CHAPTER 12

SHOALING SURFACE GRAVITY WAVES: A BISPECTRAL ANALYSIS J.C. Doering* and A.J. Bowen*

Abstract

Nonlinearities (wave-wave interactions) play a vital role in many aspects of nearshore dynamics, such as wave shoaling and breaking, wave forces, wave-current interactions, radiation stress effects, and sediment transport. The importance of nonlinearities in the nearshore region cannot be overemphasized. At present, however, there is no wave theory that adequately accounts for these interactions, and field observations are sparse. Herein, the bispectrum is used to investigate the temporal and spatial variation of wave-wave interactions in cross-shore velocity for shoaling surface gravity waves in several nearshore environments. The implications for sediment movement of the sign of the observed wavewave interactions for both the real part of the velocity bispectrum (which is related to the skewness of the horizontal asymmetry) and the imaginary part of the velocity bispectrum (which is related to the skewness of the temporal derivative) are discussed. A parameterization is given for the amplitude and phase evolution of the self-self sum interactions within the wind-wave peak for both planar and barred nearshore topography. The results of this paper underline the potential importance of infragravity wave energy in determining nearshore morphology.

1. Introduction

There have been a lot of papers written about the statistics and nonlinearities of sea surface elevation and slope (Srokosz & Longuet-Higgins, 1986; Longuet-Higgins, 1982, and the many references therein). Most of this literature, however, pertains only to deep water where nonlinearities are assumed to be weak in order for the problem to remain reasonably mathematically tractable. Similar theory has not been completed for shallow water as the complexities are formidable. This is particularly true near breaking where nonlinearities are generally not considered to be weak. Until the collection of data by field programs such as the Nearshore Sediment Transport Study (NSTS), and most recently the Canadian Coastal Sediment Study (C^2S^2), observations were sparse.

As waves propagate into shallow water they undergo a dramatic transformation and eventually break. Freilich and Guza (1984) have shown that a nonlinear model including triad interactions across the wind-wave frequency band can accurately predict the Fourier Coefficients (amplitude and phase) of the wave field through the shoaling region. Elgar and Guza (1985) used the bispectrum to examine the nonlinear interactions in the observed pressure field of shoaling surface gravity waves. In addition, Elgar and Guza (1986) used Freilich and Guza's nonlinear equations to model bispectral evolution through the shoaling region. The model results were qualitatively similar to the observations. The bispectral quantities examined by Elgar and Guza (1985, 1986) were plotted against depth. However, it is clear from inspection of Elgar and Guza (1985, 1986), that depth cannot provide a general parameterization of bispectral evolution. Such a parameterization would be useful, especially for modelling sediment transport since the models of Bowen (1980) and Bailard (1981) require the skewness of the velocity field as an input. Guza and Thornton (1985) computed a variety of velocity moments, including skewness, for some NSTS data, which

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they also plotted as functions of depth. There is, however, no obvious relationship between the depth and most of the quantities they considered.

The objectives of this paper are to investigate the following aspects of the cross-shore velocity field:

- i) the temporal and spatial evolution of triad interactions through the nearshore region
- ii) the sedimentary implications of the observed wave-wave interactions
- iii) the parametric dependence of the salient interactions

2. The Bispectrum

To investigate wave-wave interactions, the bispectrum is used. For a stationary random function of time $\varsigma(t)$, the auto bispectrum is given by the Fourier Transform of the mean third-order product (Hasselmann et al., 1963).

$$B(\omega_j, \omega_k) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} S(\tau_j, \tau_k) e^{-\mathbf{i}(\omega_j \tau_j + \omega_k \tau_k)} d\tau_j d\tau_k$$
(2.1a)

$$S(\tau_j, \tau_k) = E[\varsigma(t)\varsigma(t+\tau_j)\varsigma(t+\tau_k)]$$
(2.1b)

 τ is a lag, ω is a radian frequency, and E denotes an expected or average operator. For digital data the (auto) bispectrum is given by Kim and Powers (1979) as

$$B(\omega_j, \ \omega_k) = A_{\omega_j} A_{\omega_k} A^*_{\omega_{j+k}}$$
(2.2)

where A_{ω_j} represents the complex Fourier Coefficient of radian frequency ω_j and * denotes the complex conjugate.

