CHAPTER 58

Extreme Wave Groups in Storm Seas Near Coastal Water

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We discuss statistics of wave groups containing the maximal wave height over wave records of 20 minutes. These wave groups are called extreme wave groups (EWGs). The concept of the EWG combines the two previously separately considered design factors of extreme waves and wave groups into a more severe storm seas wave action. The wave data used in this investigation consists of near 540 records of wave staff measurements in the Gulf of Mexico. Specifically, we shall present (a) the dependence of EWG properties on the bandwidth of wave spectra, (b) the mean envelope shape of EWGs, exhibiting distinct differences from the envelope shape of regular basis.

Introduction

The study of wave groupiness has gained increasing attention in the past decade, because of its importance for long-period excitations to marine fixed and/or floating structures. Statistical analyses of ocean waves in storms (Rye, 1974; Goda, 1976; Su, et al., 1982) have firmly established the dependence of wave groupiness, such as its distributions and mean length, on the associated spectral bandwidth. Theoretical analyses (Longuet-Higgins, 1984, Kimura, 1980; Goda, 1976, 1983; Nolte and Hsu, 1972) have further related these properties in terms of either the linear narrow-band Gaussian process, or Markovian process. On the other hand, the most important classical design factor for marine structures is the estimation of extreme waves which are usually assumed to occur singly in nature (see Ochi, 1982 for a review on this subject). It can be convincingly argued, however, that a wave group containing an extremely large wave, if exists, would be an even more appropriate environmental design factor than either the single extreme wave or the "regular" wave group. For the sake of easy reference, we shall call this special type of wave groups, each containing an extremely large wave (to be defined more precisely later) by the name of "extreme wave groups" (EWG) in contrast to the "regular wave group" (RWG) which is normally defined to be a sequence of successive waves whose heights all exceed some prescribed critical value (H_c).

Goda (1976) seems to be the first investigator who made a statistical comparison of these two different types of wave groups. He noted that the mean length of runs for EWG is 2.36 when the critical value $H_c=H_{1/3}$, while the corresponding mean length for RWG is only 1.4. In other words, the highest wave does not appear singly, but is frequently accompanied by two to three high waves. Furthermore, the probability for the highest wave occurring in a group is even higher than regular high waves.

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Wave Data

The first set of wave records used in this investigation was obtained by wave staffs from an offshore platform about 60 miles offshore in the Gulf of Mexico (Ocean Data Gathering Program (OOGP), Station #1) during 1969-1971 under the sponsorship of a consortium of eight oil companies (Ward, 1974). The station is located in South Pass Block 62A, at a depth of about 340 feet. The second set of wave records was measured from a North Sea Platform (Forties Field) at the water depth of 150 ft and was kindly provided to us by UKOOA.

Each wave record is 19.85 minutes long and has been digitized into 4096 data points at the rate of 3.41 samples per second. For this analysis, we selected a total of 533 wave records (from the first set), each with the significant wave height ($H_s=H_{1/3}$)$\geq$2 meters in order to emphasize storm waves. These wave records were measured mostly in winter months, with the exception of during three hurricanes occurring in summer months. The eye of one of the hurricanes, Camille, happened to pass within 10 miles of the measuring platform.

Data Classification

The individual waves with their heights (H_j) and periods (T_j) for j=1,2...N in each wave record are determined by the standard up zero-crossing method. In the set of wave records we used, the value of N lies between 100 and 240 with the average close to 200. The power spectrum E(f) for each wave record is also computed by the Fast Fourier Transform, and is used in turn for computing two parameters indicative of the average bandwidth of the power spectrum. The first parameter was proposed originally by Longuet-Higgins (1984) and called the spectral bandwidth parameter, ν, and defined by

$$\nu = (m_2 m_0/m_1^2 - 1)^{1/2} \tag{1}$$

where m_0, m_1 and m_2 are spectral moments as defined by

$$m_i = \int_0^\infty f^i E(f)df \tag{2}$$

When $\nu^2 \ll 1$, the power spectrum is said to be narrow. The second parameter was proposed by Goda (1976) and called the spectral peakedness parameter, Q_p, and defined by

$$Q_p = \int_0^\infty 2fE^2(f)df/m_0 \tag{3}$$

The value of Q_p is always greater than unity, and increases with decreasing spectral bandwidth. We have used certain specified ranges of ν and Q_p, independently, to classify the set of wave records into several subsets in order to see dependence of the statistical characteristics of wave groupiness on the spectral bandwidth.

