CHAPTER 9

Transformation of Random Breaking Waves on Surf Beat

by

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ABSTRACT

Based on a previous study by the authors of regular breaking waves in the surf zone, a model for random wave transformation across the nearshore region is developed. The results of a laboratory investigation of the effect of a steady opposing current on the wave decay process are presented and a proposed governing equation verified. Surf beat effects on wave transformation are then included in the model by representing the long wave as a temporally and spatially-varying current and mean water level. The concept of an equivalent water depth, which contains the effect of the current, is introduced and then included in a stochastic form in the random wave model. Surf beat is found to noticeably increase the decay of the root mean square wave height, especially in the inner surf where the beat is strongest. Comparison of the models to two field data sets show very good agreement for Hotta and Mizuguchi (1980), but rather poor for Thornton and Guza (1983). Possible explanations for the unexpected behavior of the second data set, pertaining to filtering, are discussed. Finally, a possible explanation for the dependence of random wave decay on deepwater steepness, noted by Battjes and Stive (1985), is presented.

INTRODUCTION

The major goal of this effort is to predict the transformation of the probability density function (pdf) of wave height as random waves cross the nearshore region and surf zone. Secondary goals include investigation of the interaction of currents with the shoaling and breaking process, as well as the effects of mean wave steepness, beach slope, and surf beat on random wave transformation. Most previous invesigations, e.g. Collins (1970), Kuo and Kuo (1974), Battjes and Janssen (1978), and Thornton and Guza (1983), assume the Rayleigh pdf (or somewhat contrived modifications thereof), is valid in the surf zone. However, this assumption is not supported by laboratory and field data, with the possible exception of the measurements taken

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during the Nearshore Sediment Transport Study (NSTS) and analyzed by Thornton and Guza (1983).

The models described herein start with the Rayleigh pdf well outside the surf zone, but then numerically transform the pdf over beach profiles of arbitrary shape using the authors' regular wave model (Dally, Dean, and Dalrymple, 1984,85). A laboratory study of regular breaking waves on opposing currents is briefly reviewed and the resulting governing equation presented. Surf beat is then included in the numerical model by introducing an equivalent water depth approach. Finally, it is shown how some "observed" behavior of laboratory and field data reported in the literature might be a manifestation of 1) the analysis technique used, and/or 2) assuming a Gaussian sea in the surf zone, i.e. assuming $H_{rms} = \sqrt{8m_o}$ where H_{rms} is the root mean square wave height and m_o is the area under the energy spectrum.

RANDOM BREAKING WAVE MODEL

Formulation The basic concept of the random wave model is, starting with a known histogram of wave height at some offshore location, transform each representative wave in the histogram as if it were regular, i.e. assume there is no wave-wave interaction and let each wave shoal, reach incipient breaking, break, reform, etc. independently. Because wave period will be required to determine incipient breaking, the joint distribution of wave height and period for waves in deep water derived by Longuet-Higgins (1983) is adopted as an initial condition. This joint pdf, which yields a marginal pdf for wave height that is nearly Rayleigh in shape, is discretized into a histogram of 3600 bins, each with a representative wave height, period, and probability weight, w. Each representative "wave" is then transformed according to an improved version of the authors' numerical regular wave model.

In this new version, the approximate solution to the dispersion relation for linear waves provided by Nielsen (1984) is utilized to calculate the change in wave height due to shoaling, and the incipient breaking condition is explicitly defined as described in Moore (1982). This empirically based condition is a function of deepwater height, wave period, and local beach slope and is a hybrid of the expressions of Weggel (1972) and Komar and Gaughan (1972). The breaker decay is then calculated using the authors' previous scheme (1984,85) which was calibrated to laboratory data and is applicable on beach profiles of arbitrary shape.

On realistically-shaped bottom profiles, the question arises as to the definition of the effective beach slope, especially when bar/trough systems are present. Based on laboratory tests conducted at the University of Florida, it was observed that the bottom slope just seaward of the break point more directly affected the breaking characteristics in the trough than the local bottom configuration. Consequently, the beach slope used in determining incipient breaking is calculated by averaging the slope over the section just seaward of the point of interest, for a distance of one wave length. The negative slopes occurring on the landward side of a bar are treated as zeros in the averaging process.

Finally, at each location on the profile, the 3600 transformed waves are then ordered according to wave height from smallest to largest using a fast, "heap", sorting routine (Williams (1964)). From this ordered set of heights and associated probability weights, the behavior of any desired statistically representative wave can be calculated and monitored across the surf zone.