The bispectrum can be expressed in a normalized form known as the bicoherence (Kim and Powers, 1979).

$$b^{2}(\omega_{j}, \omega_{k}) = \frac{|B(\omega_{j}, \omega_{k})|^{2}}{E[|A_{\omega_{j}}A_{\omega_{k}}|^{2}]E[|A_{\omega_{j+k}}|^{2}]}$$
(2.3)

Note that Cauchy-Schwarz's inequality ensures $b^2 \leq 1$ for (2.3); this is not true for the bicoherence expression given by Haubrich (1965). The variance of bicoherence estimates has been shown to be less than $\frac{2}{4.o.f.}$ (Kim and Powers, 1979).

The 95% level for zero bicoherence is defined by Haubrich (1965).

$$b^2 \simeq \frac{6}{d.o.f.} \tag{2.4}$$

Although the bicoherence indicates the wave-wave interactions that are significant above some chosen level, it does not provide any information regarding the <u>relative</u> contribution of these interactions to the nonlinearity of the record as measured by skewness. This information is, however, provided by the real part of the bispectrum. Integrating the real part of the bispectrum yields an estimate of skewness.

$$E[\varsigma^{\mathbf{3}}(t)] = \sum_{\omega_j} \sum_{\omega_k} \Re e\{B(\omega_j, \ \omega_k)\}$$
(2.5)

Prior to the work of Masudo and Kuo (1981) no physical interpretation had been attached to the imaginary part of the bispectrum. However, they showed that the imaginary part of the bispectrum is related to the <u>vertical</u> asymmetry of the waves. Moreover, Elgar and Guza (1985) showed that the imaginary part is related to a measure of the skewness of the temporal derivative of a time series (i.e., the Hilbert transform).

$$E\left[\left(\frac{\partial\varsigma}{\partial t}\right)^{3}\right] = \sum_{\omega_{j}} \sum_{\omega_{k}} \omega_{j} \omega_{k} \omega_{j+k} \Im m\{B(\omega_{j}, \omega_{k})\}$$
(2.6)

Lastly, from (2.5) and (2.6) it is clear the bispectrum can be written in terms of an amplitude and phase

$$B(\omega_j, \ \omega_k) = |B(\omega_j, \ \omega_k)| e^{i\theta(\omega_j, \ \omega_k)}$$
(2.7a)

where

$$\theta(\omega_j, \ \omega_k) = \tan^{-1} \left\{ \frac{\Im m[B(\omega_j, \ \omega_k)]}{\Re e[B(\omega_j, \ \omega_k)]} \right\}$$
(2.7b)

3. Data Base

The data for this investigation were collected from four nearshore environments: Pte. Sapin, New Brunswick (C^2S^2) , Stanhope Lane, Prince Edward Island (C^2S^2) , Queensland, Nova Scotia, and Leadbetter Beach, California (NSTS). These data provide a range of beach slopes (including a barred environment), wave periods, depths, and (significant) wave heights — see table 1 for a summary. Because of space limitations a detailed analysis of only 1 run (run 52 from Pte. Sapin) is presented. Doering and Bowen (submitted) have shown that a bispectral analysis of cross-shore velocity and pressure for the February 2, 1980 data from Leadbetter are very similar. Therefore, the reader is referred to Elgar and Guza (1985) for a detailed analysis of the Feb. 2 Leadbetter data.

The velocity data from these four environments were collected using 2-axis, 4 centimeter diameter, spherical probe, electromagnetic current meters manufactured by Marsh-McBirney Incorporated (model 512/OEM). Corrections were made for the filter characteristics of the current meter electronics where necessary.

4. Observations & Discussion

Figure 1 shows the beach profile and location of the current meters deployed at Pte. Sapin, New Brunswick during C^2S^2 . The cross-shore velocity spectra for current meters C26 (h = 3.2m, line a), C23 (h = 2.9m, line b), C07 (h = 2.0m, line c) and C16 (h = 1.7m, line d) for run 52 at Pte. Sapin are given in Figure 2. Note the sharpening of the spectral peak and the relative increase in both harmonic and infragravity wave energy as the waves shoal (lines $a \rightarrow d$).