Definition of Extreme Wave Group (EWG)

For the purpose of our analysis which is constrained by the availability of each wave record being about 20 minutes long, the extreme wave group (EWG) is defined as the wave group containing the largest wave height in the wave record. For convenience of data analyses, we first assume that the EWG may cover up to seven waves and can be expressed as

$$W_{-3},\ W_{-2},\ W_{-1},\ W_0,\ W_1,\ W_2,\ W_3$$

where W_0 is the wave with the largest wave height, the three waves with negative subscripts are waves preceeding W_0, and those with positive subscripts are waves following W_0. The corresponding wave heights, periods, wavelengths, and steepness are denoted by H_i, T_i, L_i and S_i;

$$S_i = a_i k_i = (H_i/2) \times (2\pi/L_i) = \pi H_i/L_i \qquad (4)$$

where a_i is the wave amplitude and k_i the wave number (see Figure 1).

Results

We shall present the computed results which show the mean values (denoted with an overbar) of H_i, T_i and S_i for subsets of wave records with prescribed ranges of Q_p or ν . The majority of the wave records had the values of Q_p between 1.3 and 4.0, or has the values of between 0.40 and 1.0. Based on this distribution, we have classified the wave records into the following subsets: (1) for Q_p;

$$\begin{aligned}
&(a)\ 1.0 \le Q_p \le 2.0\\
&(b)\ 2.0 \le Q_p \le 3.0\\
&(c)\ 3.0 \le Q_p \le 4.0\\
&(d)\ 1.0 \le Q_p \le 5.0 \text{ (the entire data set)}
\end{aligned} \qquad (5)$$

and (2) for ;

$$\begin{aligned}
&(a)\ 0.40 \le \nu \le 0.50\\
&(b)\ 0.50 \le \nu \le 0.60\\
&(c)\ 0.60 \le \nu \le 0.70\\
&(d)\ 0.60 \le \nu \le 0.80.
\end{aligned} \qquad (6)$$

Table 1 gives the mean values of the wave heights, periods and steepness of the extreme wave groups for each of the three subsets according to the spectral classification of Q_p, plus the entire data set. Note that the wave heights given here have been normalized by the root-mean square of wave surface displacement $(m_0)^{1/2}$ of each corresponding wave record. The wave periods are normalized by the mean wave period of the corresponding wave record.

Table 1. Extreme Wave Group Analysis for Gulf of Mexico Wave Data. $H_{1/3} \geq 2.0$ m. Based on Q_p classification.

	j = -3	-2	-1	0	+1	+2	+3
(a) $\overline{H}_j$	2.335	2.510	3.016	6.284	3.389	2.238	2.322
$\overline{T}_j$	1.093	1.041	1.217	1.369	1.315	1.035	1.003
$\overline{S}_j$	0.152	0.142	0.144	0.186	0.122	0.150	0.143
(b) $\overline{H}_j$	2.491	2.735	3.943	6.335	3.931	2.677	2.361
$\overline{T}_j$	1.024	1.083	1.369	1.289	1.364	1.105	1.014
$\overline{S}_j$	0.154	0.137	0.125	0.195	0.138	0.136	0.146
(c) $\overline{H}_j$	2.667	2.997	4.325	6.352	4.435	3.156	2.704
$\overline{T}_j$	1.085	1.177	1.314	1.233	1.318	1.163	1.059
$\overline{S}_j$	0.122	0.117	0.133	0.208	0.128	0.142	0.136
(d) $\overline{H}_j$	2.506	2.753	3.864	6.325	3.929	2.699	2.418
$\overline{T}_j$	1.047	1.093	1.334	1.291	1.348	1.104	1.022
$\overline{S}_j$	0.153	0.134	0.129	0.196	0.134	0.139	0.143

We shall start with case (d) in Table 1, which includes the entire set of wave records in order to see the overall mean characteristics first, before discussing their dependence on spectral bandwidth. From the mean values of wave heights, H_j, one can see the envelope of the EWG is rather symmetric with respect to the highest wave height, H_0. The mean wave heights for the three central waves are $H_0 = 6.3$ and $(H_{-1}, H_1) = 3.9$, while the mean wave heights for outlying waves; $(H_{-2}, H_2) = 2.7$ and $(H_{-3}, H_3) = 2.5$, are much smaller. Recalling that the mean wave height for a linear narrow band Gaussian process is $H = \sqrt{2\pi} = 2.506$ (Arhan et al., 1976), we can infer, based on the height distribution alone, that (W_{-1}, W_0, W_1) is definitely distinctive from $(W_{-3}, W_{-2}, W_2, W_3)$.

