

CHAPTER 67

MAGNITUDE OF THE β -FACTOR UNDER WAVE ACTION

by

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Abstract

The sediment transport due to waves and currents depends on the distribution of sediment concentration and on the distribution of the velocity over the water depth. Our knowledge of both phenomena for practical applications is still rather poor. Some results of wave flume tests concerning the distribution of sediment concentrations due to wave action will be discussed. It turns out that the sediment size of the bottom material has a rather unexpected effect hereupon. With respect to the velocity distribution only some qualitative remarks can be made at the moment.

1. Introduction

The sediment transport (per m) due to waves and currents parallel to the coast can be described in principle with:

$$S = \int_0^h \overline{v(z)} \overline{c(z)} dz \quad (1)$$

where: S : sediment transport rate
 $\overline{v(z)}$: time averaged current velocity (function of z)
 z : height above the bed ($z = 0$ being the bottom)
 $\overline{c(z)}$: time averaged sediment concentration (function of z)
 h : water depth

[For simplicity reasons the over-bars in $\overline{v(z)}$ and $\overline{c(z)}$ have been omitted further.]

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Eq.(1) is a simplification of the general description according to:

$$S = \frac{1}{t_0} \int_0^{t_0} \int_0^h v(z,t) c(z,t) dz dt \quad (2)$$

where: t_0 : integration time (long with respect to the wave period T)
 $v(z,t)$: current velocity (function of z and t)
 $c(z,t)$: sediment concentration (function of z and t)
 t : time.

For the cross shore sediment transport description Eq.(2) should be applied in principle. The orbital motion affects the parameter $v(z,t)$ continuously and thus in fact the simplifications according to Eq.(1) cannot be made. However, Stive and Battjes (1984) do apply an equation like Eq.(1) also in cross shore transport cases with some success. [The use of Eq.(1) instead of Eq.(2) has great advantages since our knowledge of the parameter $c(z,t)$ on the wave period scale is nearly completely insufficient; see Bosman (1986).]

In longshore sediment transport cases the orbital motion hardly affects (due to refraction) the parameter $v(z,t)$ in Eq.(2), thus the simplification of Eq.(1) seems allowed. Apart from possible applications in cross shore transport cases, we restrict ourselves in the present paper to Eq.(1) (the longshore transport application).

According to Eq.(1) two topics affect the actual rate of sediment transport, viz.:

- $v(z)$ - distribution over the water depth
 - $c(z)$ - distribution over the water depth
- (see Fig.1).

In arbitrary coastal engineering applications $v(z)$ as well as $c(z)$ are highly dependent on the boundary conditions like wave height H, wave period T, water depth h, bottom particle size D_{50} and average current velocity v . Although some proposals for possible relationships can be found in literature, general accepted (and soundly proved) formulations are not yet available. That is, in fact, not surprising since many rather simple basic questions cannot be answered up till now. Some examples of these questions are:

- what is the effect of the grading of the bottom material on the $c(z)$ distribution?
- do we understand the behaviour of different particle sizes in a same wave and current generated 'turbulence field'?

Nielsen (1979) already mentioned the effect of the rate of grading of the bottom material on the apparent diffusion coefficient distribution over the water depth [$\varepsilon_s(z)$ - distribution]. Van de Graaff and Roelvink (1984) showed, with the help of some practical examples, that even in quite normal graded cases ($D_{90} / D_{10} \approx 2$), completely different