The information required to run the model consists of 1) root mean squared wave height and band width parameter, (ν) at the starting location, and 2) the bottom profile. If the spectrum is not available for calculating the moments required to determine the band width parameter (Longuet-Higgins, 1983) a value of ν=0.3 is used for gentle swell conditions and ν=0.6 for "confused" or storm conditions. It should be noted that the concept of addressing random wave transformation in the surf zone by monitoring a set of regular waves was also utilized by Mase and Iwagaki (1982). However, their model was limited to planar beaches, and differs significantly in the details.

Verification The random wave model was tested against the field data sets of Hotta and Mizuguchi (1980) and Thornton and Guza (1983). Hotta and Mizuguchi filmed a series of sixty wave staffs established at 2 to 3 m intervals across the surf zone. The measured bottom profile contained a large bar/trough formation, but unfortunately the cameras documenting the trough area failed during the experiment. Nevertheless, it is still an excellent data set. The bottom profile and transformation of several statistically representative waves (H_{rms}, $H_{1/3}$, $H_{1/10}$) are displayed in Figure 1. The model-predicted results generated using the values for the empirical coefficients as found in the laboratory calibration of the regular wave model (K=0.15, Γ=0.40) are also shown. The agreement is quite satisfactory, except for $H_{1/10}$ and perhaps $H_{1/3}$ in the region just seaward of the surf zone. It is believed this discrepancy is because linear theory underpredicts the rate of shoaling in shallow water, especially for waves of low deepwater steepness. Using a nonlinear theory would increase the peak value of the statistically representative wave and shift its location seaward. Even though a gap in the data in the trough region prevents verification of the wave reformation aspects of the model, the favorable comparison landward of this section lends support to the stable wave assumption of the original regular wave model (1984,85). It is noted that the random wave model predicted that the majority of the waves reformed in the trough, consistent with visual observations made during the experiment.

Figure 2 displays comparison of the transformation of the histogram of wave height as reported by Hotta and Mizuguchi (1980) with results of the model. Wave height has been nondimensionalized by the local average wave height, $\bar{H}$. Note that the Rayleigh distribution represents the actual initial histogram fairly well (station 57) but would not compare well at stations in the inner surf zone. The basic shape of the predicted pdf appears correct, especially for $(H/\bar{H}) > 1.0$. However, in the inner surf zone the model overpredicts the number of waves near $(H/\bar{H}) = 1.0$ and underpredicts for $(H/\bar{H}) < 1.0$.