The results of a bispectral analysis of these data are plotted as 3-D plots in Figures 3-6. The following notes apply to all the 3-D plots in this paper:

- 1) For an auto bispectrum only $\frac{1}{8}$ of the (ω_j, ω_k) frequency plane is unique (Kim and Powers, 1979). However, plotting the entire 1st quadrant allows the backside of some of the peaks to be seen. For this reason, the entire 1st quadrant $(\omega_j, \omega_k > 0)$ has been plotted.
- 2) The origin is located at the left corner.
- 3) The two axes defining the frequency plane show the non-radian frequency, f_1 , $f_2 = \frac{\omega}{2\pi}$, and both run from 0 to 0.5 Hertz.
- 4) There is symmetry about a 45° line $(f_1 = f_2)$.

To facilitiate visualizing the bicoherence plots, only the bicoherence values above the 95% level for zero bicoherence ((2.4)) are shown. The "slab" thickness indicates the 95% value. The four panels of figure 3 show the bicoherence plots for current meters C26 (panel 1), C23 (panel 2), C07 (panel 3) and C16 (panel 4) for run 52 at Pte. Sapin. In panel 1,

| Environment /Symbol | Beach slope | Run | Date dd/mm/yy | d.o.f. | T_p [s] | ${f Depth(s)}\ [m]$ | H_s [m] |
|------------------------|----------------|------|------------------|--------------|--------------|---------------------|--------------|
| Pte. Sapin ▲ | 0.005 | 52 | 03/11/83 | 2048 | 6.6 | 3.2-1.7 | .39 |
| | | 62 | 05/11/83 | 1536 | 7.9 | 3.1-1.6 | .69 |
| Leadbetter = | 0.04 | | 02/02/83 | 2 048 | 15.8 | 5.9 - 1.2 | .41 |
| Queensland + | 0.06 | 131 | 26/06/79 | 512 | 8.0 | 1.4 | .16 |
| | | 133 | 26/06/79 | 512 | 8.0 | 1.2 | .20 |
| | | 141 | 26/06/79 | 512 | 8.0 | 1.0 | .20 |
| | | 142 | 26/06/79 | 512 | 8.0 | 0.7 | .15 |
| | | 151 | 26/06/79 | 512 | 8.0 | 0.6 | .13 |
| Stanhope | t | 8.1 | 18/10/84 | 2560 | 5.7 | 2.5 | .91 |
| | • | 8.2 | 19/10/84 | 1536 | 5.5 | 2.7 | .67 |
| | | 8.4 | 19/10/84 | 1536 | 5.2 | 2.3 | .52 |
| | | 9.1 | 19/10/84 | 1536 | 5.0 | 2.1 | .43 |
| | | 10.5 | 24/10/84 | 2560 | 4.9 | 2.2 | .29 |
| | | 11.6 | 25/10/84 | 2560 | 4.7 | 2.5 | .32 |
| | | 12.1 | 25/10/84 | 2560 | 6.6 | 1.7 | .76 |
| | | 12.2 | 26/10/84 | 2560 | 8.5 | 1.8 | .87 |
| | | 13.2 | 26/10/84 | 2048 | 7.5 | 1.8 | .74 |
| | | 14.3 | 27/10/84 | 1536 | 4.7 | 2.0 | .31 |

TABLE 1 - DATA SUMMARY

 \dagger determined using linear theory; for the shore normal transect of sensors on Pte. Sapin and Leadbetter Beaches value given was computed using the deepest sensor.

‡ beach is concave with several shore parallel bars.



FIGURE 1. Beach profile and location of the current meters deployed at Pte. Sapin, New Brunswick.



FIGURE 2. Cross-shore velocity spectra for current meters C26 (a), C23 (b), C07 (c) and C16 (d) deployed for run 52 at Pte. Sapin. d.o.f. = 100.