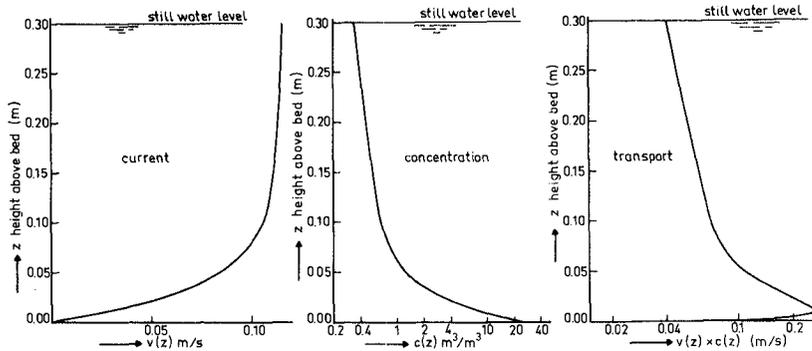


Fig.1 Velocity \times concentration = transport

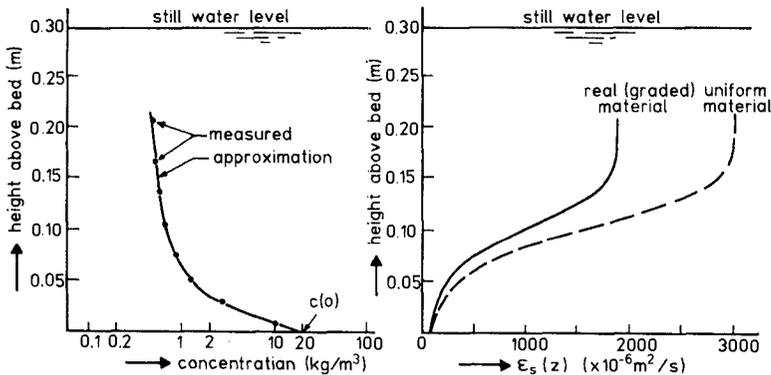


Fig.2 Effect of grading on $\epsilon_s(z)$ distribution

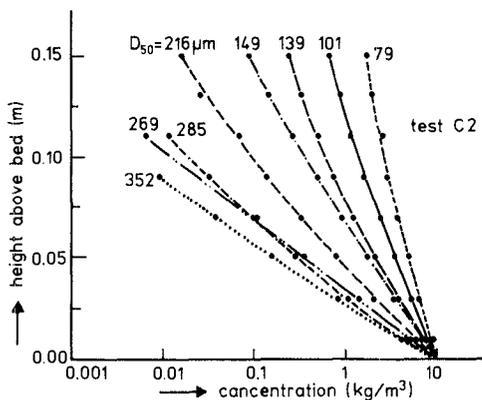


Fig.3 Concentration as function of diameter

$\epsilon_s(z)$ distributions are found when the actual grading is taken into account in comparison with the $\epsilon_s(z)$ distribution as calculated under the assumption of uniform bottom material.

Van de Graaff and Roelvink also mentioned a single test result from which it became clear that different particle sizes react quite unexpectedly on the same turbulence field (cf. second question). Since the answer on the second question affects the results arising from the first question, one should know the answer on the second question first.

In the present paper some results, of mainly experimental research in laboratory wave flumes, will be discussed. The tests are aimed to disclose the relationship between the diffusion coefficients holding for the fluid and for the sediment:

$$\epsilon_s = \beta \epsilon_f \quad (3)$$

where: ϵ_s : diffusion coefficient for sediment
 ϵ_f : diffusion coefficient for fluid
 β : factor.

It will turn out (see Section 2) that β depends (among others) on the sediment size.

Knowledge of the β -factor is important if one likes to know the $c(z)$ distribution as mentioned earlier in the present Section.

In Section 3 some remarks will be made on the possible $v(z)$ distribution for the combination of waves and currents.

2. The β -factor

A uniform flow, a pure horizontal oscillatory flow (e.g. in a wave tunnel), regular waves, irregular waves and a combination of waves and currents, in all cases, together with a sandy bottom, sediment suspensions will be formed in the water column.

When a steady state is considered, convective processes as well as diffusion effects will maintain the concentration distribution. For the time being the concentration distribution, even under wave conditions, will be described by the well-known diffusion equation:

$$w c(z) + \epsilon_s(z) \frac{\partial c(z)}{\partial z} = 0 \quad (4)$$

where: w : fall velocity
 $c(z)$: time averaged concentration at level z above the bed
 $\epsilon_s(z)$: diffusion coefficient for the sediment (function of z)
 z : vertical upward directed ordinate; the bottom being $z = 0$.

It should be stressed that the parameter $\epsilon_s(z)$ in Eq.(4) is, in fact, an auxiliary parameter in the description of the sediment concentration distribution. It is nearly for sure that not only diffusion processes maintain the concentration distribution.

