ON THE GROWTH OF WATER WAVES BY TURBULENT WIND

H. Mitsuyasu and T. Kusaba
Professor and Research Associate

Abstract

The growth of water waves by turbulent wind is studied on the basis of typical results of recent studies. It is shown that the energy input from wind to a spectral component is not much affected by the existence of the other spectral components, and the dimensionless growth rate $\beta/f$ is uniquely determined by $u^*/c$ if the friction velocity $u^*$ measured over the water surface is used for the analysis. It is also shown that steep monochromatic waves without wind action show very complicated spectrum, but, under the wind action, they tend to be a continuous spectrum which has a spectral peak near the frequency of the lower side band and satisfy the 3/2 power law.

1. Introduction

Since the very famous studies of Miles (1957) and Phillips (1957), great many studies have been made on the growth of water waves by turbulent wind. Particularly many reliable results of field and laboratory measurements have been reported on the growth rate of water waves by turbulent wind (Snyder et al. 1981, Plant 1982, Hsiao & Shemdin 1983).

However, there still remains several fundamental problems that are not fully understood. For example, the following questions are still difficult to answer; Is the growth of dominant wave affected by high frequency waves overlapping on the dominant wave ? Is the energy transfer from wind to a certain spectral component affected by the existence of the other spectral components ? What is the role of wave breaking in the process of wave growth ?

In order to answer these questions and to clarify the fundamental process of wind-induced growth of water waves, discussions are made on the basis of typical results of our recent studies. Although individual studies have been published elsewhere (Mitsuyasu & Honda 1982, Kusaba & Mitsuyasu 1984, Kusaba & Mitsuyasu 1986), important results are combined and discussed in the present paper.

1) Research Institute for Applied Mechanics, Kyushu University, Kasuga 816, Japan.
2. Experiments

The following three laboratory experiments have been conducted;

Experiment 1; The wind-induced growth of mechanically-generated monochromatic waves with relatively small steepness has been measured in a wind wave flume both for ordinary tap water and for water containing a surfactant (Mitsuyasu & Honda 1982). In the former water, wind waves are generated and overlap on the mechanically-generated waves, while in the latter water, no wind waves develop on the surface of the mechanically-generated waves. Thus we can measure the wind-induced growth of monochromatic waves with or without the effects of wind waves overlapping on the monochromatic waves. Thirty different waves were used in the experiment; their periods were in a range 0.6 sec ~1.3 sec and their steepness without wind action were in a range 0.01 ~0.06. The wind speed was changed successively as 5, 7.5, 10, 12.5 (m/s).

Experiment 2; Similar measurements have been made for mechanically-generated composite waves which have two different frequency components to clarify the effects of other spectral components on the energy transfer from wind to a certain spectral component (Kusaba & Mitsuyasu 1984). As shown in Table-1 two kinds of composite waves were used for the experiment. One has two component waves with much different period, i.e., T =0.6 sec and T =1.2 sec. Another one has two component waves with similar periods, i.e., T =0.6 sec and T =0.7 sec.

Experiment 3; The wind-induced growth of unstable monochromatic waves with relatively large steepness has been measured in the same wind-wave flume for ordinary tap water to clarify the effect of nonlinear instability on the wind-induced growth of water waves (Kusaba & Mitsuyasu 1986). The properties of the waves without wind action are shown in Table-1. The wave (7)(T =0.5 sec, H =1.7 sec, H/L=0.044) is a stable one which is used for the comparison, e.g., to verify the empirical relation derived from the results of the Experiment 1, or to compare its growth properties with those of the unstable waves (8) and (9).

All of the measurements have been done in a wind-wave flume 0.8 m high, 0.6 m wide and with a usual test-section length of 15 m (Fig.1). Waves were measured with resistance type wave gauges at eleven stations (eight stations for Experiment 3) of 1 m interval. Experimental conditions are summerized in Table-1, where Ur is a reference wind speed measured at the inlet of the test section. Power spectra of every waves were obtained through a FFT method and the spectral energy of the fundamental component was analyzed in the studies. The friction velocity of the wind was
determined from the vertical wind profiles measured above the water surface in the test section. More detailed descriptions of the experiment are referred to the individual papers (Mitsuyasu & Honda 1982, Kusaba & Mitsuyasu 1984, Kusaba & Mitsuyasu 1986).

