CHAPTER 65

Grouping Waves and Their Expression on Asymptotic Envelope Soliton Modes

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ABSTRACT

An approach, that treats natural sea states with remarkable groupiness as random sequences of envelope solitons, is suggested here to explain their dynamical and statistical properties from the viewpoint that wave packets contained in the states have their own characteristics and should be regarded as elementary modes. Some examinations are made on its applicability to the temporally observed waves. And the approach is shown to be effective also for waves with a non-zero nonlinearity and finite spectral band-width. Further, a formulation based on envelope solitons is made on the wave drifting force and is shown to be useful for analyzing the time series of drifting forces.

INTRODUCTION

The wave grouping such as runs of consecutive high waves or small. waves in nature has been shown to occur more often and to contain more waves than would be expected if waves are completely random. The potential consequences are often quite significant in both offshore and coastal activities. It is well-known, [for example, Spangenberg, 1980], that many moored offshore structures and floating vessels suffer a great influence of the wave grouping and their responses become maxima when the period of the wave groups is equal to or lies in the vicinity of the natural periods of the moored systems. With the progress of ocean development and coastal activities, the concern on the wave grouping has been grown and various investigations have been made theoretically and experimentally on wave grouping characteristics. However, usual investigations in engineering sides are based on the assumption of a narrow banded Gaussian process and lack the fact that the wave grouping is due to weak but non-zero nonlinearity and should be treated as a nonlinear phenomenon, so that they have limits in evaluating dynamical characteristics of grouping waves.

Wave systems that depend on the band-width and nonlinearity may exhibit component dispersion ranging from that given by the linear dispersion to that of an effectively nondispersive phase-locked system in which wave components propagate essentially with a single speed. This demonstrates that the Fourier spectrum representation and the

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associated Gaussian random wave theory do not comprise an entirely satisfactory analysis technique forsea states with non-zero nonlinearity. Recent researches on waves with a weak nonlinearity and narrow band-width [for ex. Yuen & Lake, 1982] made clear that the description of grouping waves contained in the wave field provided by the nonlinear Schrödinger(NLS) equation and the Zakharov equation is qualitatively and quantitatively correct and nonlinear effects on the formation of wave groups are significant. It has been already stated [Sobey & Colman, 1982] that the envelpe soliton bears a remarkable resemblance to the common appreciation of wave groups. However, waves in field have spectra with a finite band-width and nonlinear aspects such as peaking of wave crests and flattening of troughs, so that it is difficult to apply directly the NLS equation to them and some trials[for ex. Stiassinie and Shemer, 1984] using the Zakharov equation has not been applied successfully in describing them.

The nature of wave groups in field is a little bit made clear, although their natural occurrence is not in doubt. It is not also clear whether the wave groups are spatially stable or not and they can be regarded as wave packets with soliton properties, that is, envelope solitons or not. However, it is the well-known fact that a nonlinear wave packet, that is, envelope soliton is a stable grouping although linear wave packets are destroyed eventually by dispersion and are spatially unstable. It is, therefore, natural to suppose that waves after having travelled a long distance are generally in a stable state and may have a coherent structure making some stable mode elementary excitation and that the stable wave group contained in the waves can be regarded as a kind of such elementary excitation. This viewpoint is quite different from usual ones interpreting the wave group as a consequence of a mere superposition of some carrier waves and a stochastic sequence of zero-crossing waves. If the wave group has its own characteristics and can be treated as elementary mode of waves with remarkable groupiness, an approach for treating the wave group as elementary mode may become possible for explaining properties of the grouping waves. This approach may be expected to be effective for analyzing the time series of the wave drifting force caused by the existence of wave groups.

In this study, by investigating wave grouping characteristics of waves observed at various locations, we show that they remarkably depend on nonlinear effects and the frequency of carrier waves associated with each wave group is not necessarily identical with the peak frequency of the spectrum and distributes over a finite band-width. Then, all wave groups accompanied with the waves are treated as envelope solitons and their envelope profile is represented as a random train of asymptotic envelope solitons. Some examinations are made on the applicability of the theoretical result to the waves observed temporally at fixed positions and the envelope profiles. Further, this representation is extended to the formulation of the wave drifting force.