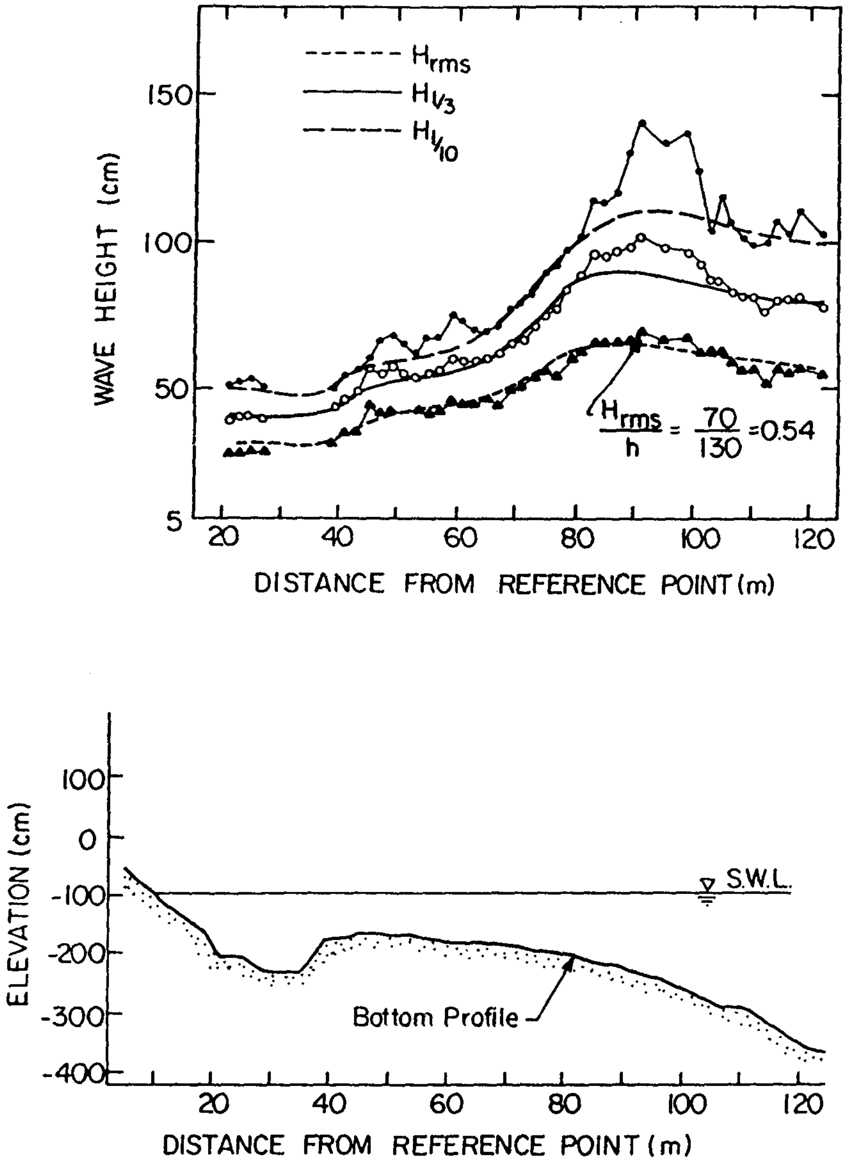

Figure 1. Comparison of transformation of statistically representative waves between model and field data of Hotta and Mizuguchi (1980).

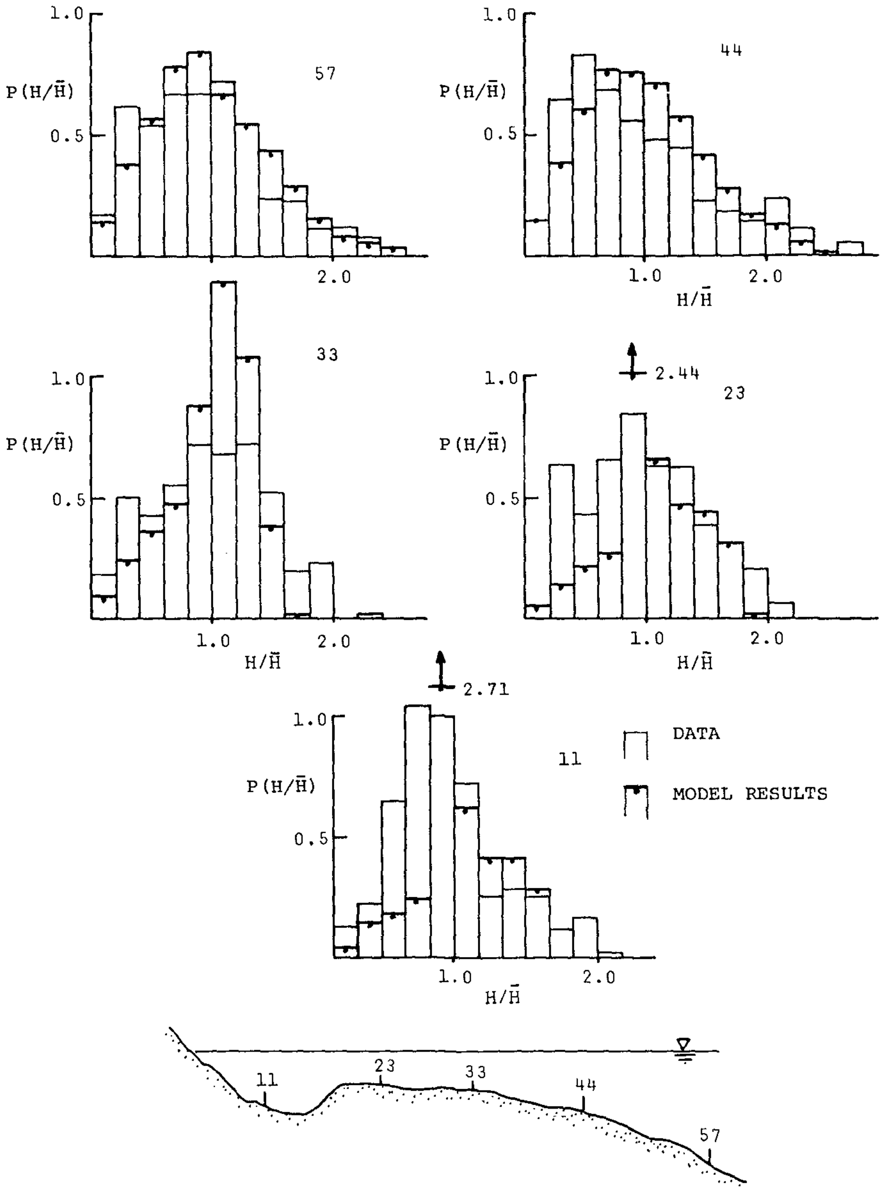

Figure 2. Comparison of transformation of histogram of wave height between model and field data of Hotta and Mizuguchi (1980).