The mean wave periods for the three central waves are $(T_{-1}, T_0, T_1) = 1.3$, while the outlying waves are $(T_{-3}, T_{-2}, T_2, T_3) < 1.1$. Since the expected value of the wave periods for a linear random process would be unity, by definition, we find again that the three central waves (W_{-1}, W_0, W_1) are qualitatively different from the other waves. We also note that $T_0 = 1.29$ is appreciably smaller than $(T_{-1}, T_1) = 1.34$.

The mean wave steepness for the highest wave is $S_0 = a_0k_0 = 0.2$, while the other waves are about 0.14. (The wave steepness for the highest Stokes waves is 0.443.) So, the highest wave turns out to be also the steepest, in the mean, a very significant feature indeed.

Taking together the above observed properties of both wave heights and wave periods, we may suggest that each extreme wave group, in the mean, contains three waves. Furthermore, if we use a threshold wave height $H_c = H_{1/3} = 4m_0^{1/2}$ for determining a wave group, then the mean (run) group length of the extreme wave group would be slightly less than 2.5. (Since $H_{-1} = 3.86$ and $\overline{H_{+1}} = 3.93$ are only slightly smaller than the threshold value of 4.0, W_{-1} and W_{+1} will have slightly less than 50% of chance exceeding the prescribed threshold.) This estimated value is thus in good agreement with the value of 2.36 found by Goda (1976). The mean wave group length for EWG is considerably larger than that of regular wave groups ($\approx$1.5) (Goda, 1976; Su and Bergin, 1983).

Next, we shall examine the effects of spectral bandwidth on the above group characteristics by comparing cases (a), (b) and (c) against (d) in Table 1. As the spectral bandwidth becomes narrower, i.e., Q_p increasing from 1.0 to 4.0, or from case (a) to case (c), H_{+1} is found to increase from about 3.2 to 4.4, and H_0 increases from 6.28 to 6.35, and S_0 increases from 0.186 to 0.208, and T_0 decreases from 1.37 to 1.23. Therefore, one may conclude that the specific features noted for the extreme wave group from the entire data set (case d) becomes even more prominent as the wave power spectra becomes narrower as parameterized by Q_p. In other words, for narrower spectral bandwidth, the mean group length will become longer, and the highest waves become steeper, and, increasingly, the wave period of the central waves become shorter than those of the surrounding waves.

It is well known that power spectra have narrower bandwidth in the fetch-limited growth stages than in either the fully saturated stages or in decaying stages. (Here we have excluded the consideration of swells which could have extremely sharp narrow spectra for which the beat phenomenon can be suitably applied.) One thus expects to find most clearly the unique feature of the extreme wave group in the rapidly growing sea.

Table 2 gives the statistics of the extreme wave groups as given in Table 1 with the exception that the wave records are grouped by the spectral bandwidth parameter, ν . We found much the same group characteristics as noted in Table 1 in all three cases, (a), (b) and (c), for ν = 0.4-0.5, 0.5-0.6, and 0.6-0.8, respectively, but the effects of spectral bandwidth based on ν is not as clear as based on Q_p. One possible reason for this relative sensitivity between the classifications of Q_p and ν is as follows: by its definition, Q_p emphasizes the contribution from the peak of the spectrum due to the square of E(f), while ν , by its definition, considers equally the contribution from the higher frequency range due to the factor of f^2 in the m_2 computation. In our determination of individual waves by the zero-crossing method which acts roughly as a low-pass filter, these individual waves thus obtained are primarily energy-containing

Table 2. Extreme Wave Group Analysis for Gulf of Mexico Wave Data. $H_{1/3} \geq 2.0$ m. Based on classification.