GROUPING CHARACTERISTICS OF OBSERVED WAVES

1. Spectral Band-Width and Nonlinearity

Field data were collected from various observatories locating off Gobo coast (G1-8, Oct. 1979) facing the Pacific ocean, off Ogata coast (O1-12, Mar. 1981) facing the Japan sea, off Caldera Port in Costa Rica (C1-4, May 1981) facing the Pacific ocean and at Lake Biwa (B1-10, Oct. 1975). The data were obtained by the temporal observations of water surface displacements at fixed positions. Table 1 summarizes values of wave parameters calculated from these data. In the table,

Data	h (M)	f p	ν	1B i	k₀h	Ur	GF	p	J ₁	J2
B 1	4.0	. 293	. 991	. 037	1.34	2.03	. 337	. 463	2.47	5.29
B 2	4.0	. 293	. 989	. 174	1.34	1.94	. 396	. 506	2.63	5.27
B 3	4.0	. 317	. 990	. 310	1.73	2.01	. 398	.481	2.64	5.38
B 4	4.0	. 305	. 990	. 260	1.62	2.18	. 402	. 494	2.49	5.25
B 5	4.0	. 281	.988	. 166	1.43	1.96	. 393	. 459	2.85	6.28
B 6	4.0	. 305	.988	. 143	1.62	1.61	. 374	. 447	2.48	5.57
B 7	4.0	. 322	.987	.070	1.77	1.00	. 367	. 474	2.49	5.38
B 8	4.0	. 3 2 2	.989	. 096	1.77	1.24	. 364	. 485	2.75	5.57
B 9	4.0	.440	. 981	. 191	3.13	0.27	. 371	. 518	2.92	5.62
B10	4.0	. 440	.982	. 136	3.13	0.26	. 384	. 459	2.49	5.49
G 1	28.0	. 0,68	.723	.020	0.80	3.12	. 444	. 478	4.13	8.29
G 2	28.0	.064	. 772	.073	0.74	3.12	. 468	. 456	2.38	5.41
G 3	28.0	.072	.765	.069	0.85	3.32	. 546	. 5 2 2	2.77	5.45
G 4	28.0	.061	.768	. 093	0.70	2.94	. 351	. 557	3.25	6.50
G 5	28.0	.064	.745	.005	0.74	3.68	. 471	. 457	4.00	7.14
G 6	28.0	.064	.720	. 124	0.74	2.99	. 397	. 4 2 3	2.73	7.22
G 7	28.0	.061	. 759	.049	0.70	1.99	. 347	. 482	2.41	4.33
G 8	28.0	.065	·.780	1.08	0.75	2.92	. 391	. 468	3.36	6.70
C 1	18.5	.053	. 920	. 323	0.48	9.09	. 411	. 492	3.56	7.38
C 2	18.6	.057	.910	. 252	0.52	9.90	. 336	.529	4.27	8.36
C 3	18.2	.057	. 898	.501	0.51	9.94	. 455	. 484	3.21	6.94
C 4	18.1	.053	.904	. 689	0.47	15.5	. 446	. 492	4.00	7.93
01	6.2	. 107	. 981	. 393	0.56	8.78	. 319	. 488	2.05	4.27
02	6.9	.096	.803	. 813	0.53	11.2	.350	. 441	3.00	6.67
03	6.9	. 114	. 811	.701	0.64	10.1	. 367	. 493	2.77	5.67
04	6.9	. 175	.977	. 489	1.08	4.71	. 650	. 492	4.80	
05	6.9	.175	.977	.476	1.08	6.45	.770	. 481	4.80	
06	7.0	.175	.977	. 297	1.09	5.69	.700	. 485	4.80	
07	6.2	.096	. 803	.968	0.50	5.68	.740			
08	5.3	.096	.803	1.12	0.46	5.07	.780			
09	5.3	.096	. 803	.845	0.46	4.02	.740			
010	6.2	.114	.811	.721	0.60	7.64	.720			
011	5.3	.114	.811	. 826	0.55	6.23	.730			
012	5.3	. 114	.811	.618	0.55	7.29	.730			

