TWO-DIMENSIONAL EMPIRICAL EIGENFUNCTION MODEL FOR THE ANALYSIS AND PREDICTION OF BEACH PROFILE CHANGES

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ABSTRACT

A two-dimensional empirical eigenfunction model is pro-
posed for the analysis and the prediction of beach profile
change due to longshore and cross-shore sediment transports.
Beach profile data from Redhill coast, Taiwan, measured
every two months at 150 meters interval along the detached
breakwaters are analyzed and the relative importance from
two directions is investigated. Furthermore, by employing
the method of Markov process and linear regression, a pre-
diction model is formulated which takes into account the
effect of breaking waves, bottom sediment and radiation
stress of waves. This 2-D model is shown to be effective in
the analysis and the prediction of beach changes near the
coastal structures.

INTRODUCTION

The movement of coastal sediment can be decomposed into
longshore component and cross-shore component. Longshore
transport is defined as sediment transport parallel to the
beach, while cross-shore transport is defined as transport
perpendicular to the beach. These components are both sig-
nificant in response to seasonal variations in the forcing
parameters such as waves, tides, winds and currents. There-
fore, beach profile changes due to longshore and cross-
shore transport should be separated and represented as a
time series in order to obtain a detailed understanding.

Winant et al.(1975) proposed a method to describe beach
changes in terms of empirical eigenfunctions. The useful-
ness of the eigenfunctional representation was confirmed as
a concise method of representing beach profile changes

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(Aubrey, 1978; Aranuvachapun and Johnson, 1979). Uda and Hashimoto (1982) proposed a new model to analyze the beach changes due to longshore and cross-shore sediment trans- ports. They used longshore eigenfunctions and cross-shore eigenfunctions to describe temporal variations of beach profile changes. However, the cross-shore eigenfunctions were taken as a time average in their analysis and only the longshore eigenfunctions changing with time. This implies that the beach profile change is independent of cross-shore sediment transport.

The aim of this paper is to extend Uda and Hashimoto's concept and proposes a 2-D empirical eigenfunction model for the analysis and the prediction of beach profile changes for temporal and spatial variation where both longshore and cross-shore sediment transport are significant. Field ob- servation of beach profiles obtained from Redhill, Taiwan, are examined and the relative importance of beach changes from two orthogonal modes is also given. In addition, a prediction model is formulated by employing the method of Markov process and linear regression. The results of pre- diction model are compared with the existing models and measured data.

THEORETICAL FORMULATION

One-Dimensional Empirical Eigenfunction Method

The method has been described by Winant et al. (1975). In brief, an eigenfunction is expressed in the form of

$$h_{xt} = \sum_{n} e_{nx} C^{*}_{nt} \tag{1}$$

where the h_{xt} are the beach profile data, e_{nx} represent the spatial eigenfunctions, C^{*}_{nt} represent temporal eigenfunc- tions, and n represent the variation modes.

The spatial eigenfunctions form an orthogonal set as

$$\sum_{n} e_{nx} e_{mx} = \delta_{nm} \tag{2}$$

where δ_{nm} is Kronecker delta.
The spatial correlation matrix A is expressed as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & a_{ij} & & & . \\ . & . & . & & & . \\ a_{n1} & a_{n2} & a_{nj} & . & . & a_{nn} \end{bmatrix} \tag{3}$$

In which, the elements of matrix A are defined by

$$a_{ij} = \frac{1}{N_x N_t} \sum_{t} h_{it} h_{jt} \tag{4}$$

where N_x is the number of data points per profile, N_t is the

number of measured times. The matrix A possesses a set of
eigenvalues λ_n and a set of corresponding eigenfunctions e_{nx}
which are defined by matrix equation

$$A\ e_{nx} = \lambda_n\ e_{nx} \tag{5}$$

The temporal eigenfunctions are calculated with

$$\overset{*}{C_{nt}} = \sum_x h_{xt}\ e_{nx} \tag{6}$$

By defining $C_{nt} = \overset{*}{C_{nt}}/(\lambda_n\ N_x\ N_t)^{1/2}$, it can be shown
that C_{nt} form an orthogonal set, and eq.(1) is rewritten as

$$h_{xt} = \sum_n (\lambda_n\ N_x\ N_t)^{1/2}\ e_{nx}\ C_{nt} \tag{7}$$

Analysis of 2-D Empirical Eigenfunction Method

Following Uda and Hashimoto (1982), we extend the 1-D
empirical eigenfunction method to the 2-D bed elevation
$h(x,y,t)$, which is a function of offshore distance x, and
longshore distance y at a certain time t. The expression of
$h(x,y,t)$ is as follow

