CHAPTER 88

TIME SCALE FOR MODELING BEACH CHANGE

by

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1. ABSTRACT

A time scale in the similarity of beach change between model and prototype in transitional beach processes from an initial even slope to an equilibrium is developed using a series of small- and large-scale experiments in which the experimental conditions were set up with the scale-model relationship by the authors (1984). The time scale is obtained empirically as a function of experimental scale. Applied the proposed time scale and the scale-model relationship to model experiments, similarity of morphological beach change such as shoreline change and relative breaker point was well reproduced within the allowable range of experimental error. A semi-theoretical time scale is obtained from the continuity equation, the sediment transport rate, and the scale-model relationship of equilibrium beach profile in two-dimensional beach change. The relation between experimental and semi-theoretical time scale is discussed.

2. INTRODUCTION

Experimental technique for reproducing beach changes during time-dependent storm waves is desirable for beach erosion control and design of coastal structures. Some investigators have attempted to find the scale-model relationship and the time scale for beach change, however, scale-model relationship and time scale in transitional beach processes are not established well. Most of those studies had developed scale-model relationships by using major parameters which govern effectively beach changes. Noda (1972) discussed how the scale-distortion in vertical and horizontal scales and properties of beach material in models influence on the scale-model relationship. Vellinga (1982) carried out a number of small- and large-scale model tests for the dune erosion in Duch coast, Netherland caused during huge waves, and proposed a scale-model relationship with distortion and time scale by summarizing its results. Also, by applying the dynamic similarity condition and the dimensionless fall velocity parameter demonstrated by Dean (1973), Hughes (1983) studied the

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scale-model and time scale relationship. He examined the applicability of the proposed relationship to distorted model tests of coastal dune erosion in Panahadle, Florida due to a hurricane. As the result of the tests, the beach profile nearby the shoreline was only reproduced. Hallermeir (1985) proposed a distorted scale-model relationship which was introduced using the parameter composed of sediment and the fluid characteristics, and also discussed it including the scale-model relationships by Hughes and Vellinga. He was examined his relationship using two-dimensional large-scale models which were conducted at the Coastal Engineering Research Center, US Army. The scale-model relationships by Noda, Vellinga and Hughes include an effect of distortion in vertical and horizontal scales. These scale-model relationships have not been examined by verification tests.

The authors (1984) proposed an empirical scale-model relationship by considering the degree of experimental error which appears in model experiments. This scale-model relationship is that gives the relation between experimental scales such as wave properties, water depth in a wave tank which can be determined by the Froude law, and the grain-size scale of beach sand. By using the authors' scale-model relationship, we can be similar the beach profiles and breaking wave properties in the only equilibrium of model and prototype. Then the time scale which can hold the similarity between model and prototype during the transitional beach processes from an initial even slope to an equilibrium must be found.

3. SCALE-MODEL RELATIONSHIP FOR EQUILIBRIUM BEACH PROFILE

To find the scale-model relationship for two-dimensional equilibrium beach profile a series of small- and large-scale experiments was carried out in the following conditions:

(1) The vertical and horizontal length scales, such as scales for beach profile and water depth in a wave tank were taken the same (i. e., undistorted or geometric similarity). Both the vertical and horizontal length scales are subjected to Froude law.

(2) Wave characteristics such as wave height in deep water and period also are determined by the Froude law.

(3) The beach sediment in small-scale models was used sand or silica-sand with the same specific gravity as the sand used in the large-scale models. The grain-size scale $\lambda_d = (d)_m / (d)_p$, is chosen independently of the experimental scale, $\lambda_l = 1/n$, as mentioned in (1) and (2).

In accordance with the abovementioned (1), (2) and (3) small- and large-scale model experimental conditions are given. These experiments were carried out using the large two-dimensional wave tank (78m long, 1m wide and 1.2m deep) and the medium one (28m long, 0.5m wide and 1m deep). By arranging results of small- and large-scale model experiments, the similitude zone which indicates a good agreement between the model and prototype beach profiles at equilibrium was found as shown in Fig. 1. The scale-model relationship in Fig. 1 was rearranged for the conditions of $(d/H_0)_p < 0.01$ and $H_0/L_0 = 0.007 \sim 0.042$. We have also reported that the scale-model relationship shown by the dot-dashed line as the central trend curve of the similitude zone can be expressed by the following empirical relationship

$$\frac{1}{\lambda_d} = 1.7^{a} \left(\frac{1}{\lambda_l}\right)^{b} \quad (1)$$

where a=1, b=0.83 for the range of experimental scale $1 \geq \lambda_l \geq 1/2.2$, and a=0, b=0.2 for $\lambda_l > 1/2.2$.