Thornton and Guza (1983) analyzed measurements taken by resistance wire wave staffs, electromagnetic current meters, and pressure sensors during a NSTS field experiment. A typical sample of their results for the transformation of H_{rms} is presented in Figure 3 along with the local beach profile. The model comparison is also displayed, and one can see that it shows a significant discrepancy from the data. It should be pointed out that the point where H_{rms} begins to decay in the data seems to occur much farther seaward than is realistic. The mean water depth at this point was reported as 279 cm and the local ratio of H_{rms} to h was 72/279 = 0.26, which is less than half the value of the Hotta and Mizuguchi data (70/130 = 0.54). Also, it is noted that in histograms of wave height provided by Thornton and Guza, the largest wave never exceeded three times the deepwater H_{rms}, i.e. the largest wave observed anywhere during the test was approximately 168 cm in height. If this wave was just starting to break with an incipient condition H/h of say 0.78, the water depth would be about 215 cm, which is 64 cm shallower than the depth where H_{rms} began to decay.

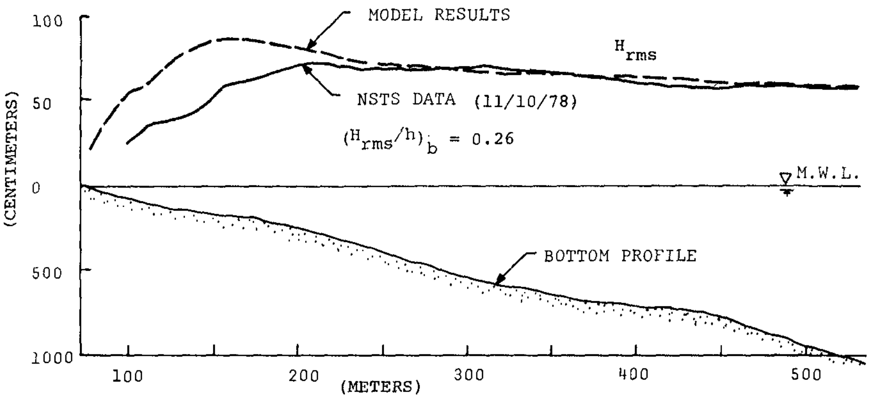

Figure 3. Comparison of transformation of H_{rms} between model and field data of Thornton and Guza (1983).

From the above observations, it is apparent that some phenomenon not included in the model and not present during Hotta and Mizuguchi's experiment was causing wave decay to occur sooner than expected. This phenomenon may be real, or an artifice of the measurement/analysis technique. A real phenomenon to which the majority of the remainder of this paper is devoted, is the interaction of wind waves with long waves such as surf beat. It is hypothesized that during the phase of the beat where the long wave water particle velocity opposes the incident waves and/or the mean water level is depressed, shoaling is accelerated and breaking initiated sooner than would otherwise occur. The opposite phase will tend to stretch the short waves and suppress breaking, but because shoaling is a conservative property while breaking is not, increased decay in average energy density (i.e. H_{rms}) should be expected. The wave conditions during the NSTS experiment were characterized by long period, very low steepness, groupy swell, and significant long wave energy was documented, at least in the inner surf zone (Guza and Thornton, 1985). During the Hotta and Mizuguchi experiment, the waves were comparable in height to NSTS, but had a peak period less than half. They were not characterized as groupy, and only a small amount of long wave energy was documented.

Another possible source of the unexpected behavior of the NSTS data may lie in the manner the data was filtered during analysis. Although the raw data was taken at a rate of 64 Hz, it was immediately low-pass-filtered to a Nyquist frequency of 1.0 Hz, which may not be sufficient to resolve the wave height for very peaked waves, nor the front face of a broken wave. Hotta and Mizuguchi (1980) recorded on film at 5.0 Hz and did not filter before determining wave height, plus they had waves that were less peaked than NSTS and could therefore better resolve the wave height. These points are discussed further at the end of the paper.

