \frac{\partial}{\partial x_i}\left(K_i \frac{\partial M_{\circ i}}{\partial x_i}\right) \tag{7}$$

where, τ_{bj} is a time averaged bottom friction term by waves and currents.

$$\overline{\tau_{bj}} = f_w \overline{\left(\frac{1}{2}\frac{M_{\circ j}}{h+\bar{\eta}} + \frac{M'_j}{h+\eta}\right)\left|\frac{1}{2}\frac{M_{\circ j}}{h+\bar{\eta}} + \frac{M'_j}{h+\eta}\right|} \tag{8}$$

where, f_w is a bottom friction coefficient on the bottom.

2) For waves

$$\frac{\partial \eta'}{\partial t} + \frac{\partial M'_i}{\partial x_i} = 0 \tag{9}$$

$$\frac{\partial M'_j}{\partial t} + \frac{\partial}{\partial x_i}\left(\frac{M'_i M'_j}{h+\eta'}\right) + g(h+\eta')\frac{\partial}{\partial x_j}\eta'\delta_{ij} + (\tau_{bj} - \overline{\tau_{bj}})$$
$$+ \frac{\partial}{\partial x_i}\left(\frac{M_{\circ i} M'_j}{h+\eta'}\right) + \frac{\partial}{\partial x_i}\left(\frac{M'_i M_{\circ j}}{h+\eta'}\right) + DM_j \tag{10}$$
$$= \frac{h^2}{3}\left[\frac{\partial}{\partial x_j}\left(\frac{\partial^2 M'_i}{\partial x_i \partial t}\right) - \frac{M'_i}{h+\eta'}\frac{\partial}{\partial x_i}\left(\frac{\partial^2 \eta'}{\partial x_j \partial t}\right)\right]$$

where, DM_j is the momentum dissipation term after wave breaking, it is expressed as :

$$DM_j = \frac{\partial}{\partial x_i}\left[-\nu\frac{\partial M'_i}{\partial x_i} + \frac{\overline{-M'_i M'_j} + M''_i M''_j - \overline{M''_i M''_j}}{h+\eta'}\right] \tag{11}$$

From eq.(11), we see that the momentum dissipation after wave breaking and the Reynolds stress generated by wave breaking become a governing factor in wave-induced currents because the right hand side of eq.(11) includes the same term which appears in driving force and lateral mixing terms for mean currents. DM_j is evaluated by eq.(12) after Sawaragi et al.(1974).

$$DM_j=\frac{\partial}{\partial x_i}\left[K_i^*\frac{\overline{M_i'M_j'}}{(h+\eta')^2}\cdot(h+\eta')\right] \tag{12}$$

where, the momentum dissipation coefficient K_i has a value of 0.4 ~ 0.6, the 5th and 6th term in the left hand side of the eq.(10) indicates the effects of wave-induced interaction. The effect of the bottom friction is neglected in the wave equation.

3. NUMERICAL MODEL

The implicit method was used, then each time and space were taken as 2-level and central difference scheme respectively. The solution was obtained effectively by the Gauss Seidel interaction method with the A.D.I. double sweep algorithm.

To obtain the numerical solution of waves, the following linear offshore and onshore boundary condition are applied.
At offshore :

$$\eta_t'=a\cdot\sin[\sigma\cdot\Delta t\cdot n + k\cdot\Delta s(i-iE)] \tag{13}$$

where, a = amplitude of waves
σ = $2\pi/T$
n = time step
k = wave number
ΔS = interval of grid

and

$$M_t'=-[\sqrt{g/h}\cdot\eta_t']\cdot h=-\sqrt{g\cdot h}\cdot\eta_t' \tag{14}$$

At onshore :

$$\begin{aligned} M'&=(1-r)\sqrt{gh}\cdot\eta' \\ \eta'&=-M'/\sqrt{gh} \end{aligned} \tag{15}$$

where, r is a reflection coefficient. In order to determine the breaking point, the applicabirity of breaking criteria proposed by some investigators to the river mouth where strong wave-current interaction takes place are examined experimentally. Then we used the most adequate criteria in the calculations.

In the computation of mean currents, open boundary conditions at the river mouth(iS) and offshore(iE) are applied. They are given as follows :

$$\text{Open Boundary}\ \begin{cases} M_{o\,is-1,j} = \text{River discharge}, & M_{o\,iE+1,j} = M_{o\,iE,j} \\ \bar{\eta}_{is-1,j} = \bar{\eta}_{is,j}\ , & \bar{\eta}_{iE+1,j} = 0 \end{cases} \tag{16}$$

As mentioned above, by calculating the waves and currents alternately,the effect of wave-current interaction is considered in waves and currents.