4. EXPERIMENTS OF TRANSITIONAL BEACH CHANGES FROM INITIAL TO EQUILIBRIUM STATE

If we set up the experimental conditions of model and prototype so that the both relationships enter into the similitude zone shown in Fig. 1 {i. e., by Eq. (1)}, we are possible to similar the only equilibrium beach profiles between the two, but impossible to similar the beach change between the two in case of the transitional beach process from an initial even slope to an equilibrium. we find out experimentally the time scale relationship so that the prototype beach profile at the specified operation time can similar to the model beach profile which was selected up among experimental results in gradually increasingly wave operation time.

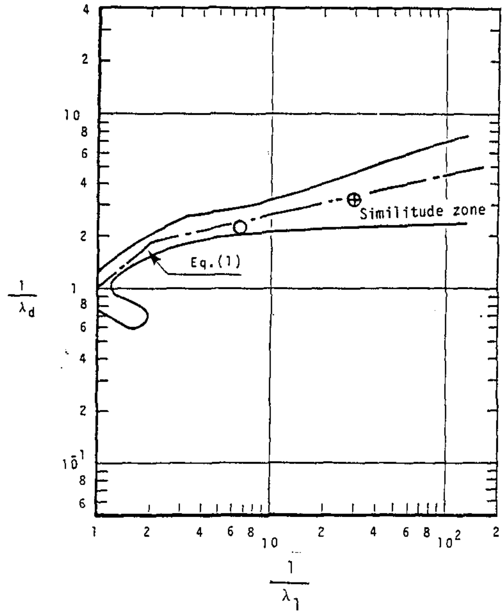
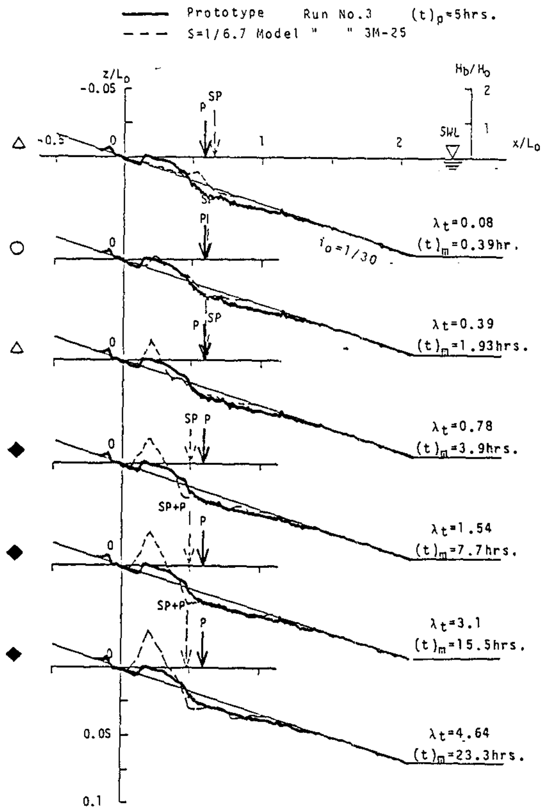


Fig. 1 Scale-model relationship by Ito & Tsuchiya for equilibrium beach changes with time scale relationship obtained by laboratory experiment.

By use of steady and regular waves in the two-dimensional large wave tank, beach changes as large-scale models (prototype) were measured intermittently until they reach from the initial even slope to an equilibrium state The experimental result and condition which Saville (1957) carried out a long time ago using a large wave tank, also is taken to be the prototype. In small-scale models, the vertical and horizontal lengths (experimental scale) such as wave properties and water depth in the wave tank were determined by the Froude law as mentioned in 3. (1) and (2). The sand and silica-sand to be used in the model were chosen according to Eq. (1) so that the grain-size scale enters within the similitude zone in Fig. 1. Model experiments were carried out until the beach profiles reach their equilibrium state for longer time, and beach changes and wave breaking properties during the experiments were measured intermittently. Model beaches in the medium wave tank were made initially the same beach slope with

Table 1 An Example of the experimental conditions in models and prototypes.