When one 'knows' the $\epsilon_s(z)$ distribution and a concentration at a certain level (e.g. the bottom concentration), the entire $c(z)$ distribution can be calculated. However, it is impossible at the moment to predict the $\epsilon_s(z)$ distribution as a function of the boundary conditions like H , T , h and v . Our knowledge is still completely insufficient to do that.

All over the world a mainly experimental approach is followed to acquire more insight in the concentration distribution. The next sequence is followed in that way:

- Measuring of $c(z)$ distributions
- Find the characteristics of the underlying $\epsilon_s(z)$ distribution
- Try to relate these characteristics to the boundary conditions
- Try to 'understand' the relationships as have been found.

In Fig.2b the resulting $\epsilon_s(z)$ distributions are shown belonging to the real measured $c(z)$ distribution as given in Fig.2a. From this example it becomes clear that it is quite important to take the real grading ($D_{90} / D_{10} = 2$ in this example) into account. Different $\epsilon_s(z)$ distributions appear whether real graded or uniform material is taken into account. Many problems will undoubtedly arise in relating the boundary conditions with the $\epsilon_s(z)$ characteristics as found when the real grading is not taken properly into account. [The present example holds for an irregular (non-breaking) wave case in a laboratory wave flume. Notice that even high in the water column apparently rather high diffusion coefficients are present. Straight forward 'theories' predict diffusion coefficients often in a restricted zone close to the bed only.] The diffusion coefficient distributions as found in Fig.2b hold strictly for the bottom sediment as applied. However, the 'fluid' may contain different diffusion coefficients [see Eq.(3)]:

$$\epsilon_s(z) = \beta \epsilon_f(z) \quad (3a)$$

where: β : factor.

Depending on the value (or better: the behaviour) of β , one is able to derive the $\epsilon_f(z)$ distribution from measured concentration distributions. In fact that $\epsilon_f(z)$ distribution should be related to the boundary conditions. That is more realistic than such a relationship with $\epsilon_s(z)$, since, depending on the behaviour of β , $\epsilon_s(z)$ might be dependent on the particle size.

What about β ? If $\beta = 1$ (as sometimes is assumed, however, without sound evidence) things become rather simple. The Delft University of Technology has carried out many tests in wave flumes to disclose the β behaviour. In these tests the same mixing activity should be generated by the waves holding for different particle sizes. The dimensions of the bottom ripples (to be formed) will most likely affect the mixing activity. Since these dimensions depend on the particle size, tests with 'natural' sandy bottoms will not fulfill the requirements. In the tests, therefore, artificial roughness elements have been mounted on the flume bottom. During the tests a small amount of sand was maintained on the bottom. The amount was that small that no 'own'

ripples could be formed, but large enough to shape a measurable sediment concentration distribution over the water column. The concentrations have been derived from 'time- and bed averaged' suction samples [see Bosman (1982)]. The suction direction was perpendicular to the orbital plane. In that case the actual concentration differs from the measured one. However, if the intake velocity of the water-sand mixture is large enough compared with the orbital motion, a constant factor is found between measured and actual concentrations [Bosman et al. (1987)]. In the present paper concentration *distribution* aspects are considered rather than *absolute values*, so the application of correction factors is not necessary.

Up till now 3 series of tests have been carried out in several wave flumes (Delft Hydraulics and Delft University of Technology). In Table I the most important parameters of the tests have been summarized.

Test	Wave characteristics			Ripple characteristics			Number of bed materials
	h (m)	H (m)	T (s)	λ (m)	η (m)		
A1 DH	0.30	0.080	2.0	0.07	0.017	*	8
B1 DUT	0.30	0.081	1.5	0.08	0.020	+	5
B2	0.30	0.128	1.5	0.08	0.020	+	5
B3	0.30	0.092	2.5	0.08	0.020	+	5
B4	0.30	0.136	2.5	0.08	0.020	+	5
C1 DUT	0.30	0.064	1.7	0.08	0.020	*	8
C2	0.30	0.125	1.7	0.08	0.020	*	8
C3	0.30	0.044	2.3	0.08	0.020	*	7
C4	0.30	0.072	2.3	0.08	0.020	*	8
C5	0.30	0.142	2.3	0.08	0.020	*	7

Table I Characteristics of tests

Remarks : DH Delft Hydraulics
 DUT Delft University of Technology
 * triangular shape
 + sinusoidal shape plus steel angle sections
 λ ripple length
 η ripple height.