two (unique) salient peaks are noted. The peak centered at (0.15Hz, 0.15Hz) indicates nonlinear phase coupling between fundamental and first harmonic frequencies of the incident waves; this interaction is hereinafter denoted (f, f). The peak at (0.15Hz, 0.02Hz)indicates coupling between the fundamental and a long wave. Physically, this peak suggests that neighboring frequencies within the fundamental peak interact to form a beat or wave group, which forces a long wave, at the difference frequency; this interaction is hereinafter denoted $(f, \Delta f)$. After propagating ~ 100m further shoreward, over essentially a flat bottom, panel 2 shows that very little additional coupling occurred. This is also evident from figure 2 (lines a and b). However, panel 3 indicates that additional coupling did occur after propagating $\sim 120m$ further shoreward over a gently sloping seabed. In particular, the bicoherence of the (f, f) peak increased, and broadened slightly. In addition, a new peak is present at (f, 2f), indicating coupling between the fundamental and the first harmonic. The shallowest station (panel 4), which is ~ 36m shoreward of CO7 (panel 3), reveals several new interactions. The peaks at (2f, 2f), (f, 3f) and (3f, 3f) indicate sum interactions between the first harmonic and itself, the fundamental and the second harmonic, and the second harmonic with itself, respectively. Of particular interest though, are the peaks at $(2f, \Delta f)$ and $(3f, \Delta f)$. These peaks suggest that interactions within the (forced) harmonic peaks result in (harmonic) groups, which are coupled to a (forced) long wave beneath the group.



FIGURE 3. Bicoherence for current meters C26 (h = 3.2m), C23 (h = 2.9m), C07 (h = 2.0m) and C16 (h = 1.7m) for run 52 at Pte. Sapin. Only values above the 95% level for zero bicoherence (indicated by the thickness of the slab) are shown.

Figure 4 shows the real part of the bispectrum for the same four sensors as figure 3. Inspection of these four panels readily yields several points. First, the observed skewness of these cross-shore velocity time series is essentially due to the (f, f) interaction. Second, the sign of the skewness of the (f, f) interaction is <u>opposite</u> to that of the $(f, \Delta f)$ interaction. These observations of velocity bispectra are consistent with the pressure observations of Hasselmann *et al.* (1963) and Elgar and Guza (1985). For sediment transport this implies that the skewness of the horizontal velocity field arising from the $(f, \Delta f)$ interaction pushes sediment in an opposite direction to the skewness arising from the $(f, \Delta f)$ interaction. The sedimentary significance of these two interactions was noted by Wells (1967), who used Biesel's second-order solution for a discrete number of gravity wave trains to investigate the spatial variation of skewness across a beach. Wells suggested that a neutral line of zero skewness exists separating two regions, one of positive skewness in shallow water (skewed onshore), the other of negative (or seaward) skewness in deep water; he notes that, "sand in deeper water can actually be transported in a seaward direction". This mechanism was recently employed by Shi and Larsen (1983) to explain the seaward transport of fine sand and silt on the continental shelf. It is worth noting the shallow water portion of Wells' results are suspect as he extended his computations into relatively very shallow water where the underlying assumptions of the theory are no longer met.



FIGURE 4. Real part of the bispectrum for current meters C26 (h = 3.2m), C23 (h = 2.9m), C07 (h = 2.0m) and C16 (h = 1.7m) for run 52 at Pte. Sapin.

Figure 5 shows the imaginary part of the bispectrum for the same four sensors as figures 3 and 4. From (2.6) it is clear that for a velocity time series, figure 5 provides information related to the wave-wave interactions that give rise to the skewness of horizontal acceleration. Note that the relative strength of the wave-wave interactions observed from these plots is biased because each imaginary estimate was not multiplied by the respective triple product of radian frequencies; as a result, the strength of the high(er) frequency interactions is deemphasized. For panel 1 the (f, f) interaction is negative, indicating that the horizontal acceleration of this interaction is negatively skewed, therefore suggesting back-

ward tilted waves. This is, of course, not the usual direction that shoaling waves are tilted. As the waves shoaled further (panels 2 to 4) the (f, f) interaction becomes increasingly more positive (i.e., forward tilting). Although partially obscured, the $(f, \Delta f)$ interaction for panels 1 to 3 is negative, indicating that the horizontal acceleration of this interaction is skewed seaward. However, at the shallowest station, panel 4, the $(f, \Delta f)$ interaction of this interaction is positively skewed. This is not the usual direction that the horizontal acceleration of this interaction of this interaction is skewed, which suggests that the amplitude and phase of the Δf Fourier coefficients may include a partially reflected component.



FIGURE 5. Imaginary part of the bispectrum for current meters C26 (h = 3.2m), C23 (h = 2.9m), C07 (h = 2.0m) and C16 (h = 1.7m) for run 52 at Pte. Sapin.