Run No.	Experimental scale λ_t	Wave height in deep water Ho (cm)	Period T (sec)	Wave steepness in deep water Ho/Lo	Water depth h (cm)	Initial beach slope i_0	Grain-size scale size λ_d	Sand grain in d_{50} (mm)	Mark shown in Fig.1
3	Proto.	20.6	3.00	0.015	100.0	1/30	Proto.	0.94	○
3M-25	1/6.7	3.5	1.16	0.017	15.0	1/30	1/2.24	0.42	
T-56	Proto.	171.2	5.6	0.035	442.0	1/15	Proto.	0.46	⊕
T-59	1/30	5.8	1.02	0.036	14.7	1/15	1/3.07	0.15	

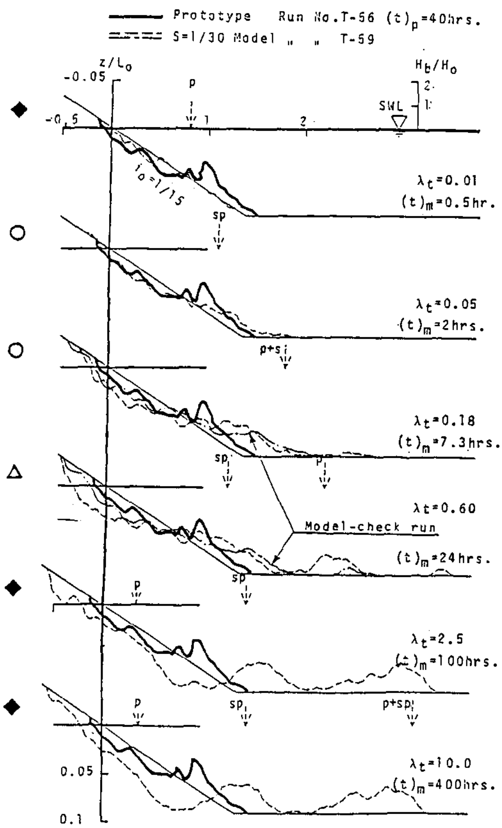


(a) Transitional beach change in model and prototype.

Fig. 2 Similarity comparison of the prototype beach profiles at the given time with temporarily varying model beach profiles (to be continued).

the corresponding prototype. An example of the experimental conditions determined by the manner mentioned is shown in Table 1.

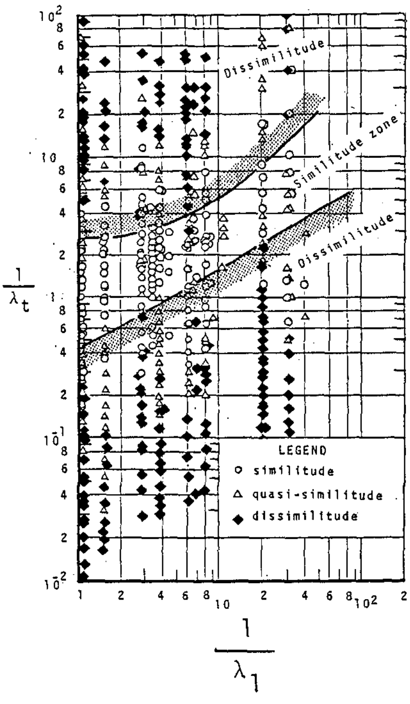
As is seen in Fig. 1, these experiments are satisfied the similarity condition as indicated by the open circle for Run No. 3M-25 to Run No. 3 and the cross-circle for Run No. T-59 to Run No. T-56 {by Saville (1957)}. Therefore, the only equilibrium beach profile between the two will become similar, but the similarity in transitional processes is uncertain. To define the similarity in transitional processes, prototype beach profile and breaking wave properties at the time, $(t)_p$=5hr, are compared with those in scale models during the times, $(t)_m$=0.39~23.3hr, as shown in Fig. (2). The position and the length



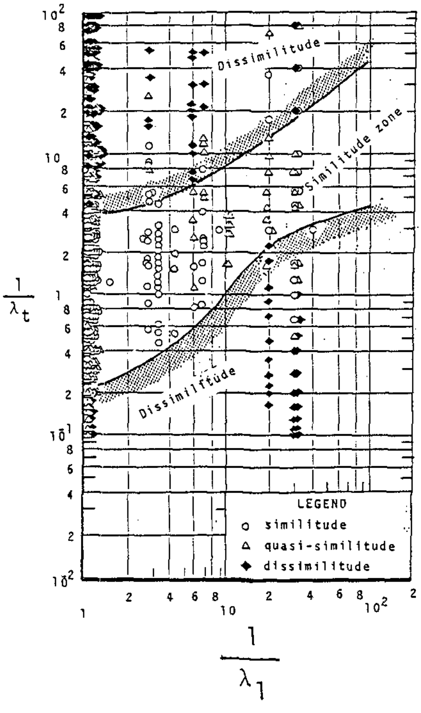
(b) Equilibrium beach profile in prototype.