Table 1 Wave parameters of observed waves

h denotes the mean water depth, $f_{\rm p}$ the peak frequency of the power spectra, $_{\rm V}$ the spectral band-width parameter defined by

$$\nu = [1 - m_2^2 / (m_0 m_4)]^{1/2} , \qquad m_n = \int_0^\infty f^n S(f) df , \qquad (1)$$

in which f is the wave frequency, $k_{\rm p}$ the wave-number corresponding to $f_{\rm p}$ through the linear dispersion relation, $\sqrt{\beta_{\rm l}}$ the skewness, $U_{\rm r}$ the Ursell number, GF the Groupiness Factor defined by Funke and Mansard 1979, p the probability of the wave-height H of zero-up crossing wave exceeding

the average wave-height \bar{H} , and \bar{j}_1 the average run length of the zeroup crossing waves exceeding \bar{H} . It is easily noticed from this table that all values of v exceed 0.7 and are near unity rather than zero, that is, observed waves to be used here have not a narrow but finite spectral band-width. The values of skewness $\sqrt{\beta_1}$ of the observed waves except for G5 exceed 0.01 and indicate that their water surface displacements are influenced by nonlinearity. The values of U_r also show that the observed waves have a finite nonlinearity in comparison with a frequency dispersion. Hence, it may be said from these results that the observed waves can not be analyzed on the basis of a narrow-banded Gaussian process.

2. Wave Grouping Characteristics

GF and qj₁ are used here as the criteria representing wave grouping characteristics. q is the complementary number of p and is defined as 1-p. The value of qj_1 is unity when the time series of the wave-heights of zero-up crossing waves are completely random. Since a modulational instability occurs when kh>1.36 in the wave field provided by the NLS equation and then envelpee solitons are formed, kh is used here as the instability criterion relating to the formation of envelope solitons. Figure 1 shows the relation of GF and $q\bar{j}_1$ vs. kh. The value of q], is slightly inclined to increase with the increasing of the value of 1/kh, although varying remarkably in the region of kh<1.36. While, the value of GF is almost independent of kh. If wave groups possessed in one wave record have the identical carrier frequency with the peak frequency $f_{\rm p}$ and are governed by the single NLS equation, their envelope profiles shoud be flattened when the value of kh falls below 1.36. That is, as the value of kh gets to be below 1.36 and further decreases, GF should decrease and $q\bar{j}_1$ should increase. However, such a definite trend can not be found out and the outward profile of wave groups is supposed to be still stable even under the value of kh being below 1.36. It, therefore, may be said that wave groups possessed in the waves with a finite nonliearity and finite spectral band-width are still stable in the region, kh<1.36 and that the wave grouping characteristics are almost independent of the value of kh calculated from the peak frequency fn and



Fig. 1 Relation of kh vs. GF and $q\bar{j}_1$

should be investigated in relation to nonlinearity.

Figure 2 shows the relation of GF and $q\bar{j}_1$ vs. the Ursell number U_r . It is easily noticed from the figure that the wave grouping characteristics evaluated by $q\bar{j}_1$ and GF definitely depends on the ursell number, that is, both values of $q\bar{j}_1$ and GF increase as the value of U_r increases until reaching about 8 and then they suddenly begin to decrease just after the value of U_r exceeds about 8. This states that the degree of groupiness of random waves with a finite nonlinearity and spectral band-width depends on the nonlinearity itself. It, further, is imagined that the wave groups once formed are conserved as stable modes and may be regarded as elementary modes of waves with a non-zero nonlinearity when U_r is less than 5 or 6 and that their associated carrier waves turn to a train of solitons when U_r exceeds about 8 and then are made stable.