 $F_{I}(Hz)$

 $F_2(Hz)$

F, (Hz)

For progressive waves, in shallow water, it is known that sea surface elevation and the horizontal velocity in the predominant direction of propagation are strongly coherent for wind-wave frequencies (Guza and Thornton, 1980). Skewness of the horizontal acceleration resulting from vertical asymmetry of the waves, therefore, is related to the skewness of the vertical velocity. Vertical asymmetry of the waves may be an important mechanism for suspended sediment transport, since an upward skewed vertical velocity (under forward tilting waves) could balance gravitational settling, thereby maintaining suspension for advection by the mean flow (e.g., horizontal skewness).

F₂(Hz)

To parameterize bispectral evolution, it is easiest to express the bispectrum as an amplitude and phase. Figure 6 shows the amplitude plots for the four previously considered sensors. Clearly, the (f, f) and $(f, \Delta f)$ interactions dominate these plots. A first-order parameterization of bispectral evolution therefore necessitates parameterizing the amplitude and phase of these interactions through the shoaling region.



FIGURE 6. Amplitude of the bispectrum for current meters C26 (h = 3.2m), C23 (h = 2.9m), C07 (h = 2.0m) and C16 (h = 1.7m) for run 52 at Pte. Sapin.

It can be readily shown that the skewness arising from a second-order Stokes wave (a fundamental and its phase-locked, in phase harmonic) can be parameterized in terms of the Ursell number (Ursell 1953)

$$\lim_{kh \to 0} \frac{\eta^{(2)}}{\eta^{(1)}} \sim \frac{3}{4} \frac{ak}{(kh)^3} = U_r$$

In shallow water the Ursell number can be expressed as

$$U_{r} = \frac{3}{4} \frac{g}{8\pi^{2}} \frac{H_{s}T^{2}}{h^{2}}$$

where g is the acceleration due to gravity, H_s is the significant wave height, T is the period, and h is the local depth. Note that taking the same limit for $\frac{u^{(2)}}{u^{(1)}}$ also yields the

Ursell number. The Ursell number would seem to be a logical quantity to choose for the parameterization of the (f, f) interaction observed in cross-shore velocity. Figure 7 shows the normalized (i.e., divided by the appropriate band integrated variance to the $\frac{3}{2}$ power) amplitude of the (f, f) interaction for run 52, and all the other data listed in table 1, as a function of U_r . Least squares regression yields for this interaction



Nrm. Ampl.
$$(f, f) = .48 + .31 \log(U_r), r^2 = .92$$
 (4.1)

FIGURE 7. Normalized (NRM) amplitude for the self-self sum interaction of the windwave peak as a function of the Ursell number (U_r) for Pte. Sapin run 52 \blacktriangle , run 62 \blacklozenge , Stanhope Lane \bigcirc , Leadbetter Beach =, and Queensland Beach +.

All of the Queensland data, which is strongly dominated by backwash, and the one Stanhope run from breaking wave conditions (circle near lower right of figure 7) were excluded from the regression. The relatively high correlation between these two quantities implies several points. First, the majority of the energy at the first harmonic is forced (ie. free energy would beat down r^2). Second, there is a relative increase in first harmonic energy as the waves shoal. Lastly, the growth of the (f, f) interaction is dependent on depth, but apparently not on slope as U_r contains no beach slope information. Figure 7 also suggests that the relatively long period incident waves, which are commonly observed on the Pacific coast, develop much stronger nonlinearities before breaking than the shorter period waves of the Gulf of St. Lawrence or Atlantic coast.

It can be shown for a wave consisting of a fundamental and a phase-locked harmonic that the value of the (f, f) biphase represents the phase difference, δ_1 , between the fundamental and its first harmonic. Iwagaki and Sakai (1972) give a solution, correct to secondorder, for the evolution of the phase shift δ_1 . Their expression, which is a function of the dimensionless quantity $\frac{\beta}{\sqrt{k_{\infty}h}}$, was found to be poorly correlated with the present data. However, figure 8 shows that the (f, f) biphase was found to be well correlated with the relative depth (i.e., <u>local</u> estimate of kh). The two lines suggest, not surprisingly perhaps, that the evolution of the (f, f) biphase is different for planar (dashed line) and barred (dotted line) beaches. This was not the case for the evolution of the (f, f) normalized amplitude. Least squares regression gives,

planar:
$$\theta(f, f) = 64 + 179 \log(kh), r^2 = .93$$
 (4.2a)

barred :
$$\theta(f, f) = 35 + 211 \log(kh), r^2 = .97$$
 (4.2b)



FIGURE 8. Biphase of the self-self sum interaction, $\theta(f, f)$, of the wind-wave peak as a function of kh for Pte. Sapin run 52 \blacktriangle , run 62 \diamondsuit , Stanhope Lane $\textcircled{\bullet}$, Leadbetter Beach \blacksquare , and Queensland Beach +.