Fig. 2 Similarity comparison of the beach profile of prototype at the specialized wave operation time with temporarily varying model beach profiles.

of allows in this figure indicate the breaking position and relative breaker height, H_b/H_0, and "P", "SP" and "SP+P" symbols indicate breaker types, plunging, spilling and intermediate between them, respectively. By considering the characteristics of experimental error, similarity of beach profiles and wave breaking properties are classified into three categories, *similitude* ("○"-mark), *quasi-similitude* ("△") and *dissimilitude* ("◆"). It is clearly recognized from Fig. 2 (a) that the judgement for similarity depends on the time scale, $\lambda_t=(t)_m/(t)_p$, and that the arrange of experiments which satisfy the relationship for the time-dependent similarity can easily be found. Also, Fig. 2 (b) shows the similarity comparison of the time-dependent beach profiles of the model, Run No. T-59 in Table 1 with Saville's prototype, Run No. T-56, in which the beach change is almost in equilibrium. By this way, in the case where the beach change of prototype is in transitional stage, non-equilibrium, the judgement results for similarity including the other experimental results are rearranged by λ_t^{-1} and λ_l^{-1} as shown in Fig. 3 (a). Furthermore, in



(a) Time scale for beach changes of model and prototype in transitional process.



(b) Time scale for beach changes of prototype only in equilibrium.

Fig. 3 Relationship between the time scale λ_t and the experimental scale λ_l for similarity in beach changes of model and prototype.

the case where the beach change of prototype is in equilibrium, the similarity comparison between the prototype (Run No. 56 in Table 1) and the time-dependent beach change of model (Run No. T-59) is shown in Fig. 3 (b), in which the results of other experimental results are also included. Median grain sizes of the sand and the silica-sand used in the model and prototype range of $(d)_m$=0.15~0.42mm and $(d)_p$=0.15~1.62 mm, respectively. The similitude /dissimilitude zones where the time scale relationship is/not achieved, are shown in Fig. 3 (a) and (b). The similitude zones are in Fig. 3 (a) and 3 (b) are not same, but a overlapped similitude zone can be found in Fig. 4. The similitude zone in Fig. 4 may be expressed by

$$\frac{1}{1.6}\sqrt{\lambda_l} \leq \lambda_t \leq \frac{1}{0.65}\sqrt{\lambda_l} \qquad (2)$$

It is noted in the figure that the dotted line which indicates the average tendency of Eq. (2) agrees closely with the Froude time scale

$$\lambda_t=\sqrt{\lambda_l}=\frac{1}{\sqrt{n}} \qquad (3)$$

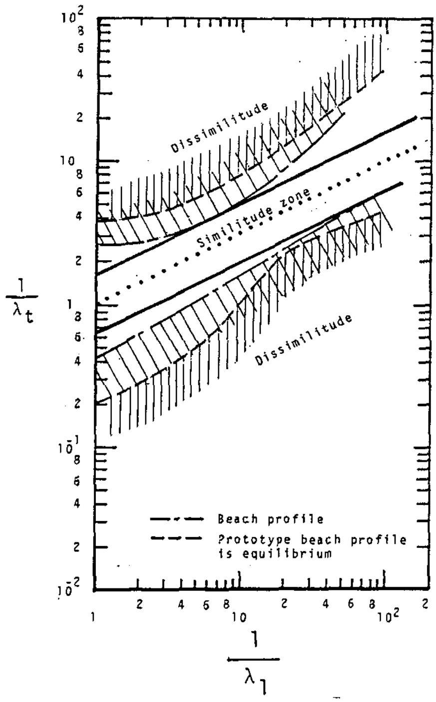


Fig. 4 Empirical time scale for modeling beach change.