Figure 3 shows the relation between the run length j_1 and the ratio T/\bar{T} . Where, T is the zero-up crossing period averaged over the

consecutive high waves constituting one wave group. \overline{T} is the zero-up crossing period averaged over one record of the observed waves. If all wave groups included in the observed wave can be described by the multi-soliton solution of the single NLS equation, each wave group has the identical carrier frequency with the peak frequency and T agrees with \overline{T} . However, it is seen that the ratio T/\overline{T} is not necessarily unity and there is a definite correspondence between j_1 and T/\overline{T} . The period of carrier waves associated with one wave group becomes larger as the run length j1 increases, that



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is, the wave group grows up. This says that each wave group accompanies a carrier wave with its own wave period independent of other wave groups.

ASYMPTOTIC ENVELOPE SOLITON MODE REPRESENTATION

1. Fundamental Assumptions

The observed results shown in the preceding chapter revealed that the wave grouping depends on U_r rather than kh and can not be sufficiently explained by the modulational instability under a narrowband spectrum. This states that the wave groups are formed on the basis of nonlinear dynamics and behave as stable wave packets. Hence, we suggest an approach to treat them as envelope solitons governed by the plural NLS equations, although we can not verify its validity. The motive of this approach is based on the viewpoint that waves with remarkable groupiness have a dynamical structure making envelope soliton elementary excitation. Under the viewpoint, in order to represent them as a sequence of envelope solitons of which amplitudes and phase constants are random variable dependent on an initial probability, the following assumptions are made.

i) All wave groups possessed in observed waves behave as envelope soltons and are composed of the multi-envelope soliton solutions of the plural NLS equations.

ii) The envelope profile of the waves observed temporally at a fixed position can be approximately represented as a random sequence of asymptotic envelope solitons.

A schematic diagram of the sea state corresponding to these assumptions is shown in Fig. 4. The diagram explains that the sea state is governed independently by the L-NLS 1=2equations having N₁envelope soliton solutions (1=1,2,...L) and the observed waves are composed 1=L of a train of envelope solitons with the number of ΣN_1 .

2. Formulations

Since the wave data are obtained by the temporal observations at fixed positions as mentioned above, the NLS equation of spatial evolution type is required to analyze them and is expressed as follows:



Fig. 4 Schematic diagram of wave field represented as random train of envelope solitons

$$-i\partial A/\partial X + \alpha \partial^2 A/\partial \xi^2 + \beta |A|^2 A = 0$$

where

$$\begin{array}{c} \alpha = (1/2)[c_{g}/c - (gh/c^{2}) \operatorname{sech}^{2} Kh(1 - Kh \tanh Kh)], \\ \beta = (\cosh 4Kh + 8 - \tanh^{2} Kh)/(16 \sinh^{4} Kh) \\ - (2c \cosh^{2} Kh + c_{g})^{2}/[2 \sinh^{2} 2Kh(gh - c_{g}^{2})], \\ c = [(g/K) \tanh Kh]^{1/2}, \\ c_{g} = (c/2)[1 + 2Kh/\sinh 2Kh], \\ X = \varepsilon^{2} Kx, \quad \xi = \varepsilon 2\pi f(x/c_{g} - t), \end{array}$$

$$(3)$$

in which K is the wave-number corresponding to the central frequency f of a narrow-band spectrum, ϵA the amplitude of a carrier wave nondimensionalized by K, h the mean water depth, c the wave-celerity, c_g the group-velocity, ϵ the small parameter denoting the degree of nonlinearity, x the horizontal coordinate, t the time and g the acceleration of gravitation.