The Queensland data was once again excluded from the (planar environments) regression analysis. The strong backwash, noted previously, is evident in the very different biphase evolution. However, the Stanhope data point which was excluded from (4.1) has been included in (4.2b). This suggests (rather lightly as it is based on only one point) that $\theta(f, f)$ is not affected by wave breaking as strongly as as normalized amplitude was observed to be. The similar evolution of $\theta(f, f)$, over the steep Leadbetter Beach and gently Sloping Pte. Sapin beach suggests that $\theta(f, f)$ is <u>not</u> strongly dependent on beach slope (β). The work of Flick (1981) indicates that $-90 \leq \delta_1 \leq 0$. Therefore, equations (4.2a) and (4.2b) should be restricted to this range.

The other peak to be parameterized is the $(f, \Delta f)$ interaction. The observed evolution of normalized amplitude and phase suggests that the estimated Fourier Coefficients of the low frequency (Δf) component of this interaction are complicated by reflection and possibly other sources of low frequency energy. Clearly, the estimates of amplitude and phase from a sensor beneath a partially reflected wave depends on where the sensor is located (ie. node \rightarrow anti-node). Since discerning the composition of the low frequency energy in these data sets is in itself a formidable task, a parameterization of this interaction is not considered here.

5. Conclusions

Current meter data, collected from four nearshore environments, were used as an input to the (auto) bispectrum to investigate wave-wave interactions in the cross-shore velocity field of shoaling surface gravity waves. Bicoherence values indicate that as shoaling begins coupling occurs first within the wind-wave peak, i.e., (f, f) and $(f, \Delta f)$ interactions. As shoaling progresses coupling occurs between the fundamental-harmonic (f, 2f), and eventually within and between the harmonic peaks, e.g., (2f, 2f), $(2f, \Delta f)$ and (2f, 3f). The real and imaginary parts of the velocity bispectrum indicate that the skewness and asymmetry, respectively, of the (f, f) interaction is usually of opposite sign to the $(f, \Delta f)$ interaction.

Bispectrum amplitudes clearly show that the (f, f) interaction, and to a lesser extent the $(f, \Delta f)$ interaction, are typically much stronger than the other triads. Least squares regression indicates that the normalized amplitude of the (f, f) interaction is well parameterized $(f^2 = .92)$ by the Ursell number for planar and barred environments; the data provide no evidence for any strong return of energy to the fundamental frequency (as necessary, for example, for the Fermi-Pasta-Ulam instability) which might lead to wavelike horizontal variability in the (f, f) interaction, one of the suggested mechanisms for bar formation. A suitable parameterization was not obtained for the smaller amplitude $(f, \Delta f)$ interaction, as the observed amplitudes and phases of this interaction were quite erratic. This suggests that the Fourier Coefficients of the Δf component are contaminated by reflection and possibly other forms of infragravity wave motion. These observations highlight the potential role that low frequency motion may play in determining seabed topography (e.g., the shore parallel bars observed at Stanhope Lane).

The biphase of the (f, f) interaction, $\theta(f, f)$, was observed to evolve differently for planar and barred environments, but for both types of beach profiles was well parameterized $(r^2 \ge .93)$ as a function of kh. This implies that, like normalized amplitude, $\theta(f, f)$ does not appear to depend strongly on the beach slope. Observations indicate that the Pacific coast data (NSTS) tend to exhibit not only stronger nonlinearities prior to breaking than the east coast data (C^2S^2) , but also $\theta(f, f)$ values that are much closer to -90° . This implies that the NSTS data develops stronger (vertical) asymmetry before breaking than the C^2S^2 data, which suggests that wave skewness/asymmetry cannot by itself be used as a breaking criteria. Since skewness/asymmetry depend on a combined functional form of kh and U_r (which is a function of kh and $\frac{a}{h}$) suggests that another or additional parameter is required to determine the inception of breaking; kh and $\frac{a}{h}$ apparently do <u>not</u> provide this information.

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