$$h(x,y,t) = \sum_k e_k(x,t)\ e_k(y,t) \tag{8}$$

In this expression, $e_k(x,t)$ are the cross-shore eigenfunc-
tions, $e_k(y,t)$ the longshore eigenfunctions, k the variation
modes. Eq.(8), obviously, represents the variation of the
beach profile configuration in terms of longshore and cross-
shore distances at the specific time t. In some cases, such
as the profile configuration around a coastal structure or a
rhythmic topography, it is desirable to use the data of all
profiles for the analysis of spatial characteristic beach
changes instead of the single beach profile.

The cross-shore eigenfunctions form an orthogonal set
as

$$\sum_x e_n(x,t)\ e_m(x,t) = \delta_{nm} \tag{9}$$

In order to generate the cross-shore eigenfunctions,
like the correlation matrix A, a symmetric matrix B is
formed with elements of

$$b_{ij} = \frac{1}{N_x\ N_y} \sum_y h(i,y,t)\ h(j,y,t) \tag{10}$$

where N_y denotes the total number of measured points along-
shore. The matrix B are real matrix with a set of eigen-
values λ_k and a set of corresponding eigenfunctions $e_k(x,t)$

$$B\ e_k(x,t) = \lambda_k\ e_k(x,t) \tag{11}$$

The longshore eigenfunctions $e_k(y,t)$ which define the
longshore changes of beach profile are evaluated as

$$e_k(y,t) = \sum_x h(x,y,t) \, e_k(x,t) \tag{12}$$

In the analysis of beach profile changes due to longshore and cross-shore sediment transports, Uda and Hashimoto (1982) took the cross-shore eigenfunctions $e_k(x,t)$ to be independent of time and only let the longshore eigenfunctions $e_k(y,t)$ changing with time. This means that the nature of variations doesn't depend on cross-shore sediment transport. This paper allows $e_k(x,t)$ and $e_k(y,t)$ to vary with distance and time respectively. The expressions of $e_k(x,t)$ and $e_k(y,t)$ are given as follows,

$$e_k(x,t) = \sum_n (\lambda_n \, N_x \, N_t)^{1/2} \, e_k^n(x) \, C_{kx}^n(t) \tag{13}$$

$$e_k(y,t) = \sum_m (\lambda_m \, N_x \, N_t)^{1/2} \, e_k^m(y) \, C_{ky}^m(t) \tag{14}$$

where $e_k^n(x)$: cross-shore spatial eigenfunctions
              $C_{kx}^n(t)$: cross-shore temporal eigenfunctions
              $e_k^m(y)$: longshore spatial eigenfunctions
              $C_{ky}^m(t)$: longshore temporal eigenfunctions

Formulation of the Prediction Model

In order to relieve forecast errors, the variation of beach profile subtracted the mean beach profile from the original data is used. By this method, the set $h(x,y,t)$ is expressed as

$$h(x,y,t) = \bar{h}(x,y) + \sum_k e_k'(x,t) \, e_k'(y,t) \tag{15}$$

where $\bar{h}(x,y)$ is the temporal mean of the beach profile. A prime on the eigenfunctions indicates that these eigenfunctions are similar in data sets where the mean profile has not been removed. According to eqs.(13) and (14), $e_k'(x,t)$ and $e_k'(y,t)$ can be expressed, respectively, in the form of

$$e_k'(x,t) = \sum_n (\lambda_{nx} \, N_x \, N_t)^{1/2} \, e_k^{n'}(x) \, C_{kx}^{n'}(t) \tag{16}$$

$$e_k'(y,t) = \sum_m (\lambda_{my} \, N_x \, N_t)^{1/2} \, e_k^{m'}(y) \, C_{ky}^{m'}(t) \tag{17}$$

The fluctuations of beach profile are calculated as

$$h'(x,y,t) = h(x,y,t) - \bar{h}(x,y) \tag{18}$$

$h'(x,y,t)$ was used to generate eqs.(16) and (17) by processing the procedure of the analysis of 2-D empirical eigenfunction method.

The predictands for prediction model always consist of a representation of the beach profile time series. Accordingly, it is necessary to forecast the next time step eigenfunctions $C_{n(i+1)}$ and let the spatial eigenfunctions $e_k(x)$ and $e_k(y)$ to be constants.