The asymptotic N_1 -soliton solution of eq.(2) is written as

$$\frac{\varepsilon A_l}{K_l} = \sum_{n=1}^{N_l} a_n \operatorname{sech} \left[\sqrt{\beta/2\alpha} \cdot a_n (2\pi K f/c_g) \left\{ x - c_g (t - \delta_n) \right\} \right]_l \exp\left(-i\beta_l a_n^2 K x \right) \right]_{(4)}$$

where the parameters with the subscript l are the quantities related to the envelope soliton with the central frequency f_1 , δ_n the phase constant determining the position of the soliton on the temporal coordinate, and a_n the amplitude of the envelope soliton. If all the spacings between two consecutive wave groups are enough wide to make asymptotic approximation possible, the asymptotic expression based on the above assumptions becomes possible. Hence, the leading order profiles η of the observed waves with remarkable groupiness are expressed by using eq.(4) and defining the phase constant σ_1 of carrier waves:

$$\eta = \sum_{l=1}^{L} \frac{|\varepsilon A_l|}{K_l} \cos(K_l x - 2\pi f_l t + \sigma_l)$$
⁽⁵⁾

Similar expressions of horizontal water particle velocity and water pressure, which are necessary to calculate the wave drift force etc. reflecting wave grouping characteristcs, can be easily obtained.

$$u = \sum_{l=1}^{L} |\varepsilon A_l| c_l \left\{ \frac{\cosh K_l(h+z)}{\sinh K_l h} \right\} \cos(K_l x - 2\pi f_l t + \sigma_l)$$
(6)

$$\frac{p}{\rho g} = \sum_{l=1}^{L} \frac{|\varepsilon A_l|}{K_l} \left\{ \frac{\cosh K_l(h+z)}{\cosh K_l h} \right\} \cos(K_l x - 2\pi f_l t + \sigma_l) - z \tag{7}$$

The envelope profile R(t) of observed waves is expressed as a sequence of M-asymptotic envelope solitons, if eqs.(4) and (5) are used.

$$R(t) = \sum_{m=1}^{M} R_m , \qquad R_m = a_m \operatorname{sech} \left[\mathcal{I}_m(t - \delta_m) \right] ,$$

$$\mathcal{I}_m = 2\pi f_m K_m a_m (\sqrt{\beta/2\alpha})_m .$$
(8)

The above equation states that both the envelope profile and the associated carrier waves are governed by the values of the random variables, a_m , f_m and δ_m . Hence, by investigating statistical characteristics of these random variables, we can make a statistical description both for the envelope profile and carrier waves and a simulation of random waves with a given groupiness. Probability distribution function $F(R_c)$ of the envelope profile R(t) is defined as the probability of R(t) being below the critical value R_c and expressed as

$$F(R_{c}) = p[R \le R_{c}] = \sum_{m=1}^{M} \frac{1}{\gamma_{m}} \times \int_{0}^{R_{c}} \ln\left\{ \left[1 + \sqrt{1 - (R_{m}/a_{m})^{2}} / (R_{m}/a_{m})\right] dR_{m} / \sum_{m=1}^{M} (1/\gamma_{m}) \right\}$$
(9)

Since the differentiation of F(R_c) by R_c yields the probability density function f(R), a statistical approach becomes possible for runs of consecutive high waves by using f(R). The averaged run length j_1 making R_c the critical value is expressed as

$$F^{*}(R_{c}) = 1 - \left\{ \sum_{m=1}^{M} \frac{2}{\gamma_{m}} \left[\ln \left(1 + \sqrt{1 - Y_{m}^{2}} \right) - \ln Y_{m} \right] \right\} / T_{0}, \qquad Y_{m} = R_{0} / (2a_{m}) , (10)$$

$$j_1 = [(2f_m/\tau_m)\{\ln(1+\sqrt{1-Y_m^2}) - \ln Y_m]] , \qquad (11)$$

where brackets [] denote the integral representation.