Fig.3 shows an example of a set of measured concentration distributions (test C2 of Table I). All test results have been normalized, yielding an (arbitrary) bottom concentration of 10 kg/m^3 . The measuring points

have been approximated by a mathematical function according to the method as has been described by Van de Graaff and Roelvink (1984).

Fig.3 shows, as could be expected, a fan-shaped set of concentration distribution lines. The finer material goes 'easier' into suspension than the coarser material. Taking into account the (slight) grading of the bottom material, the apparent $\epsilon_s(z)$ distributions of each of the bottom materials can be calculated with the help of Eq.(4).

Due to inevitable experimental scatter, quite identical $\epsilon_s(z)$ curves for the different particle sizes cannot be expected. However, in reality distinct trends could be observed. In Fig.4 some examples of $\epsilon_s(z)$ values (holding for a constant z level above the bed) are shown as a function of the fall velocity of the bed material. The values of w in Fig.4 correspond with the D_{50} values of the bed materials.

A theoretical basis is still lacking for this, but the measuring points in figures like Fig.4 can be approximated by a straight line. If it is allowed to extrapolate that line to $w = 0$ m/s, the intersection point can be considered as ϵ_f , the diffusion coefficient holding for the fluid. This consideration is rather speculative since the diffusion process of water particles will undoubtedly differ from that of very small sediment particles. Density differences between water and particles being a reason.

Lines like drawn in Fig.4 can be represented by (a least square fit procedure):

$$\epsilon_s = \epsilon_f + m w \quad (4)$$

where: ϵ_s : diffusion coefficient (sediment)
 ϵ_f : diffusion coefficient (fluid)
 m : slope of approximation line
 w : fall velocity.

Eq.(4) can be compared with Eq.(3):

$$\epsilon_s = \beta \epsilon_f \quad (3a)$$

From Eqs.(3a) and (4) follows:

$$\beta = 1 + \alpha w \quad (5)$$

where: α : m/ϵ_f ; non-dimensionless parameter
 (α in s/m)

It can be seen from Eq.(5) that, depending on the α value (direction of slope m), $\beta > 1$ as well as $\beta < 1$ can be found. Furthermore the actual β value depends on the fall velocity of the bed material.

For many points in a vertical ϵ_f and α values have been determined for each of the tests. Each ϵ_f - α combination has been plotted in Fig.5. A lot of scatter can be seen. It is, however, clear that for relatively small ϵ_f values positive α values are predominant. For large ϵ_f values negative α values occur.

Fig.6 shows a typical $\epsilon_f(z)$ distribution as could be derived for test B2 from Table I. Notice again the relatively large $\epsilon_f(z)$ values high in the vertical; these values are large in comparison with the $\epsilon_f(z)$ values close to the bed.