5. APPLICABILITY OF THE TIME SCALE RELATIONSHIP PROPOSED

We set the experimental scale and grain-size scale of model for prototype by Eq. (1), and determine the wave duration operation time between the two by Eq. (3) which indicates the average tendency of similitude zone in Fig. (4). In case of this experimental manner, the relationship of the wave duration time in the model, $(t)_m$, and prototype, $(t)_p$, is given by

$$(t)_m=\frac{(t)_p}{\sqrt{n}} \qquad (4)$$

and the relationship of wave periods is

$$(T)_m=\frac{(T)_p}{\sqrt{n}} \qquad (5)$$

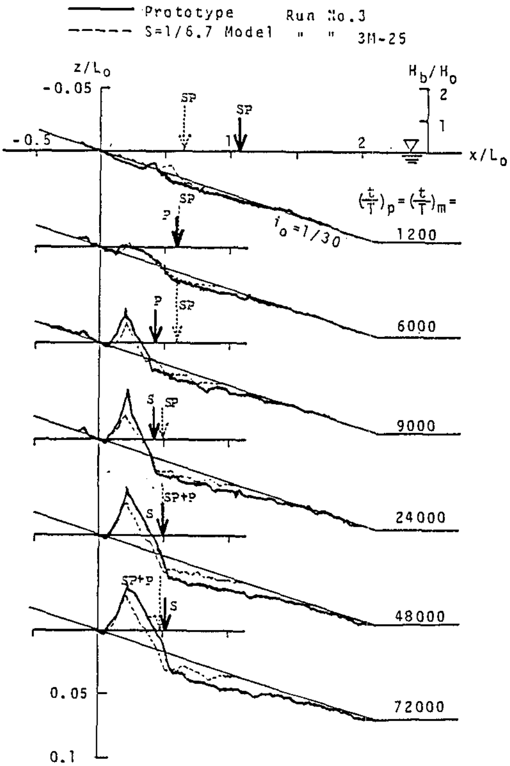


Fig. 5 Similarity of model and prototype beach changes reproduced by applying the time scale relationship.

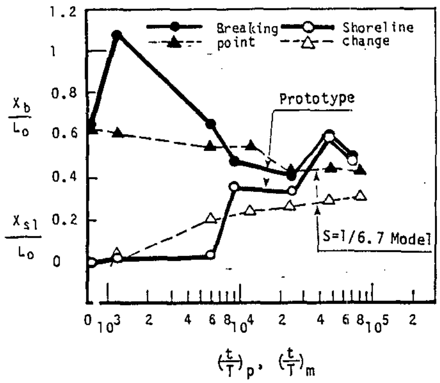


Fig. 6 Time changes of shoreline change and relative breaking position in the similarity experiments.

From Eqs. (4) and (5), we obtain

$$\left(\frac{t}{T}\right)_m = \left(\frac{t}{T}\right)_p \tag{6}$$

An example of the comparison between the beach change reproduced by model and prototype is shown in Fig. 5. At the same dimensionless times by Eq. (6), this figure illustrates changes of the beach profile and the breaking wave properties from an initial even slope to an equilibrium. Fig. 6 also shows in detail the reproduction of the shoreline change, X_{sl}/L_0, and the movement of breaking point, X_b/L_0 in dimensionless form. The discrepancy between the two shown in the figure is less than the range of experimental error. From Figs. 5 and 6, it is recognized that the time scale relationship of Eq. (2) is available for the laboratory model for reproducing the transitional beach change in prototype.

6. TIME SCALE BY SEMI-THEORETICAL METHOD

We can introduce the time scale in beach change processes using the three relationships for two-dimensional beach change; 1) the continuity equation of beach change

$$\frac{\partial z}{\partial t} + \frac{1}{1-\Lambda}\frac{\partial q_x}{\partial x} = 0 \tag{7}$$

where q_x is the sand transport rate in the offshore direction, t the time, Λ the bottom sediment porosity, and z and x the vertical and horizontal coordinates, respectively; 2) the sand transport rate; and 3) the the scale-model relationship of equilibrium beach profile expressed by Eq. (1).