3. Application to observed waves

In order to represent the envelope profile of the observed waves using eq.(8), the values of a_m , δ_m and f_m governing an envelope soliton must be determined from each wave group. Hence, deriving the linear envelope function $R_0(t)$ based on the narrow-band spectrum from the wave record band-pass filtered so as to eliminate both frequencies higher than 1.5f_p and lower than 0.5f_p, we first determine the number of envelope solitons possessed in the record and the values of δ_m from the crest positions expressed by the function $R_0(t)$. And then, we determine both values of a_m and f_m so as to minimize the error energy between the linear envelope function $R_0(t)$ and the nonlinear envelope function R(t) defined by eq.(8).

$$E = \int_{0}^{T} \{R(t) - R_{0}(t)\}^{2} dt$$
⁽¹²⁾

That is, both values of \mathbf{a}_{m} and \mathbf{f}_{m} are determined by solving the following simultaneous equation with 2M-dimensions.

$$\partial E/\partial a_m = 0$$
, $\partial E/\partial f_m = 0$. $m = 1, \dots, M$ (13)

Figure 5 shows some comparisons between the profiles of the observed waves and the envelope profiles represented as a train of the envelope solitons of which parameters were calculated as mentioned above. The comparisons state that the expression based on eq.(8) can be applied to the observed waves with a non-zero nonlinearity and remarkable groupiness instead of the linear envelope function $R_o(t)$ although it utilizes $R_o(t)$ to determine the values of a_m , δ_m and f_m .



Fig. 5 Comparisons between profiles of the observed waves and the envelope profiles calculated by the present approach

Figure 6 shows comparisons between the envelope profile 2R(t) calculated by eq.(8) and the temporal variations in wave-heights of zero-up crossing waves. It is seen that the properties of the time series of zero-up crossing waves can be easily evaluated on the basis of eq.(8). This states that the statistical approach briefly shown above has a possibility of its application to the observed waves.

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Fig. 6 Comparisons of a train of wave-heights of zero-up crossing waves with the envelope profile represented by the present approach

Figure 7 shows the relation between f_m/f_p and $(\bar{a}/a_m)/k_ph$. Here, f_m is the frequency of carrier waves associated with an envelope soliton corresponding to one wave group and \bar{a} is the averaged amplitude of envelope solitons included in one record of the observed waves. It is noticed that a functional relation equivalent to the regression curve, defined by

$$\left. \begin{array}{c} f_m / f_p = 0.38 + 2.55 x - 0.67 x^2 + 0.07 x^3 \\ x = \tilde{a} / K_p h a_m \end{array} \right\}$$
(14)

exists between f_m/f_p and $(\bar{a}/a_m)/k_ph$, nevertheless,the values of a_m and f_m are determined only by the conditions shown in eq.(13). This states that grouping waves in field are governed by some dynamics slightly similar to the assumpsions mensioned above and the value of f_m can be estimated using eq.(14).

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Fig. 7 Relation between peak frequencies of the observed waves and carrier frequencies associated with envelope solitons

As shown above, it is seen that the assumptions i) and ii) hold at least for the observed waves used here and the schematic diagram drawn in Fig.1 reflects partially a correct statement of the physics of the waves with a weak nonlinearity and finite band-width. This states that the approach treating envelope solitons as elementary modes can be applied to the waves with remarkable groupiness. The approach, which may be called as the envelope soliton mode approach, suggests that there is a possibility to explain simply dynamical and statistical properties of the waves and to simulate numerically waves with a expected groupiness. Further, it is expected that the time series of grouping waves can be analyzed by making clear the statistical characteristics of random variables a_m and δ_m , instead of usual approach based on the run theory of zero-crossing waves and the linear envelope function.

DRIFTING FORCE BASED ON ENVELOPE SOLITONS

1. Formulations

It is well-known that the wave grouping is closely related to the slow drift oscillation of moored systems. Pinkster[1975] formulated the wave drifting force under the assumption of a narrow-banded Gaussian process and showed that the slow drift oscillation is due to the resonance between the natural frequency of a moored system and the peak frequency of the spectrum of the drifting force. However, waves with remarkable groupiness have generally a weak nonlinearity and finite spectral band-width and are not suitable for Pinkster's approach based on a narrow-banded Gaussian process. Further, Pinkster's approach is not effective for evaluating the time series characteristics of the drifting force because the spectral analysis itself is not useful for the investigation on time domain. While, the envelope soliton mode approach suggested in the preceding chapter is applicable to such waves and effective for investigating the time series characteristics.