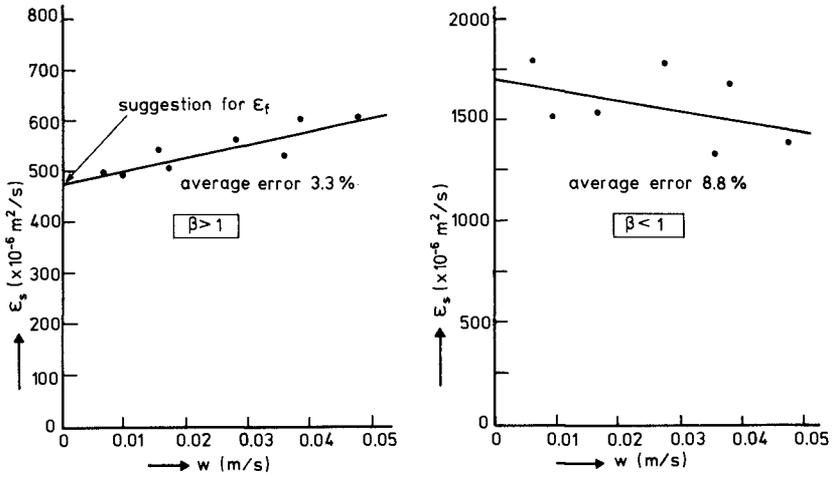


Fig.4 ϵ_s as function of fall velocity

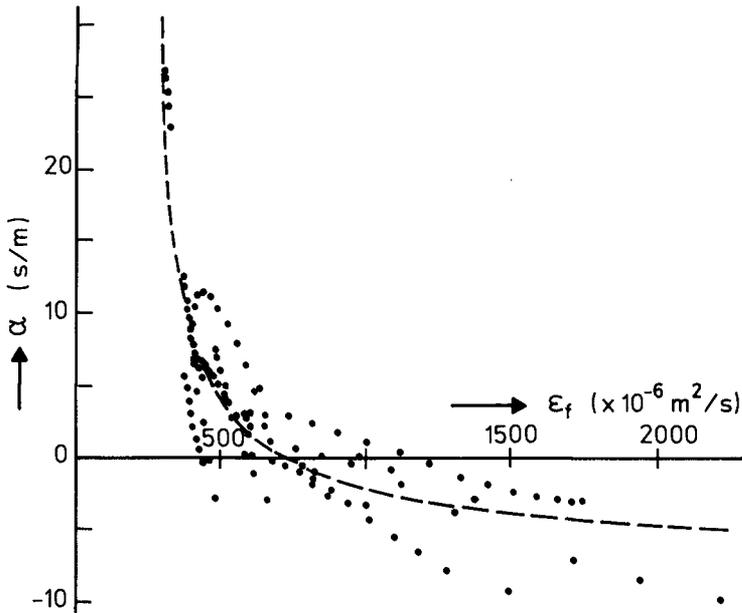


Fig.5 Effect of ϵ_f on α value

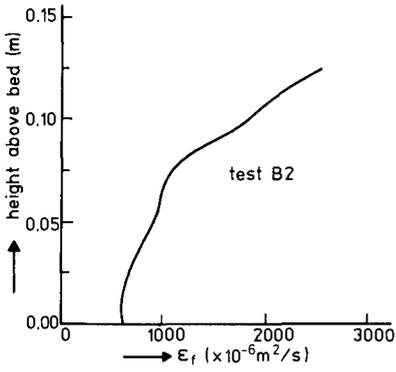


Fig.6 Typical ϵ_f distribution

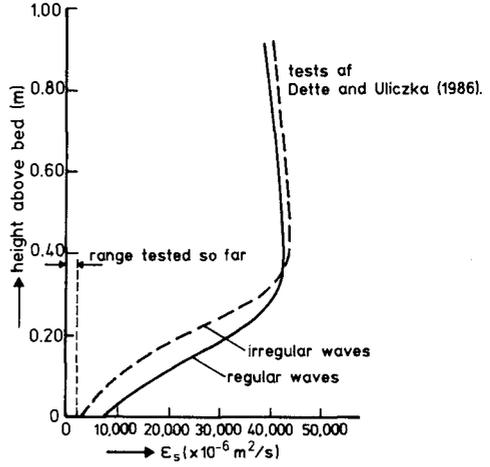


Fig.7 ϵ_s for large scale tests

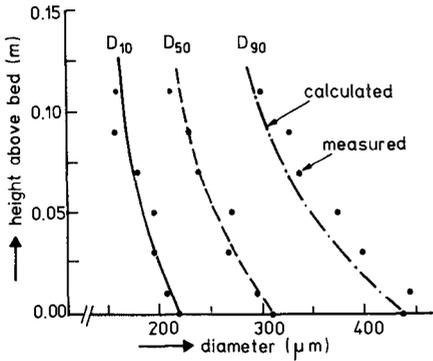


Fig.8 Degradation

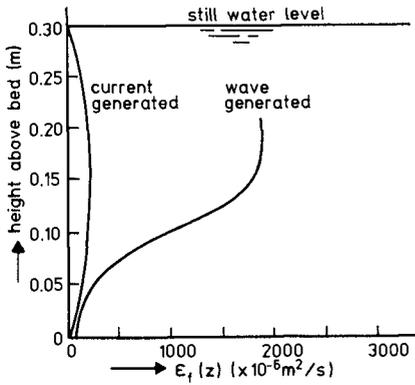


Fig.9 Wave- and current generated ϵ_f distributions

A convincing explanation for the results of Fig.5 with respect to α (and thus β) cannot yet be given. However, some remarks can be made. Probably the behaviour of α as a function of ϵ_f is related to the position above the bed. The relatively smaller ϵ_f values are normally present close to the bed (cf. Fig.6). Due to the 'rippled' bottom, eddies are formed in the bottom layer. These eddies (laden with bottom particles) move as a whole in an upward direction during some phases of the orbital motion. During these phases distinct convective transport processes occur through which, also in the tests with coarse bottom material, the particles can reach rather high levels. Such a process has the same effect as if rather high diffusion coefficients occur for coarse material, thus leading to $\beta > 1$. Far from the bed (in most cases the zone with relatively large ϵ_f values) the bottom generated eddies have lost their energy. Probably there occurs a more or less 'normal' diffusion concept according to Eq.(4). In that case $\beta = 1$ could be expected. At this moment the authors cannot formulate a reliable explanation for $\beta < 1$ for these high ϵ_f values.

The research in the behaviour of β is far from ended. Up till now only some first ideas have been found. The range of ϵ_f values as have been encountered during the present test series is, in fact, very restricted. For practical applications in future, insight in the behaviour of α for far larger ϵ_f values should be acquired. That can be demonstrated with the results of some tests by Dette and Uliczka (1986) carried out in the large wave flume of Hannover ($H \approx 1.5$ m).