The Markov process based on Sonu (1973) and three forc-
ing parameters were used here to estimate the temporal ei-
genfunctions $C'_{n(i+1)}$ in the prediction model.   The simplest
linear equations can be described as

$$C'_{1(i+1)} = \overline{a_{11}}C'_{1i} + \overline{a_{12}}C'_{2i} + \ldots + \overline{a_{1n}}C'_{ni} + \overline{a_{1(n+1)}}F_{i+1}$$
$$C'_{2(i+1)} = \overline{a_{21}}C'_{1i} + \overline{a_{22}}C'_{2i} + \ldots + \overline{a_{2n}}C'_{ni} + \overline{a_{2(n+1)}}F_{i+1}$$
$$\vdots$$
$$C'_{n(i+1)} = \overline{a_{n1}}C'_{1i} + \overline{a_{n2}}C'_{2i} + \ldots + \overline{a_{nn}}C'_{ni} + \overline{a_{n(n+1)}}F_{i+1}$$

(19)

or

$$
\begin{bmatrix} C'_{1(i+1)} \\ C'_{2(i+1)} \\ \vdots \\ C'_{n(i+1)} \end{bmatrix}
=
\begin{bmatrix} \overline{a_{11}} & \overline{a_{12}} & \ldots & \overline{a_{1n}} & \overline{a_{1(n+1)}} \\ \overline{a_{21}} & \overline{a_{22}} & \ldots & \overline{a_{2n}} & \overline{a_{2(n+1)}} \\ \vdots & \vdots & & \vdots & \vdots \\ \overline{a_{n1}} & \overline{a_{n2}} & \ldots & \overline{a_{nn}} & \overline{a_{n(n+1)}} \end{bmatrix}
\begin{bmatrix} C'_{1i} \\ C'_{2i} \\ \vdots \\ F_{i+1} \end{bmatrix}
$$

(20)

where $\overline{a_{11}} \ldots \overline{a_{n(n+1)}}$ are constant coefficients, F_{i+1} are
the dimensionless forcing parameters for the next time step.

Three forcing parameters corresponding to breaking
waves,   features of bottom sediment transport and radiation
stresses of waves were taken into   account in the present
method.   The reason why these three parameters are chosen is
that they have been used in the past with degrees of success
to describe nearshore process.   Based on Battjes (1974), the
characteristics of breaking waves can be described by surf
similarity parameter ζ defined by

$$\zeta = \tan \beta / (H/L_0)^{1/2}$$

(21)

where $\tan\beta$ is the bottom slope, H wave height, L_0 wave
length in deep water.   The features of bottom sediment
transport can be classified by the parameter C proposed by
Gourlay (1968)

$$C = H_0 / wT$$

(22)

where w is the fall velocity of sediment determined by the
characteristics of sand and kinematic viscosity of the fluid
and T is the wave period.   The radiation stresses of in-
clined waves are written as (Bowen,1969)

$$
S =
\begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix}
=
\begin{bmatrix} S_{rr}\cos^2\alpha + S_{ss}\sin^2\alpha & (S_{rr}-S_{ss})\sin\alpha \cos\alpha \\ (S_{ss}-S_{rr})\sin\alpha \cos\alpha & S_{rr}\sin^2\alpha+S_{ss}\cos^2\alpha \end{bmatrix}
$$

(23)

with

$$S_{rr} = E(2n-1/2) \;,\; S_{ss} = E(n-1/2) \;,\; E = 1/8\; \rho gH^2$$

where r is the direction of wave propagation, s is the
direction normal to the wave propagation, x is cross-shore
coordinate,   y is longshore coordinate, α is the angle be-

tween wave crest and beach, n is the ratio of wave group
velocity to wave celerity, E is wave energy, ρ is density
and g is gravity acceleration.   A dimensionless wave energy
was introduced for the consistent dimension of forcing
parameters

$$E_0 = (1/8 \ \rho gH^2) \ / \ (1/8 \ \rho gH_0^2) \qquad (24)$$

It is convenient to write eq.(19) in general matrix
notation as

$$P = \overline{A} \cdot D \qquad (25)$$

where P is the (m·N) matrix of m quantities to be predicted,
D is the (n·N) matrix of n data parameters and $\overline{A}$ is an (m·n)
coefficient matrix, N is the total number of observations of
these quantities which are to be used in the prediction. The
optimal form of the coefficient matrix $\overline{A}$ was determined by
using the linear regression:

$$\overline{A} = C_{PD} \ C_{DD}^{-1} = (P \cdot D^T) \ (D \cdot D^T)^{-1} \qquad (26)$$

where C_{PD} denotes the covariance matrix between the predict-
and and data, C_{DD} is the auto-covariance matrix of the data,
and T is the transpose operator.