From Eq. (7), the time scale ratio in beach change between model and prototype is expressed by

$$\frac{t_m}{t_p} = \frac{\Lambda_m}{\Lambda_p}\,\frac{(q_x)_p}{(q_x)_m}\,\frac{z_m}{z_p}\,\frac{x_m}{x_p} \tag{8}$$

Denoting the scale ratio of model parameter to prototype parameter by $\lambda_{parameter}$, the experimental conditions mentioned in 3. (1) ~ (3) can be presented as

$$\left.\begin{array}{lll}
\lambda_l = \lambda_x = \lambda_z = \lambda_{i_0} & & \text{(Geometrical similitude)} \\
\lambda_H = \lambda_{L_0} & \multirow{2}{*}{\Big\}} & \text{(Froude law)} \\
\lambda_T = \lambda_x^{1/2} & & \\
\lambda_s = 1 & & \left(\begin{array}{l}\text{Specific gravity of beach material}\\ \text{is assumed to be the same in}\\ \text{model and prototype}\end{array}\right) \\
\lambda_\nu = \lambda_\rho = 1 & & \left(\begin{array}{l}\text{Properties of fluid are the same}\\ \text{in model and prototype}\end{array}\right)
\end{array}\right\} \tag{9}$$

$$\left.\begin{aligned}\lambda_\Lambda &= 1\\ \lambda_g &= 1\end{aligned}\right\}$$

Rearranging Eq. (8) with Eq. (9), the following relationship is yielded

$$\lambda_t = \lambda_{q_x}^{-1}\ \lambda_l^2 \tag{10}$$

For introducing the time scale using Eq. (10), the relationship which specifies exactly the on-offshore sand transport rate in beach change is necessary. Such a formulation of sand transport has not been obtained completely, but empirical relationships of sand transport in a bed load were proposed by Madsen and Grant (1976) and Tsuchiya et al. (1984) from experimental results. These relationships are only applicable in the offshore zone, not surf zone. Tsuchiya et al. proposed the relationship of Kalinske- Brown type

$$\frac{q_x}{U^* d} = K\ (\tau^* - \tau^*_c)^2 \tag{11}$$

where

$$\tau^* = \frac{U^{*2}}{sgd} \qquad \text{(Shields parameter)} \tag{12. a}$$

$$\tau^*_c = \frac{U^{*2}_c}{sgd} \sim 0.05 \qquad \text{(Threshold Shields number)} \tag{12. b}$$

$$K = 1.7\left(1 + \frac{300d}{2a_w}\right) \qquad \text{for } \frac{2a_w}{d} > 100 \tag{12. c}$$

where U^* is the shear velocity at the bed, U^*_c the critical shear velocity, a_w the orbital radius of water particle, and K the constant depending on the ratio of orbital diameter to grain size, $2a_w/d$. The shear velocity at the bed is expressed by

$$U^* = \sqrt{\frac{f_w}{2}}\ Ub_{max} \tag{13}$$

where f_w is the wave friction factor, Ub_{max} the one half of the maximum horizontal velocity of water particle in the vicinity of the bed $\{\pi H/(2T\sinh 2\pi h/L)\}$. By Swart (1976), the friction factor, f_w, is expressed by

$$\left.\begin{aligned} f_w &= 0.0025\exp\{5.21\left(\frac{a_w}{k_s}\right)\} && \text{for } \frac{a_w}{k_s} > 1.57\\ f_w &= 0.3 && \text{for } \frac{a_w}{k_s} < 1.57\end{aligned}\right\} \tag{14}$$

where k_s is the roughness length of the bed which is nearly equal to the sand size. Consequently, the sand transport rate in a bed load by

Eq. (11) becomes

$$q_x = K U^{*5} s^{-2} g^{2} d^{-1} - 0.08 K U^{*3} s^{-1} g^{-1} + 0.04^{2} K U^{*} d \tag{15}$$

Substituting Eq. (13) into Eq. (15) and taking the scale ratio of the sand transport rate, we obtain

$$\left.\begin{aligned}
\lambda_{qx} &= \lambda_K \ \lambda_{f_w}^{5/2} \ \lambda_d^{-1} \ \lambda_H^{5} \ \lambda_T^{-5} \ \lambda_{\sinh\frac{2\pi h}{L}} \\
\lambda_{qx} &= \lambda_K \ \lambda_{f_w}^{3/2} \ \lambda_H^{3} \ \lambda_T^{-3} \ \lambda_{\sinh\frac{2\pi h}{L}}^{-3} \\
\lambda_{qx} &= \lambda_K \ \lambda_{f_w}^{1/2} \ \lambda_d \ \lambda_H \ \lambda_T^{-1} \ \lambda_{\sinh\frac{2\pi h}{L}}^{-1}
\end{aligned}\right\} \tag{16}$$