Fig.7 shows the ϵ_s distribution as have been derived from two series of sediment concentration distribution measurements. The analyses have been carried out with the assumption of $\beta = 1$. Fig.7 shows ϵ_s values in a total different range compared with these found during the tests as discussed so far.

More or less as a by-product of the calculation methods as have been used, the degradation over the water column of the bottom material can be computed. Fig.8 shows a confrontation between computation results and measurements. The rather good fit seems to support to some extent the diffusion-type description of the tests.

3. The $v(z)$ distribution

From the analyses as have been discussed in Section 2 also so-called $\epsilon_f(z)$ distributions are found (cf. Fig.6 and Fig.7 to some extent). The shape of these distributions is in many cases quite different from 'theoretical' $\epsilon_f(z)$ distributions as can be found in literature. An $\epsilon_f(z)$ distribution is frequently used to derive the horizontal velocity distribution over the water depth, viz.:

$$\tau(z) = \rho \epsilon_f(z) \frac{\partial v(z)}{\partial z} \quad (6)$$

where: $\tau(z)$: shear stress at level z above the bed
 ρ : density of the fluid
 $\epsilon_f(z)$: diffusion coefficient (fluid) at level z

$v(z)$: horizontal current velocity at level z
 z : vertical upward directed ordinate;
the bottom being $z = 0$.

The well-known logarithmic velocity distribution for uniform current is, for instance, found with a triangular $\tau(z)$ distribution [$\tau(z=0)$:maximum; $\tau(z=h)$:zero] and a parabolic $\epsilon_f(z)$ distribution (ϵ_f reaches its maximum value for $z = \frac{1}{2} h$).

The test results as have been discussed in Section 2 yield $\epsilon_s(z)$ [and $\epsilon_f(z)$] distributions 'generated' by waves only. Waves are obviously able to maintain sediment concentrations at levels rather high above the bed. It can be *assumed* (only an assumption can be made for the time being since a sound proof and verification is still missing) that waves are also very effective in transferring shear stresses in cases of a combination of waves and currents. If that assumption is true, a practical calculation example can clarify the consequences.