Fig. 1  Schematic diagram of wind-wave flume (units in m).

| TABLE - 1 |
| EXPERIMENTAL CONDITIONS |

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Water depth d(m)</th>
<th>Wind speed Ur(m/s)</th>
<th>Waves</th>
<th>H(cm)</th>
<th>H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.335</td>
<td>5, 7.5, 10, 12.5</td>
<td>0.6-1.3, 0.01, 0.06</td>
<td>thirty different waves</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.355</td>
<td>7.7</td>
<td>(1) 0.6, 2.0, 0.036 (2) 1.2, 2.0, 0.011 (3) composite waves of (1) and (2) (4) 0.6, 1.1, 0.02 (5) 0.7, 1.5, 0.02 (6) composite waves of (4) and (5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.320</td>
<td>10</td>
<td>(7) 0.5, 1.7, 0.044 (8) 0.5, 3.2, 0.082 (9) 0.5, 4.1, 0.105</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. RESULTS AND DISCUSSIONS

3.1 Wind-induced growth of monochromatic waves with or without overlapping short waves.

Monochromatic waves superimposed by high frequency wind waves grow exponentially with fetch. Their exponential growth rates, $\beta$, show a quadratic relation to the friction velocity of the wind, $u^*$,

$$\beta / f = 0.34 (u^*/c)^2,$$

where $f$ is the frequency of the waves and $c$ is the corresponding phase velocity. Almost the same relation holds for the monochromatic waves with smooth surface, i.e., without overlapping wind waves, if the measured friction velocity of the wind is used for the relation (Fig. 2).

![Fig. 2 Comparison of $\beta / f$ for tap water with that for water containing surfactant. The solid line is (1) and the broken line is (2).](image)

For the latter case, the measured growth rate is smaller than that for the former case, but the friction velocity of the wind is also small, and the same relation between $\beta / f$
and $u_*/c$ holds for both cases. A kind of $u_*/c$ similarity is satisfied for the growth of water waves by turbulent wind. We can say, from these results, that the effect of steep short waves overlapping on longer waves are to increase the growth rate of the longer waves through the increase of the downward momentum flux of the wind.

In Fig. 2 the broken line corresponds to the empirical relation from Snyder et al. (1981):

$$\beta / f = 0.04u_*/c - 1.7 \times 10^{-3}, \quad (2)$$

which is obtained from their original form for the growth-rate parameter $\Im \gamma$, $\Im \gamma = (0.2 - 0.2)(U_s/c - 1)$, using the relation $\beta / f = 2\pi(p_\alpha/p_\omega) \Im \gamma$ and assuming $p_\alpha/p_\omega = 1.2 \times 10^{-3}$ and $U_s = 23u_*$. Figure 2 shows that the empirical relation (1) gives slightly larger growth rate than that given by (2) of Snyder et al. (1981) in a region near $u_*/c = 0.1$.

3.2 Wind-induced growth of composite waves.

Figure 3 shows the growth rates of the composite waves, where the growth rates of each component waves alone are also shown with the same symbol. It can be seen that the
growth rates of component waves of the composite waves follow approximately to the empirical relation (1). It should be noted, however, that one of the data for the surfactant water deviate greatly from the empirical relation (1). This data correspond to the wave (5)\((T=0.7 \text{ sec})\) coexisting with the wave (4)\((T=0.6 \text{ sec})\) for the water containing surfactant*). For this composite wave, since the periods of the component waves are fairly close, the waves show beat structures and their amplitude increase at the loop. Therefore, wave instability or small scale breaking might have happened, which has decreased greatly the growth rate.

Although further studies are needed to clarify the deviation of some data, we can say that the energy transfer from the wind to a certain spectral component is not much affected by the other spectral components if the wave steepness is not large. This is a reason why the empirical relation (1) for monochromatic waves is similar to the relation of Plant (1982), that has been obtained by using various wave data including the waves with continuous spectrum.

3.3 Wind-induced growth of unstable waves

Steep monochromatic waves become gradually unstable even without wind action. Since Benjamine & Fair (1967) made a pioneering study on the nonlinear instability of the Stokes waves, great many studies have been done on this problem; Comprehensive review has been given by Yuen & Lake (1982). However, many of the previous studies are concerned with the instability of steep waves without wind action.