Furthermore, combining Eqs. (10) and (16), and arranging those by Eq. (9), the time scale relationship can be given as

$$\left.\begin{aligned}
\lambda_t &= \lambda_K^{-1} \ \lambda_{f_w}^{-5/2} \ \lambda_d \ \lambda_l^{-1/2} \\
\lambda_t &= \lambda_K^{-1} \ \lambda_{f_w}^{-3/2} \ \lambda_l^{1/2} \\
\lambda_t &= \lambda_K^{-1} \ \lambda_{f_w}^{-1/2} \ \lambda_d^{-1} \ \lambda_l^{3/2}
\end{aligned}\right\} \tag{17}$$

It is clear from Eq. (17) that the time scale is subjected to the scale effect of $2a_w/d$ (λ_K), the scale effect of wave friction factor, λ_{f_w}, and the grain size scale, λ_d. Using the sand transport rate by Eq. (11), the time scale may be given by the condition which is satisfied simultaneously the three equations in Eq. (17).

On the other hand, Madsen and Grant proposed the dimensionless bedload sand transport rate in the half wave cycle, q_x/wd, using the Shields number, $\tau_{om}/\rho sgd$, as

$$\frac{q_x}{wd} = 12.5\left(\frac{\tau_{om}}{\rho sgd}\right)^3 \tag{18}$$

where w is the sediment fall velocity, ρ the fluid density, s the specific gravity of the sediment in water, and τ_{om} the maximum bottom shear stress by

$$\tau_{om} = \frac{1}{2} f_w \, \rho \, Ub_{max}^{2} \tag{19}$$

Combining Eqs. (13) and (19), the scale ratio becomes as

$$\lambda_{qx} = \lambda_{f_w}^{3} \ \lambda_w \ \lambda_d^{-2} \ \lambda_s^{-3} \ \lambda_H^{6} \ \lambda_T^{-6} \ \lambda_{\sinh\frac{2\pi h}{L}}^{-6} \ \lambda_g^{-3} \tag{20}$$

The similarity relationship of the sediment fall velocity , as the authors (1984) considered already, may generally be expressed

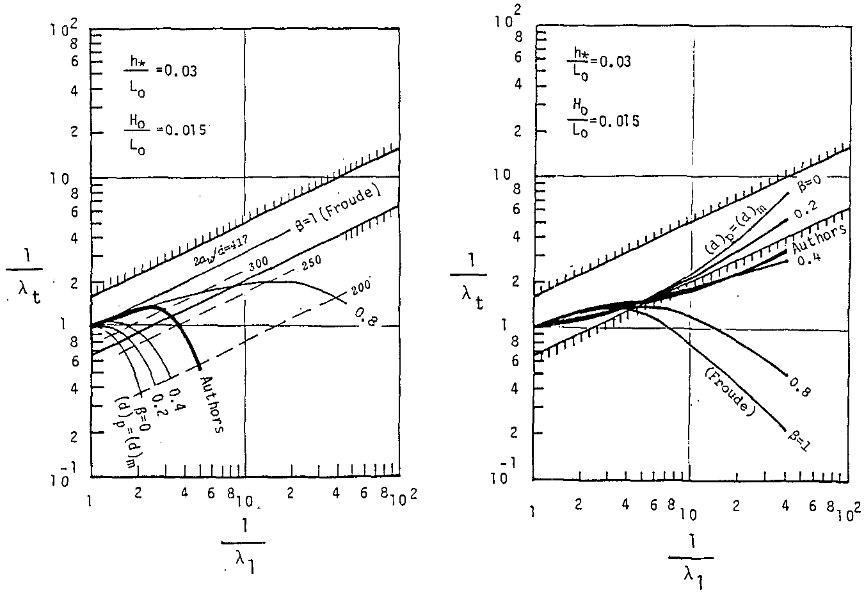
$$\lambda_w = \lambda_d^{\,n_w} \tag{21}$$

where n_w=2, 1, 2/3, and 1/2 for Stokes', Allen's, Kàrmàn's, and Newton's formulas of sediment fall velocity, respectively. Combining Eqs. (10), (20) and (21), and arranging it with Eq. (9), the time scale can be reduce to

$$\lambda_t = \lambda_{f_w}^{-3}\ \lambda_d^{\,2-n_w}\ \lambda_l^{-1} \tag{22}$$

It is recognized from Eq. (22) that the time scale relationship is influenced by the scale effects of the wave friction factor and grain size.