	j = -3	-2	-1	0	+1	+2	+3
(a) $\bar{H}_j$	2.586	2.823	3.971	6.320	3.993	2.739	2.404
$\bar{T}_j$	1.046	1.085	1.324	1.228	1.311	1.088	1.029
$\bar{S}_j$	0.131	0.132	0.122	0.213	0.125	0.140	0.129
(b) $\bar{H}_j$	2.537	2.701	3.703	6.233	3.780	2.532	2.429
$\bar{T}_j$	1.068	1.101	1.335	1.316	1.340	1.086	1.012
$\bar{S}_j$	0.148	0.120	0.109	0.169	0.120	0.128	0.140
(c) $\bar{H}_j$	2.310	2.653	3.895	6.347	3.976	2.674	2.478
$\bar{T}_j$	1.018	1.106	1.327	1.389	1.418	1.190	0.972
$\bar{S}_j$	0.140	0.135	0.125	0.177	0.119	0.122	0.159
(d) $\bar{H}_j$	2.557	2.653	3.982	6.420	4.098	2.890	2.429
$\bar{T}_j$	0.965	1.073	1.320	1.359	1.420	1.143	1.000
$\bar{S}_j$	0.139	0.145	0.133	0.193	0.122	0.139	0.156

lower-frequency waves centering around the spectral peak. Hence, Q_p will be a more sensitive parameter than ν in our analysis.

In Table 3(a,b), we show the statistical results of EWG from the wave data from the Forties Field in the North Sea for $H_{1/3} \geq 2.5$ m. Similarly, in Table 4(a,b) we show the results for the same set of wave data, except only for those records with $H_{1/3} \geq 5.0$ m, in order to emphasize the truly extremely large storm waves. Comparing the results from both Table 3 and 4, with those from Table 1, we have found very little difference among them.

Interpretation of Large Waves and Extreme Wave Groups

Several unique features of the extreme wave group, particularly for the case with narrower spectral bandwidth normally expected for growing seas, prompt us to consider a hypothesis that these extreme wave groups may be caused by nonlinear wave instabilities, rather than simply due to the linear beat phenomenon. In this section we shall present some reasoning in support of this hypothesis.

The side-band instability is characterized by two perturbations with the frequencies f_1 =r $f_0 - \Delta f$ and $f_2 = f_0 + \Delta f$, where f_0 is the original primary (unperturbed) freqeuncy of a wave train and/or wave

Table 3(a). Extreme Wave Group Analysis for North Sea Wave Data. $H_{1/3} \geq 2.5$ m.

1.00 <= QP <= 2.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 33)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.105	2.204	3.065	6.363	3.277	2.391	2.282
STD DEV	1.042	1.105	1.213	1.069	1.018	1.265	1.324
AVG PERIOD	0.911	0.994	1.272	1.383	1.361	1.039	1.080
STD DEV	0.348	0.414	0.472	0.424	0.460	0.419	0.498
AVG SLOPE	0.036	0.032	0.028	0.047	0.026	0.030	0.032
STD DEV	0.019	0.018	0.017	0.019	0.016	0.015	0.027
AVG STEEPNESS	0.112	0.102	0.087	0.149	0.083	0.094	0.100
STD DEV	0.060	0.056	0.053	0.059	0.051	0.048	0.080

2.00 <= QP <= 3.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 511)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.321	2.464	3.469	6.011	3.443	2.563	2.448
STD DEV	1.180	1.237	1.149	0.578	1.215	1.291	1.255
AVG PERIOD	0.991	1.027	1.265	1.247	1.243	1.059	1.036
STD DEV	0.395	0.385	0.340	0.200	0.343	0.386	0.382
AVG SLOPE	0.035	0.034	0.032	0.053	0.032	0.032	0.032
STD DEV	0.021	0.020	0.026	0.023	0.019	0.017	0.017
AVG STEEPNESS	0.111	0.108	0.099	0.165	0.100	0.101	0.101
STD DEV	0.064	0.063	0.056	0.059	0.054	0.055	0.054

3.00 <= QP <= 4.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 415)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.452	2.883	3.918	6.054	3.836	2.859	2.569
STD DEV	1.240	1.332	1.252	0.563	1.131	1.267	1.284
AVG PERIOD	1.017	1.104	1.286	1.216	1.299	1.118	1.046
STD DEV	0.358	0.356	0.271	0.130	0.249	0.345	0.333
AVG SLOPE	0.037	0.036	0.036	0.058	0.034	0.035	0.035
STD DEV	0.023	0.018	0.020	0.015	0.018	0.019	0.018
AVG STEEPNESS	0.117	0.113	0.113	0.183	0.107	0.110	0.112
STD DEV	0.068	0.057	0.058	0.047	0.048	0.058	0.058

Table 3(b). Continuation of Table 3(a).