4. 2-D EXRERIMENTS AND NUMERICAL CALCULATION ON WAVE-CURRENT INTERACTION.

Littoral currents and water level around a river mouth are mainly affected by the relative strength of incident waves and river discharges. To investigate the effect of river mouth depth and run off on wave transformations and water levels we performed hydraulic experiments for wave-current interaction in a two dimensional wave tank. Then the results obtained by the experiments were compared with numerical results.

1) Experimental procedure and condition

The experiment was performed in the 2-D wave basin which has a 1/20 slope connected to river channel, 65cm in width and 23m in length, and the river discharge was supplied from the weir tank located in the upper part of the river channel. Fig.2 shows the experimental wave basin.

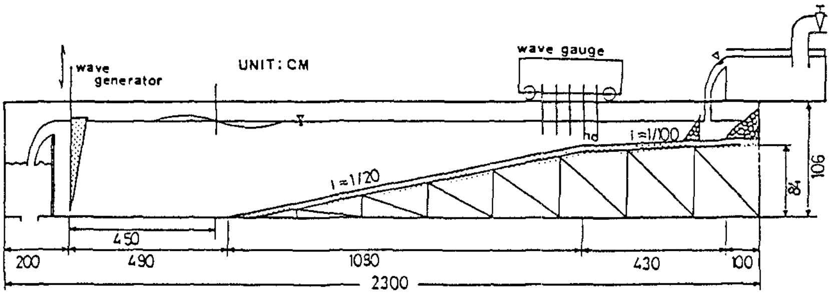


Fig.2 Wave tank and experimental set up

The wave height was measured at the intervals of 10cm by the capacitance type wave gauges equipped on platform car. Data were recorded by a data recorder with analog signals, and the data was analysed by the digital computer after the A-D conversion in 1/1000 sec interval.

We determined the experimental conditions by using a dimensional analysis. The experimental parameters are given as eq.(17).

$$\phi_3(h_o/H_o, q/(h_o\cdot\sqrt{gh_o}), i, H_o/L_o, x/L_o) = 0 \qquad (17)$$

where h_o : water depth in river mouth
H_o : wave height in offshore
T : wave period
i : slope
q : discharge per unit width
L_o : wave length in offshore
x : distance from the river mouth

Table 1 shows the experimental condition

Table 1 Experimental conditions

CASE		q_o	h_o' cm	h_o cm	H_o cm	T sec	h_o/H_o	H_o/L_o
AW	1	0	10.7	10.7	6.2	1.39	1.67	0.02
	1S	0.041	9.0					
	1L	0.069	7.7					
	2	0	6.7	6.7	6.6	1.40	1.00	
	2S	0.037	5.7					
	2L	0.062	5.1					
	3	0	6.7	6.7	8.3	1.66	0.83	
	3S	0.037	5.9					
	3L	0.062	5.4					
BW	1	0	10.7	10.7	5.6	0.89	2.00	0.05
	1S	0.041	9.0					
	1L	0.069	8.5					
	2	0	10.7	10.7	10.5	1.11	1.00	
	2S	0.041	9.0					
	2L	0.069	7.6					
	3	0	10.7	10.7	13.8	1.35	0.77	
	3S	0.041	9.1					
	3L	0.069	7.6					

$q_o = q/(h_o'\sqrt{gh_o'})$, h_o': water depth at the river mouth from S.W.L.

2) Influence of the depth at river mouth affecting on wave transformation

Fig.3 shows the relationship between wave breaking depth and break height. In the figure, NATU-A and NATU-B are the results in the uniform slope beach which have not river channel.

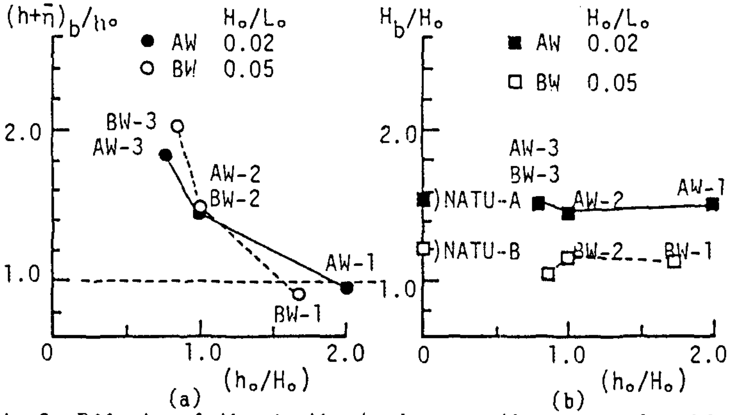


Fig.3 Effects of the depth at river mouth on wave breaking conditions

In Fig.3(b), we see that the break height is not affected by the depth at river mouth. The cases AW-1 and BW-1 of Fig.3(a) show that forced wave breaking took place in the river channel.