Now consider how the grain-size scale and the ratio of the orbital diameter of water particles to the grain size influence on the time scale. Eq. (1) may be expressed in a generalized form as



(a) Time scale calculated by using the sand transport rate by Tsuchiya et al.

(b) Time scale calculated by using the sand transport rate by Madsen and Grant.

Fig. 7 Comparison of the time scale obtained experimentally with the semi-theoretical time scale.

$$\lambda_d = \alpha\, \lambda_l^{\beta} \qquad (23)$$

where

$$\left.\begin{array}{l} \alpha = \left(\frac{1}{1.7}\right)^a \\ \beta = b \end{array}\right\} \qquad (24)$$

Fig. 7 (a) shows the relationship between the time scale and the experimental scale which was calculated numerically using Eqs. (10), (11) and (23) in the experimental conditions of Run No. 3 in Table 1 and of the beach change at the dimensionless position of $h_*/L_0=0.03$. To understand the influence of grain-size scale on the time scale, the coefficient and exponent of Eq. (23) are changed in steps as $\beta=0$ $\{(d)_m=(d)_p\}$, 0. 2, 0. 4, 0. 8, and 1. 0 when $\alpha=1$; and $\beta=b$ when $\alpha=(1/1.7)^a$ {same to Eq. (1)} From Fig. 7 (a) the influence of grain-size scale for on the time scale can therefore be understood easily. Additionally, the effect of $2a_w/d$ is shown by the broken lines in the figure. The result calculated numerically using the sand transport rate by Madsen and Grant is shown in Fig. 7 (b) taking β as parameter. It is indicated from Fig. 7 that the grain-size scale influences considerably on the time scale.

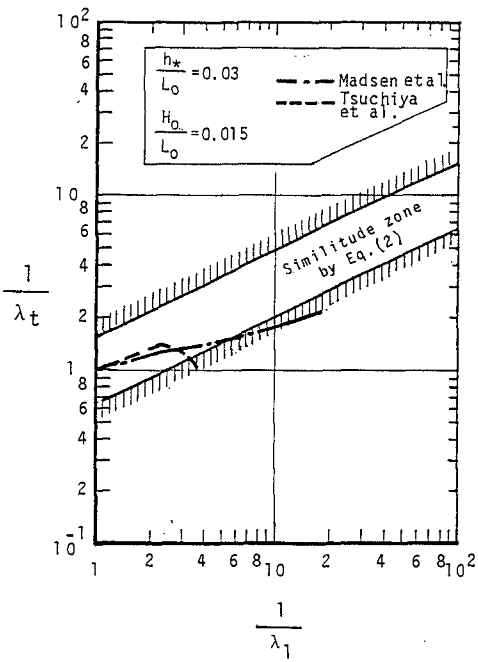


Fig. 8 Comparison of empirical time scale with semi-theoretical time scales.

Comparisons of the time scale relationship by Eq. (2) with the two semi-theoretical time scales mentioned in 6 are shown in Figs. 7 (a) and 7 (b). As is seen in these figures, there exists a quite difference between the two by the relationship of sand transport rate used in calculation. The difference in the time scale relationships is subjected to change in the exponent β in Eq. (23). The comparison of the time scale relationship by Eq. (2) with the semi-theoretical time scales which were obtained using the scale-model relationship of Eq. (1) is shown in Fig. 8. In the experimental scale range of $1 \geq \lambda_l > 1/4$, the semi-theoretical time scales agree well with the empirical time scale relationship proposed.

7. CONCLUSIONS

The main conclusions in this paper are summarized as;

(1) Based on the results of experiments which were conducted by applying the authors' scale-model relationship of Eq. (1), the empirical time scale relationship, which is held the similarity between the model and prototype in transitional beach change processes from an initial even slope to an equilibrium, was proposed by Eq. (2). The average tendency in the similitude zone for the time scale coincides nearly with the Froude time scale.

(2) The model beach processes which was set up using Eqs. (1) and (3) can well reproduce beach changes and breaking wave properties in prototype.

(3) The semi-theoretical time scale introduced by the continuity equation of beach change, the sand transport rate and the generalized scale-model relationship is not only subjected the grain size scale but the effect due to the wave friction factor and the ratio of orbital diameter of water particle to grain size of the bed material.

(4) Two semi-theoretical scales which were obtained by using the authors' scale-model relationship are a little different from those by formulas of sand transport rates, but agree well with the empirical time scale within the range of experimental scale, $1 \geq \lambda_l > 1/4$.

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