4.00 <= QP <= 5.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 107)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.496	3.040	3.980	5.972	3.902	3.103	2.689
STD DEV	1.180	1.169	1.077	0.515	1.111	1.405	1.279
AVG PERIOD	1.046	1.148	1.273	1.181	1.284	1.123	1.117
STD DEV	0.329	0.282	0.216	0.101	0.176	0.312	0.288
AVG SLOPE	0.034	0.035	0.036	0.061	0.035	0.036	0.032
STD DEV	0.016	0.018	0.014	0.015	0.015	0.016	0.016
AVG STEEPNESS	0.107	0.110	0.114	0.191	0.109	0.112	0.100
STD DEV	0.051	0.058	0.045	0.048	0.046	0.051	0.051

5.00 <= QP <= 6.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 24)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	3.112	3.279	4.525	5.962	4.314	3.219	2.699
STD DEV	1.250	1.236	0.959	0.532	0.781	1.339	1.235
AVG PERIOD	1.106	1.191	1.249	1.195	1.308	1.198	1.184
STD DEV	0.288	0.217	0.151	0.092	0.111	0.300	0.257
AVG SLOPE	0.037	0.033	0.042	0.059	0.036	0.033	0.028
STD DEV	0.016	0.013	0.013	0.013	0.011	0.015	0.013
AVG STEEPNESS	0.117	0.104	0.131	0.184	0.113	0.103	0.087
STD DEV	0.051	0.042	0.041	0.039	0.034	0.048	0.042

1.00 <= QP <= 6.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 1090)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.399	2.690	3.701	6.033	3.652	2.738	2.518
STD DEV	1.208	1.289	1.214	0.590	1.182	1.310	1.273
AVG PERIOD	1.007	1.071	1.273	1.232	1.273	1.090	1.052
STD DEV	0.373	0.366	0.307	0.182	0.299	0.365	0.359
AVG SLOPE	0.036	0.035	0.034	0.056	0.033	0.034	0.033
STD DEV	0.021	0.019	0.022	0.020	0.018	0.018	0.018
AVG STEEPNESS	0.113	0.110	0.106	0.174	0.103	0.105	0.105
STD DEV	0.064	0.060	0.056	0.054	0.051	0.056	0.056

Table 4(a). Extreme Wave Group Analysis for North Sea Wave Data. $H_{1/3} \geq 5$ m.

1.00 <= GP <= 2.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 27)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.609	2.728	3.119	6.236	3.310	2.159	2.338
STD DEV	1.420	1.382	1.850	1.176	1.494	1.170	1.192
AVG PERIOD	1.134	1.145	1.179	1.377	1.143	1.015	1.031
STD DEV	0.590	0.723	0.510	0.303	0.494	0.493	0.519
AVG SLOPE	0.039	0.041	0.062	0.073	0.050	0.039	0.039
STD DEV	0.025	0.024	0.117	0.120	0.043	0.027	0.019
AVG STEEPNESS	0.121	0.129	0.122	0.156	0.148	0.121	0.123
STD DEV	0.080	0.075	0.106	0.090	0.106	0.083	0.060

2.00 <= GP <= 3.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 203)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.311	2.567	3.400	6.064	3.187	2.578	2.487
STD DEV	1.193	1.296	1.418	0.714	1.260	1.176	1.249
AVG PERIOD	1.018	1.045	1.242	1.286	1.211	1.076	1.044
STD DEV	0.400	0.386	0.387	0.221	0.360	0.378	0.388
AVG SLOPE	0.037	0.039	0.040	0.077	0.038	0.039	0.041
STD DEV	0.021	0.022	0.048	0.196	0.031	0.032	0.073
AVG STEEPNESS	0.115	0.121	0.115	0.172	0.114	0.118	0.115
STD DEV	0.066	0.069	0.073	0.066	0.072	0.072	0.064

3.00 <= GP <= 4.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 133)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.528	2.903	3.738	6.035	3.789	2.924	2.502
STD DEV	1.235	1.224	1.207	0.608	1.182	1.177	1.317
AVG PERIOD	1.058	1.090	1.246	1.230	1.259	1.166	1.029
STD DEV	0.346	0.321	0.275	0.134	0.268	0.320	0.411
AVG SLOPE	0.036	0.040	0.038	0.065	0.038	0.049	0.051
STD DEV	0.018	0.023	0.018	0.065	0.017	0.171	0.138
AVG STEEPNESS	0.115	0.125	0.120	0.188	0.119	0.109	0.125
STD DEV	0.057	0.064	0.057	0.045	0.052	0.057	0.074

Table 4(b). Continuation of Table 4(a).