In Fig.4, the left band figure shows the evolution of the spectra of steep unstable waves without wind action and the right hand side shows that of steep unstable waves under wind action. The thin smooth curves in the figure correspond to the wind wave spectra measured independently at each fetch. From the top to the bottom, fetches are 2.51, 4.51, 6.51, 8.61, 10.51 (m) respectively.

The spectra of the steep unstable waves without wind action show many spikes which are attributed to the fundamental frequency component, higher harmonics, and their side bands generated by the wave instability. Under the wind action, however, the higher harmonics and their side band are masked with wind wave spectrum, even the side bands of the fundamental frequency component are obscured, and the wave spectrum tends to be a continuous spectrum which has a

*) The similar deviation also happened for the same composite waves on a tap water. This data is out of this figure.
Fig. 4  Evolutions of power spectra of steep unstable waves with (right figures) or without (left figures) wind action. Thin smooth curves correspond to the spectra of pure wind waves at each fetch.
spectral peak near the lower side band of the fundamental frequency component. Further development of the wave spectrum is almost the same to that of pure wind waves. These process of the transition to the continuous spectrum are considered to be largely attributed to the effect of wave breaking.

In order to clarify the change of the spectral energy, the wave spectrum was divided into five frequency regions as shown in Fig. 5, and the spectral energy in each region is studied.

![Figure 5](image)

**Fig. 5** A schematic figure for the characteristic frequency region of the power spectrum of unstable waves.

In Fig. 5, \( E_1 \) is the energy in the low frequency region, \( E_2 \) the energy of the lower side band, \( E_3 \) the energy of the fundamental frequency component, \( E_4 \) the energy of the higher side band, and \( E_s \) the energy in the high frequency region.

The change of the spectral energy in each frequency region is shown in Fig. 6. It can be seen that the energy of the fundamental frequency component \( E_2 \) decreases gradually with the increase of fetch even under the wind action, while the total wave energy \( E \) shows a nearly constant value. At short fetch, \( E_2 \) and \( E_4 \) are almost the same and increase similarly with the fetch. With the increase of fetch, however, \( E_4 \) reaches to a saturated value, while \( E_2 \) continues to increase with the fetch. The wave energy at the high frequency region, \( E_s \), shows a constant value independent of fetches reflecting a saturation of the high frequency wave spectrum for a fixed wind.
Fig. 6 The change of the energy in the characteristic frequency regions of wave spectra under wind action for wave (9), $H/L=0.105$.

Fig. 7 The change of the normalized wave energy.
It is interesting that the total wave energy $E$ shows roughly a constant value within the fetch of the present study. However, it should be noted that this study is concerned with transitional phenomena from monochromatic waves to random waves, which are caused by the wave instability and the wind action. After the transition to the random waves, the waves will develop in the same way as wind waves. Thus, the total wave energy $E$ should increase with fetch, while the dimensionless wave energy $E_{fm}/gu^*$ is expected to be constant.

In fact, Fig. 7 shows the saturation of the values of $E_{fm}/gu^*$, and the asymptotic value $E_{fm}/gu^* = 2.55 \times 10^{-4}$ is very close to the value $2.14 \times 10^{-4}$ which has been obtained by Toba (1978) as the $3/2$ power law for developing wind waves.

4. Conclusions

The following conclusions can be drawn from the present study;

(i) The growth rate of stable monochromatic waves are given by (1) irrespective of the existence of the high frequency waves, if the measured friction velocity $u^*$ is used in the relation. This is because the high frequency waves overlapping on the monochromatic waves increase both the friction velocity of the wind and the growth rate of the monochromatic waves.

(ii) The growth rate of a component wave of the composite waves is not much different from that of monochromatic waves with the same frequency, if the wave steepness is not large.

(iii) A fundamental frequency component of steep unstable waves decreases with the increase of the fetch even under the wind action, which is considered to be attributed to the wave breaking. However, the lower and higher side bands of the fundamental frequency component increase with the fetch, though the latter tends to be saturated at some fetch.

These properties are quite similar to those of steep unstable waves without wind action. In later stage, however, the spectrum of steep unstable waves under wind action tend to be continuous spectrum which is quite similar to the wind-wave spectrum.

Acknowledgement

The authors are grateful to Mr. K. Marubayashi and Mr. M. Ishibashi for their patient supports during the experiments, and to Miss. M. Hojo for her assistance in preparing the manuscript. This study is a part of Oceanographic Research Project entitled "Measurements and Simulation of Ocean Environment" and supported by the Ministry of Education.
References