The estimator of matrix P is used for both hindcast and
forecast models.   A hindcast model is defined as the es-
timate made from the same data and the predictand which are
used to form the covariance matrix, whenas a forecast model
is defined as the estimate made from the covariances formed
from other data and predictand samples.   According to Davis
(1976),   the hindcast skill can be represented by a predict-
ability index S_H:

$$S_H = 1 - \frac{E(P-\hat{P}) \ (P-\hat{P})^T}{E(PP^T)} \qquad (27)$$

in which E represents the expected value operator, $\hat{P}$ denotes
the estimated value.   The hindcast skill increases with the
higher value of S_H.   On the other hand, the forecast ability
was described by the mean-square-forecast error S_F:

$$S_F = \frac{E[h(x,y,t) \ - \ \hat{h}(x,y,t)][h(x,y,t)-\hat{h}(x,y,t)]^T}{E[h(x,y,t)][h(x,y,t)]^T} \qquad (28)$$

where $\hat{h}(x,y,t)$ is the estimated bed elevation.   A model is
predictable with lower mean-square-forecast error.

FIELD DATA

Beach Profile Data

As shown in Fig.1, Redhill coast is located in the
southern part of Taiwan and consists of eroding sandy bluffs
ranging from 6 to 10 meters high.   Fig.2 shows the detached
breakwaters built on the shoreline to prevent waves from
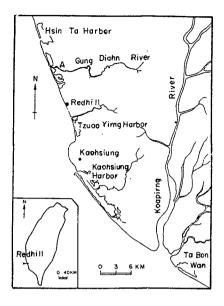reaching the eroding shore.
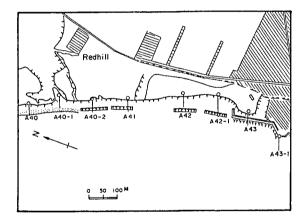
Fig. 1   Location of Redhill coast

Fig. 2   Alignment of detached breakwaters
          and observed positions

The beach sand has median grain size of 0.28mm and sub-
merged specific gravity of 1.6 Surveys of the beach profile
have been conducted at every two months from 1982 to 1984.
Each profile was measured at some intervals over a distance
about 600m offshore. A detailed measurement was done on the
foreshore part by using standard surveying method, while
depth measurements on the offshore part were taken by cable
lead.

Wave Climate of Redhill Coast

There are no wave data available at Redhill coast. The
wave data observed at Ta Bon Wan located about 48km south of
Redhill coast (Fig.1) have been used. The wave height at Ta
Bon Wan was measured at the depth of 12.5 meters using a
pressure wave gage. In winter, the prevailing direction of
incoming wave at Ta Bon Wan is NNE, and waves from various
directions were partly sheltered by land.   The cumulative
relative energy $P_E(\theta)$ is given by

$$P_E(\theta) = \frac{1}{m_0} \int_{-\pi/2}^{0} \int_{0}^{\infty} S(f,\theta) \, df \, d\theta \qquad (29)$$

where $S(f,\theta)$ is directional wave spectrum, f is angular fre-
quency, θ is the azimuth measured counterclockwise, and m_0
is the total wave energy expressed as

$$m_0 = \int_{-\pi/2}^{\pi/2} \int_{0}^{\infty} S(f,\theta) \, df \, d\theta \qquad (30)$$

For NNE directional waves, the azimuths θ_1 and θ_2 were taken
as $-45°$ and $31°$ for Redhill coast and for Ta Bon Wan, res-
pectively.   The wave height ratio k_d between Redhill coast
and Ta Bon Wan is determined by using SWOP directional
spectrum

$$k_d = \sqrt{P_E(\theta_1) / P_E(\theta_2)} = 3.15 \qquad (31)$$

In summer, since the SW directional incoming wave is almost
perpendicular to the coast, wave data from Ta Bon Wan can be
directly applied to Redhill coast without any modification.