4.00 <= QP <= 5.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 40)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.730	3.107	3.695	5.923	3.840	3.006	2.524
STD DEV	1.066	1.281	1.202	0.722	1.196	1.172	1.125
AVG PERIOD	1.137	1.168	1.328	1.235	1.277	1.148	1.086
STD DEV	0.316	0.320	0.210	0.115	0.199	0.298	0.273
AVG SLOPE	0.037	0.037	0.034	0.061	0.037	0.038	0.033
STD DEV	0.024	0.017	0.015	0.015	0.013	0.018	0.013
AVG STEEPNESS	0.117	0.115	0.107	0.191	0.115	0.120	0.102
STD DEV	0.074	0.053	0.047	0.047	0.041	0.058	0.040

5.00 <= QP <= 6.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 11)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.987	3.311	3.658	5.870	3.591	3.051	2.386
STD DEV	0.972	1.405	1.139	0.433	0.854	0.907	0.858
AVG PERIOD	1.194	1.185	1.265	1.183	1.283	1.245	1.113
STD DEV	0.258	0.140	0.090	0.073	0.075	0.153	0.283
AVG SLOPE	0.035	0.037	0.036	0.068	0.035	0.034	0.035
STD DEV	0.012	0.015	0.012	0.010	0.010	0.016	0.019
AVG STEEPNESS	0.111	0.116	0.115	0.214	0.111	0.108	0.111
STD DEV	0.037	0.048	0.036	0.032	0.031	0.051	0.059

1.00 <= QP <= 6.00

LARGEST WAVE ONLY (TOTAL NUMBER OF WAVE GROUPS = 414)

	WAVES -3	-2	-1	0	1	2	3
AVG HEIGHT	2.458	2.757	3.525	6.047	3.462	2.716	2.483
STD DEV	1.217	1.298	1.374	0.720	1.272	1.192	1.248
AVG PERIOD	1.055	1.081	1.248	1.266	1.230	1.112	1.044
STD DEV	0.391	0.391	0.347	0.197	0.328	0.362	0.394
AVG SLOPE	0.037	0.039	0.040	0.071	0.038	0.042	0.043
STD DEV	0.020	0.022	0.047	0.146	0.027	0.100	0.094
AVG STEEPNESS	0.115	0.122	0.116	0.179	0.118	0.115	0.117
STD DEV	0.064	0.066	0.068	0.061	0.066	0.066	0.065

packet under consideration. Furthermore, the Δf is related to the wave steepness, a_0k_0, of the original primary waves by

$$\Delta f = a_0k_0f_0. \qquad (8)$$

Experimental observation under laboratory-controlled conditions (Su, 1982 and 1984) showed that at the maximum modulation due to the side-band instability the amplitudes for the two side-band components, a_1 and a_2, are approximately equal to each other, and further related to the amplitude, a_0, of the primary wave at the particular moment by

$$a_1^2 = a_2^2 = 1/2a_0^2. \qquad (9)$$

Hence, the time series of the surface displacement, $\eta(t)$, of the wave train under the maximum modulation of the side-band instability can be expressed as

$$\begin{aligned}\eta(t) &= a_0 \sin(2\pi f_0 t) + a_1 \sin(2\pi f_1 t) + \\ &\qquad a_2 \sin(2\pi f_2 t) \\ &= a_0 \sin(2\pi f_0 t) \\ &\quad + \sqrt{1/2}\, a_0 \sin[2\pi(1-a_0k_0)f_0t] \\ &\quad \sin[2\ (1+a_0k_0)f_0t] \\ &= a_0\ [1+\sqrt{2}\cos(2\pi a_0k_0f_0t)]\ \sin(2\pi f_0 t).\end{aligned} \qquad (10)$$