A formulation based on the envelope soliton mode is made on the computations of the wave drifting force based on the direct integration method of pressure acting on the wetted surface of a body with reference to Pinkster's approach. Assuming as well as Pinkster that the envelope profile of the waves slowly varies and using egs.(5), (6), (7) and (8), we can represent the drifting force $F_{\rm S}$ (t) based on the present approach as

$$F_{s}(t) = \frac{1}{2} \rho g \sum_{m} \sum_{n} a_{m} a_{n} C_{FD} \operatorname{sech} \left[\gamma_{m}(t - \delta_{m}) \right] \operatorname{sech} \left[\gamma_{n}(t - \delta_{n}) \right] \cos \left(\omega_{m} - \omega_{n} \right) t ,$$
⁽¹⁵⁾

where spatial coordinate x is set to be zero for simplicity, and the parameters with the subscript m denote the quantities concerned with m-th envelope soliton with carrier frequency f_m, C_{FD} the drifting force coefficient to be determined experimentally, ω_m the angular frequency $2\pi f_m$, ρ the density of fluid.

If each envelope soliton is independent of others, eq.(15) is further simplified and is rewritten as

$$F_{s}(t) = (\rho g/2)C_{FD}R(t)^{2},$$

$$R(t) = \sum_{m=1}^{I} a_{m} \operatorname{sech} \left[\gamma_{m}(t-\delta_{m})\right]$$
(16)

This expression states that the time series characteristics of the drifting force can be explained in a direct relation with a statistical properties of random variables a_m and δ_m governing dynamically each envelope soliton and that the envelope soliton mode approach is very effective for investigating the properties of the drifting force in a time domain.

2. Characteristics

In order to investigate the properties of the drifting force expressed in eq.(15) and the relation with wave grouping characteristics, comparisons of the drifting force are made among Hsu & Blenkarn's approach[1972], Pinkster's one and authers' one. Figure 8 shows the comparison. Here, the value of the drifting force coefficient $C_{\rm FD}$ is set to be unity commonly for each case. It is found that both results of Pinkster and authors do not suffer the influences of small deviations independent of wave groups and are superior to Hsu and Blenkarn's approach in evaluating the influence due to wave groups themselves. The drifting force by the present approach agrees well with the one by Pinkster's appoach and the former may be used instead of the latter which has been put widely to practical use.



Fig. 8 Comparisons of the wave drifting force calculated by authors approach with the ones by Hsu and Pinkster

CONCLUSION

The approach, regarding an envelope soliton as an elementary mode of grouping waves and representing them as a train of envelope solitons of which amplitudes and phase constants are random variables, is suggested here and is shown to be applicable to the temporally observed waves with a non-zero nonlinearity and finite band-width. It, further, is extended to a formulation of the wave drifting force caused by the existence of wave groups and the fomulation based on the envelope soliton mode is signified to be put to practical use as well as Pinkster's one based on a narrow-banded Gaussian process.

The present study containes some debatable points such that the intuitive picture shown in Fig. 4 is not verified to be a correct statement of the physics of grouping waves and the approach can not describe their propagation process because based on the asymptotic soliton solutions of the plural NLS equations. In order to establish the envelope soliton mode approach, it is required to verify that grouping waves in field have a dynamical structure making an envelope soliton an elementary mode and overcome the above debatable points. However, it should be emphasized that the approach is useful for analyzing the problems mainly dependent on wave grouping characteristics at a fixed position, such as the slow drift oscillation of moored systems. It, further, is supposed to be effective for evaluating simply statistical properties of grouping waves and simulating numerically waves with an expected groupiness because parameters governing each envelope soliton are random variables expressing directly wave grouping characteristics and statistical quantities concerned with carrier waves and zerocrossing waves can be written as functions of the parameters.

As mentioned above, the approach suggested here remains some points to be examined. But it can be stated at least that the envelope soliton mode approach has a possibility to carry out a new statistical description on waves with a remarkable groupiness.

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