Based on Sawaragi (1982), the response of the beach to
the various kinds of incident waves can be expressed in
terms of a representative wave calculated by the following
formulas:

representative wave period: $\quad T_s = \sum_i T_i \, p_i / P_T \qquad (32)$

representative wave height: $\quad H_s^2 C_{gs} P_T = \sum_i C_{gi} H_i^2 p_i \quad (33)$

representative wave direction:
$\quad H_s^2 C_{gs} \sin\alpha_s \cos\alpha_s P_T = \sum_i H_i^2 C_{gi} \sin\alpha_i \cos\alpha_i p_i \quad (34)$

where H_i: component wave height
      H_s: representative wave height
      T_i: component wave period

T_s: representative wave period
α_i: component wave direction
α_s: representative wave direction
C_{g_i}:   representative wave group velocity
p_i: component probability
$P_T = \sum_i p_i$ = total probability
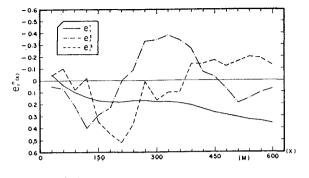
RESULTS AND DISCUSSION

Results of Beach Profile Analysis

The 2-D empirical eigenfunction analysis was conducted by using the beach profile data collected at Redhill coast from December, 1982 to December, 1984. The total number of recording times is $N_t=13$ and that of calculating points $N_x=20$, $N_y=8$.
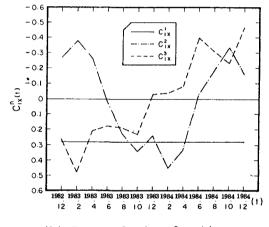
Fig.3(a) shows the cross-shore profiles of section A40-2 of the first three spatial eigenfunctions e_1^1(solid line), e_1^2 (dash-dot line) and e_1^3 (dash line). The temporal eigenfunctions are given by C_{1x}^1(solid line), C_{1x}^2(dash-dot line) and C_{1x}^3(dash line) as shown in Fig.3(b). The first spatial eigenfunction is interpreted as the mean beach function according to Winant et al.(1975). The time dependence of the mean beach function is almost constant, indicating a stable beach for Redhill coast. The second spatial eigenfunction has a minimum at the location of summer berm and a maximum in the area of winter bar. The negative value of the second temporal eigenfunction accounts for the sand migration up to the level of the summer berm, while the positive value indicates the movement of sand down beach to the winter bar and hence represents erosion on the beach. The third spatial eigenfunction has positive value in the broad region from shoreline to about 400m and negative value in the offshore region. The corresponding temporal eigenfunction is found to have positive values before December, 1983 and negative values after December, 1983.

The 2-D empirical eigenfunction method is also employed for the analysis of shoreline changes due to the construction of detached breakwaters. Based on Uda and Hashimoto (1982), the shoreline changes can be described in terms of the second longshore eigenfunctions $e_2(y,t)$. From Fig.4, we note that the second longshore eigenfunction changes with seaward advancement at the central side on which the detached breakwaters were constructed. The corresponding cross-shore eigenfunction $e_2(x,t)$ is shown in Fig.5, in which the cross-section of a detached breakwater is located about 30m in the x direction. Depositions of sand behind the detached breakwater is also found. Results of analysis reveal that the design and the construction of detached breakwaters are successful for beach preservation measures.

Fig.6(a) presents the first three longshore spatial eigenfunctions e_1^1(solid line), e_1^2(dash-dot line) and e_1^3 (dash line). The time variations of these functions as shown in Fig.6(b) are C_{1y}^1(solid line), C_{1y}^2(dash-dot line)

(a) Spatial eigenfunction

(b) Temporal eigenfunction

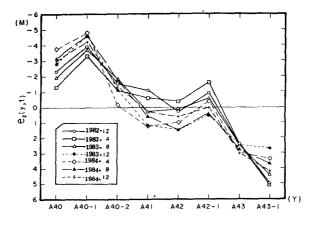Fig. 3 The first three crossshore eigenfunctions (section A40-2)

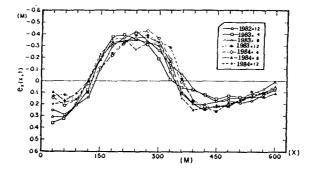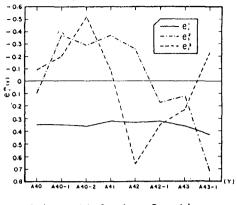Fig. 4   The second longshore eigenfunctions $e_2(y,t)$

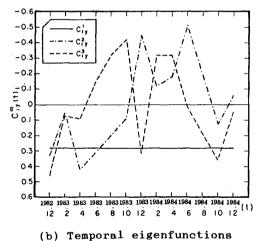Fig. 5   The second cross-shore eigenfunctions $e_2(x,t)$

(a) Spatial eigenfunctions

(b) Temporal eigenfunctions

Fig. 6 The first three longshore eigenfunctions

and C_{1y}^3 (dash line). The temporal function C_{1y}^1 shows almost no chang over the recording time. The other two eigenfunctions show no clear tendency. Perhaps more data are required for the analysis.