Note that the expression in the square bracket is the slower varying envelope of the wave group, which have a repitition period equal to $(a_0k_0)^{-1}$ number of primary wave periods. Now, the mean wave steepness for the case (c) with $3.0 \le 4.0$ in Table 1 is equal to $S = 1/3\ (0.133 + 0.208 + 0.128) = 0.156$. If we take the average $S = 0.16$, then the repetition length of the wave group is nearly 6 waves. The corresponding wave heights for the seven waves centering around the maximum of the wave envelope are then given by

$$\begin{aligned} H_0 &= a_0(1 + \sqrt{2}\cos 0^0) = 2.41\ a_0 = H_0 \\ H_{-1}, H_1 &= a_0(1 + \sqrt{2}\cos 60^0) = 1.707\ a_0 = 0.7\ H_0 \\ H_{-2}, H_2 &= a_0\ (1 + \sqrt{2}\cos 120^0) = 0.293\ a_0 = 0.121\ H_0 \\ H_{-3}, H_3 &= a_0\ (1 + \sqrt{2}\cos 180^0) = 0.41\ a_0 = 0.170\ H_0 \end{aligned} \qquad (11)$$

The most significant feature of the distribution of the above wave heights is the considerable smallness of $(H_{-3}, H_{-2}, H_2, H_3)$ in comparison to (H_{-1}, H_0, H_1). This feature agrees remarkably well with the observation of the extreme wave group consisting of three high waves, in the mean, as presented in the last section. The ratio $(H_{-1},$

$H_1)/H_0$ = 4.435/6.352 = 0.698 = 0.7 for case (d) in Table 1 agrees almost exactly with the (H_{-1}, H_1) = 0.7 H_0 of the nonlinear model. The observed fact that $T_0 < (T_{-1}, T_1)$ in Table 1 is also consistent with the experimental observation in the side-band instability.

On the other hand, let's consider a wave train consisting of two sine waves with equal amplitudes but with a difference in frequencies by an amount of $f = a_0 k_0 f_0$ the same as in the side-band instability. The time series of the corresponding surface displacement for the beat phenomenon will be

$$(t) = a_0 \sin(2\pi f_1 t) + a_0 \sin(2\pi f_2 t)$$

with $f_1 = f_0 - 1/2\Delta f$, and $f_2 = f_0 + 1/2\Delta f$. Hence,

$$(t) = a_0[2 \cos(\pi a_0 k_0 f_0 t)] \sin(2\pi f_0 t). \quad (12)$$

The envelope for the beat phenomenon is a simple cosine shape with a repetition period twice longer than the period for the side-band modulation. With the same value of $a_0 k_0$ = 0.16, the seven waves centering around the maximum of the envelope are, approximately,

$$\begin{aligned} H_0 &= 2a_0 \cos 0^0 = 2a_0 = H_0 \\ (H_{-1}, H_1) &= 2a_0 \cos 30^0 = 1.732a_0 = 0.866\, H_0 \quad (13) \\ (H_{-2}, H_2) &= 2a_0 \cos 60^0 = a_0 = 0.5\, H_0 \\ (H_{-3}, H_3) &= 2a_0 \cos 90^0 = 0 \end{aligned}$$

Note that $(H_{-2}, H_2) = 0.5\, H_0$ in this case is much larger than the corresponding values for the side-band instability. Of course, when Δf used smaller than the $a_0 k_0 f_0$, the value of H_{+2} will increase, and vice versa.

Conclusions

We have analyzed two large sets of ocean wave records collected in storm conditions for the structure of wave groups each containing the highest wave height in that wave record, called the extreme wave groups. We found that the extreme wave group predominantly consists of three high waves with heights greater than the significant wave height, and with the mean periods equal about 1.3 times the average period for the entire collection of individual waves determined by the standard up zero-crossing method, and with the mean wave steepness for the highest wave equal to about 0.20. Furthermore, as the spectral bandwidth becomes narrower, these features of the extreme wave groups become more prominent. Based on a previous knowledge of experimental studies on nonlinear side-band instability of finite-amplitude waves, the extreme wave groups are found to be explainable better as the manifestation of the maximum modulation under this type of nonlinear instability, than as the simple linear beat phenomenon.

The experimental results, statistical analyses of field data and theoretical findings, led to suggest that spilling wave breaking for large energy-containing waves near the peak frequency can occur due to nonlinear coupling of instabilities for average wave steepness near 0.14 to 0.18.

In summary, large giant waves, wave groups and wave breaking are most likely the different aspects of the same dynamical process due to the intrinsic instabilities of nonlinear steep waves.

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