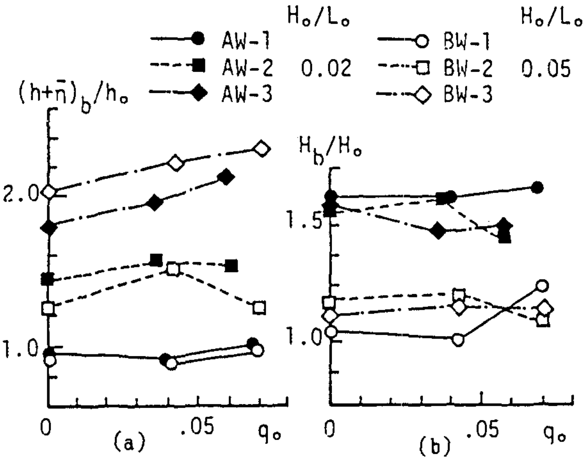


Fig.4 Effects of the river discharge on wave breaking conditions

3) Influence of currents on wave tranformation

Fig.4 shows the relation between the river discharge(q_o) and breaking condition. The results shown in Fig.4(a) show a tendency that the breaking point moves to offshore with increasing river discharge. However, in Fig.4(b), we see that the break height is not affected by the river discharge.

We also examined the applicability of wave breaking criteria to the coexistence field of waves and currents. Fig.5 illustrates the comparison of breaking heights measured by the experiment and the breaking criteria by Miche(eq.(18)) and Goda(eq.(19)).

$$H_b/L_b=0.142\ \tanh(2\pi h_b/L_b) \tag{18}$$

$$H_b/L_o=0.17\{1-\exp[-1.5\ \frac{\pi h_b}{L_o}(1+15\ \tan^{4/3}\beta)]\} \tag{19}$$

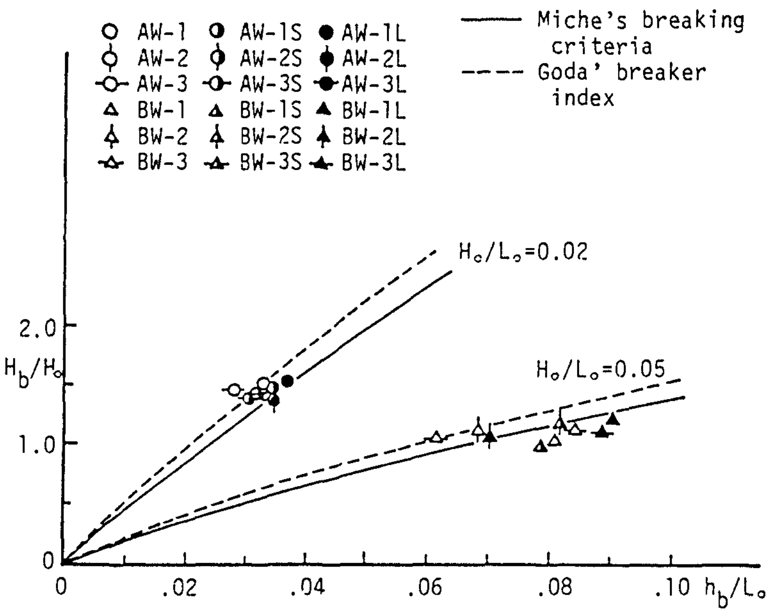


Fig.5 Comparisons between measured and predicted wave breaking conditions

From this figure, experimental results coincide well with both criteria regardless of the wave steepness.

4) Comparisons of numerical and experimental results.

In the preliminary calculations, it is found that the attenuation coefficient of the momentum after wave breaking K_i in eq.(12) of 0.6 seems to be the most adequate for the prediction of wave transformation on a natural beach. So, we use this value together with the Goda's breaking criterion in the following calculations.