The graded material $\epsilon_s(z)$ distribution of Fig.2b strictly holds for the particular sediment and for waves only. It is, however, assumed that this distribution can be considered as an $\epsilon_f(z)$ distribution (so taking $\beta = 1$) which holds also for a combination of that wave and a modest current making an arbitrary angle with the wave propagation direction. The distribution of $\epsilon_s(z)$ in Fig.2b holds for a water depth $h = 0.30$ m and a maximum orbital velocity near the bed, $\hat{u}_0 \approx 0.2$ m/s. The mean velocity of the current is assumed to be $\bar{v} = 0.1$ m/s. Fig.9 shows also the $\epsilon_f(z)$ distribution which belongs to a mere current situation with $\bar{v} = 0.1$ m/s. It can be seen that the wave generated $\epsilon_f(z)$ distribution reaches far larger values than the mere current $\epsilon_f(z)$ distribution. In the further analysis it is assumed that the current contribution can be neglected in comparison with the wave contribution. The resulting $\epsilon_f(z)$ distribution is not parabolic at all and together with a triangular $\tau(z)$ distribution a non-logarithmic $v(z)$ distribution (with $\bar{v} = 0.1$ m/s) will be found (see Fig.10).

Up till now a physical verification is not yet carried out. Tests are in preparation by the Delft University of Technology. Just to show at least the qualitative agreement, in Fig.11 a comparison has been made between the calculated curve of Fig.10 and actual measurements of Van Doorn (1981) with only roughly comparable boundary conditions. [Same water depth; same average velocity; regular waves with slightly different characteristics; artificial roughness elements instead of natural rippled bed.]

It will be clear that a velocity distribution like Fig.10 has severe consequences for the resulting sediment transport. A quite different transport will be calculated in that case in comparison with a situation where a logarithmic velocity distribution is assumed.

Since only some first thoughts are formulated, all implications cannot be overseen at the moment by the authors.

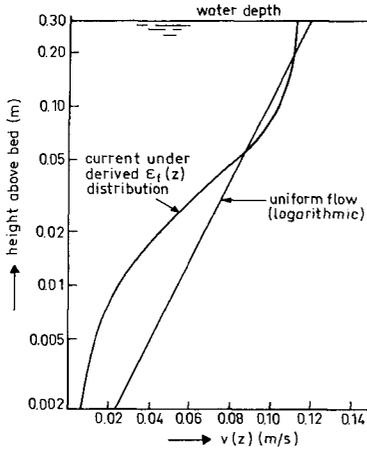


Fig.10 Velocity distributions

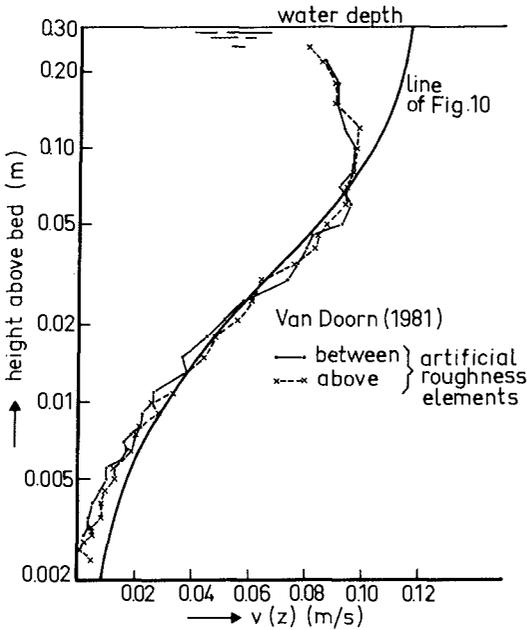


Fig.11 Measured and 'theoretical' distributions

4. Conclusions

- Wave flume tests with artificial roughness elements and different sediment sizes have revealed that the diffusion coefficient distribution for the sediment $\epsilon_s(z)$ is different for different particle sizes.
- With $\epsilon_s = \beta \epsilon_f$ this means that β differs from unity. Cases with $\beta > 1$ and $\beta < 1$ have been found, probably also depending on the 'level' of ϵ_f (the diffusion coefficient for the fluid).
- A sound explanation of the behaviour of the β -factor cannot be given at the moment.
- The wave generated diffusion coefficients are far larger than the mere current contribution.
- If a combination of waves and current is considered and the wave generated diffusion activity is also available to transfer shear stresses, a velocity distribution can be calculated which differs totally from a logarithmic distribution. If those ideas are true, the resulting sediment transport will also be affected.
- Verification tests are in preparation by the Delft University of Technology.

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