On the other hand, Winant et al. (1975) pointed out that each eigenvalue is representative of certain percentage of the mean square value of the data. Table 1 shows that the eigenvalues of longshore eigenfunctions are larger that those of cross-shore eigenfunctions, indicating that the longshore sediment transport has more effect on beach profile change than the cross-shore sediment transport does.

Prediction of Beach Profile Changes

It has been shown that the stability is an important consideration in the prediction of empirical eigenfunction method (Aubrey,1978). In this paper, two data sets of different sample lengths (N_t=10 and 12) were examined. The results of analysis show that the eigenfunctions are stable with respect to the length of the data (Wang, 1985).

Uda and Hashimoto (1982) calculated the shoreline change by one-line theory, and then the longshore eigenfunction $e_2(y,t)$ was calculated from shoreline changes. Since the nearshore zone is characterized by complex flow fields and fluid-sediment interactions, it is difficult to estimate quantitative shoreline changes by one-line simulation. The correlation between $e_2(y,t)$ and shoreline changes is not significant in present analysis as shown in Fig.7. In this paper, three parameters and computed beach configuration at each time step are used to predict temporal eigenfunctions, because they are easily to obtain from available wave data and beach profile data without any mathematical simulation.

Table 2 shows the mean square value of forecast error S_F for different models. The forecast skill of present analysis is better than 1-D model and Uda and Hashimoto's model, although its predictability S_H was lower as listed in Table 3.

Fig.8(a) and (b) depict two profiles of A40-2 and A41 and their estimates from three different models. It is seen that the prediction of 2-D model yields a better result than 1-D model and Uda and Hashimoto's model as compare to the measured profiles.

Table 1   A comparison of eigenvalues between longshore
and cross-shore eigenfunctions

component\nmode	longshore eigenfunctions			cross-shore eigenfunctions		
	一	二	三	一	二	三
1	0.124871	0.093681	0.092341	0.049939	0.038738	0.038335
2	0.000050	0.029298	0.029826	0.000029	0.009608	0.009192
3	0.000034	0.000861	0.001430	0.000022	0.001074	0.001438
4	0.000024	0.000702	0.000705	0.000005	0.000191	0.000453
5	0.000011	0.000277	0.000510	0.000002	0.000138	0.000168

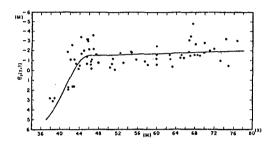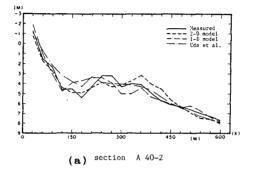

Fig. 7   A relationship between longshore eigenfunction
$e_2(y,t)$ and shoreline changes

Table 2   Mean square error S_F for three different models

Sec. No.	I − D	2 − D	Hashimoto and Uda
A 40	0.0193	0.0083	0.0215
A 40 − 1	0.0194	0.0200	0.0294
A 40 − 2	0.0151	0.0137	0.0147
A 41	0.0107	0.0071	0.0137
A 42	0.0224	0.0153	0.0233
A 42 − 1	0.0262	0.0093	0.1122
A 43	0.0107	0.0105	0.0215
A 43 − 1	0.0068	0.0142	0.0291

Table 3   Predictability index S_H for 1-D and 2-D model

	Sec. No.	S_H	Sec. No.	S_H
1-D	A 40	0.8282	A 42	0.8545
	A 40-1	0.8409	A 42-1	0.8550
	A 40-2	0.8669	A 43	0.8130
	A 41	0.9021	A 43-1	0.8005
2-D	cross-shore	0.7857	longshore	0.6089

(a) section A 40-2
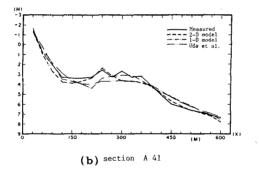

(b) section A 41

Fig. 8   Comparison of predicted results

CONCLUSION

Field data of beach profiles taken over two years at Redhill coast were analyzed by employing the 2-D empirical eigenfunction method.   The results indicate that beach profile changes due to longshore and cross-shore sediment transports are separable.   A successful detached breakwater as beach erosion countermeasure is also verified.   The prediction of 2-D model shows that the beach profile changes are predictable. It is also concluded that this 2-D model is shown to be effective in the analysis and the prediction of beach changes near coastal structures.

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