Fig.6 shows the comparisons of measured and calculated wave heights and mean water levels in the cases where incident waves broke outside the river mouth (Cases AW-3 and AW-3L). Calculated results are shown by thick lines and measured results are indicated by circles. In the figure, calculated results based on the linear model (Sawaragi et al.(1984)) are also shown by thin lines. x = 0 at the horizontal axis corresponds to the location of the river mouth.

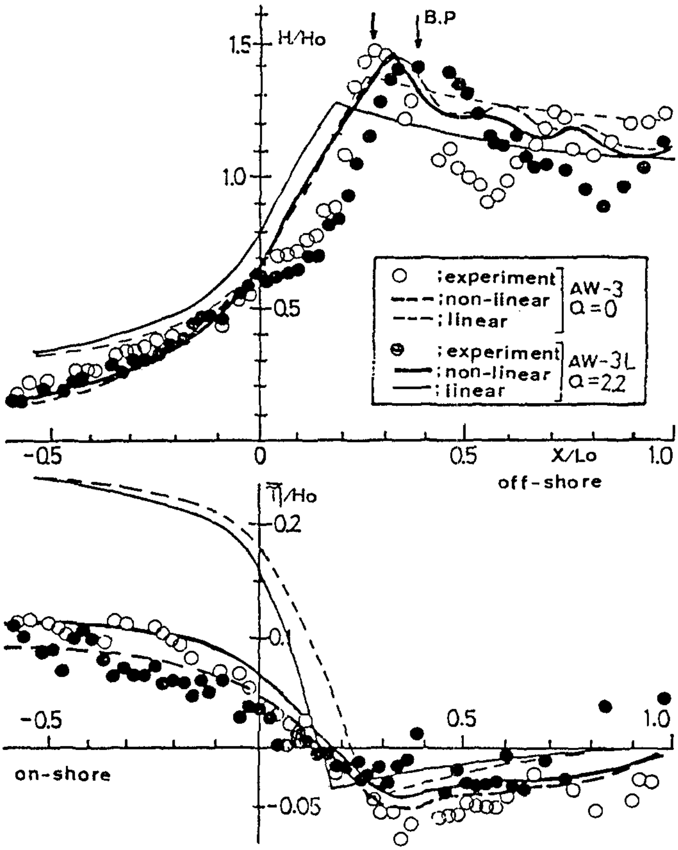
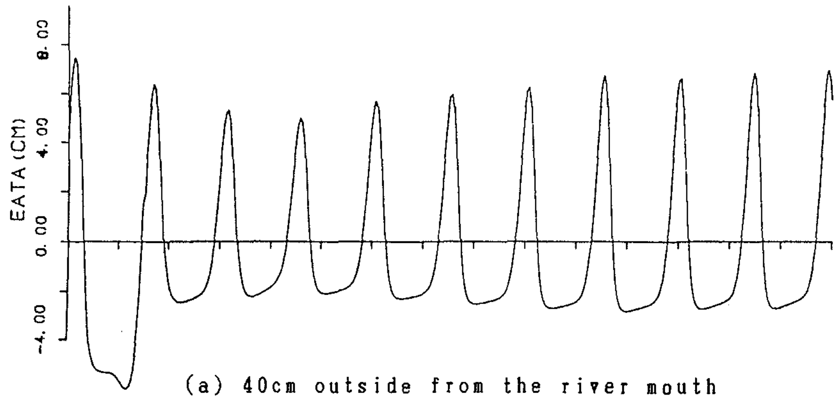


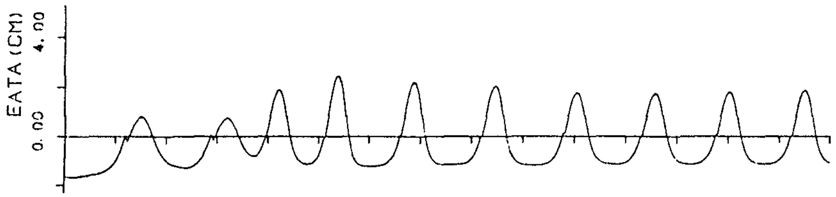
Fig.6 Comparisons between calculated and measured wave heights and mean water levels in the cases where waves broke outside the river mouth

It can be seen that although there are some discrepancies between the calculated and measured wave heights in the region of $0<x/L_0<0.3$, the calculated wave heights and mean water levels by the present model show good agreements with the experimental results regardless of the river discharge when compered with the predicted results by the linear model. Especially, the calculated mean water levels by the linear model give about two times larger estimetes of the experimental results.

Fig.7 illustrates time series of the surface displacements calculated 40cm outside and inside of the river mouth in the case of AW-3. From this figure, it is found that almost steady uniform waves are calculated after the generation of 6th wave.



(a) 40cm outside from the river mouth



(b) 40cm inside from the river mouth

Fig.7 Simulated time series of surface elevation incide and outside of the river mouth

Fig.8 shows the comparisons of measured and calculated wave heights and mean water levels in the cases where the forced wave breaking took place in a river channel (AW-1 and AW-1L).

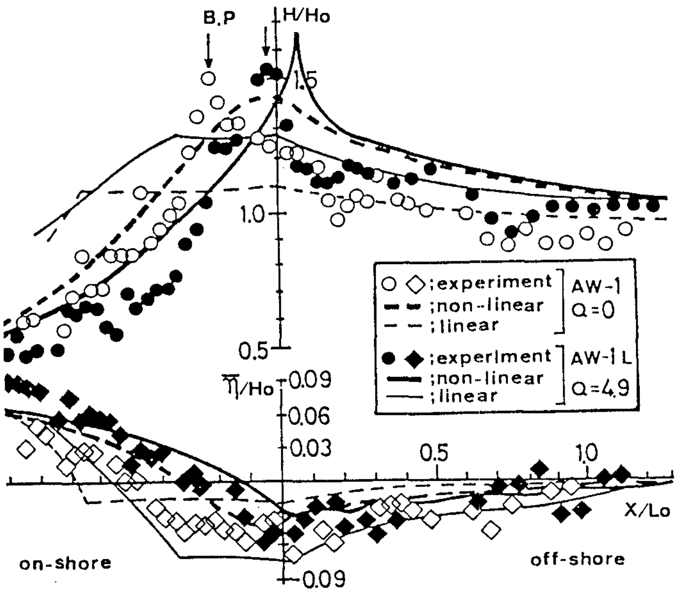


Fig.8 Comparisons between calculated and measured wave heights and mean water levels in the cases where waves broke in the river channel

In these cases, calculated wave heights by the present model no longer show good agreement with the experimental results. This disagreement comes from the difficulty in the determination of wave breaking condition in the river channel where the bottom slope is quite gentle. However, we can predict the closer experimental wave heights and mean water level by the present model than by the linear model.

As mentioned above, the linear model based on the linear wave theory overestimates the dis-placement of mean water level. On the contrary, the present model based on the non-linear wave theory offers good estimates of mean water level.

Fig.9 indicates the comparison between the excess monentum fluxes in the linear model S_{xx} and in the present model P_{xx} in thje case of AW-3. From this figure, the ratio of P_{xx}/S_{xx} becomes about 0.6 at around the break point and this results coincides with the results reported by Sawaragi et al.(1985).

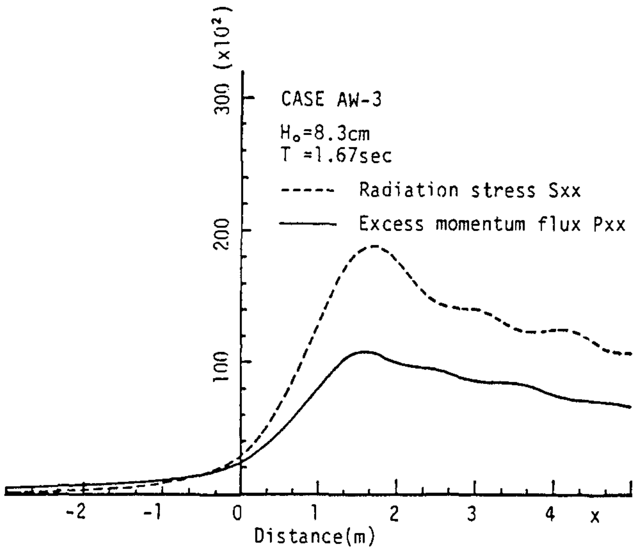


Fig.9 Comparison of P_{xx} and S_{xx}

6. CONCLUSION

In this paper, the authors applied Boussinesq type equations to simulate the wave field including wave breaking and wave-current interaction around a rivermouth.

The results simulated by the numerical model based on non-linear dispersive wave theory show good agreements with the experimental results in shoaling and wave-current interaction, and also in the prediction of wave breaking points and breaking heights it exhibits an improved results than those predicted by the linear model based on the linear wave theory.

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