REGULAR WAVES BREAKING ON STEADY CURRENTS

Because surf beat is an order of magnitude longer and an order of magnitude smaller than the wind waves that drive it, we assume that its effect on the short waves can be closely represented as that of a slowly oscillating current and mean water level. The effect of a slowly changing water level (depth) on shoaling and breaking is of course the essence of previous studies, but the effect of a current on a breaking wave has received relatively little attention in the literature. A governing equation for energy dissipation due to breaking on a steady collinear current and changing bottom is therefore proposed, and verified to some extent with a laboratory experiment.

Governing Equation Based on Conservation of Wave Action and linear wave theory, and the intuitive expression for the rate of energy dissipation due to breaking developed by the authors (1984,85), the following governing equation is proposed

$$\frac{\partial\left[H^2(U + Cg)/\sigma\right]}{\partial x} = \frac{-K}{h}\frac{Cg}{\sigma}\left[H^2 - \Gamma^2 h^2\right] \tag{1}$$

where H is wave height, σ, the intrinsic wave frequency (i.e. relative to the current), U, the current magnitude, Cg, the group velocity relative to the current, h, the water depth, and x, the horizontal coordinate in the direction of wave propagation. K is a decay coefficient (0.15-0.17) and Γ is the stable wave factor (0.4-0.5). The wave number, k, is given by the dispersion relation for linear waves on currents

$$\omega = \sigma + kU = (gk \tanh kh)^{1/2} + kU \tag{2}$$

where ω is the absolute frequency. If U is zero, these expressions reduce to the original governing equation proposed by the authors.

Verification The validity of (1) was tested by comparing numerical solutions to the results of a laboratory experiment of regular breaking waves on opposing currents. Due to space limitations the experiment will not be fully described here, but Figure 4 shows typical results of wave decay and model comparisons. The proposed governing equation appears valid, especially for mild currents. The results of this investigation are now applied to surf beat.

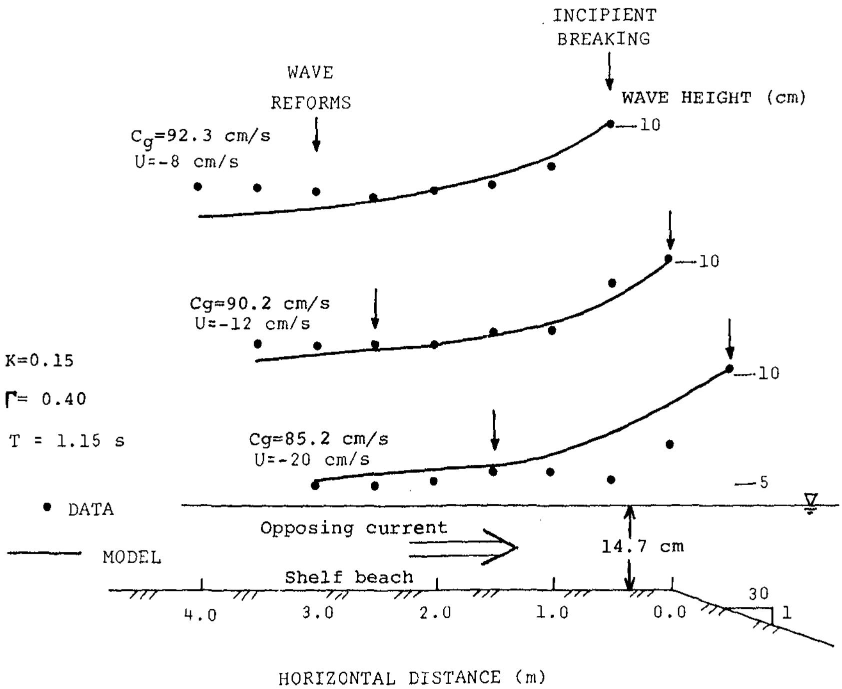

Figure 4. Sample results of wave decay on opposing currents and comparison of model (1).

RANDOM BREAKING WAVES ON SURF BEAT - THE EQUIVALENT DEPTH METHOD

The results of the laboratory investigation tend to indicate that the currents associated with surf beat are of equal or greater importance in the wave breaking process than the mean water level fluctuation. A few previous investigations have attempted to include surf beat in a random breaker model, e.g. Goda (1975) and Mase and Iwagaki (1982), but addressed only water level fluctuations.

To completely explore the problem of a short progressive breaking wave riding a partially-standing long wave, a wave tracking technique based on the method of characteristics would be required. Such a method in turn requires a time series as an initial condition, as well as complete temporal and spatial description of the surf beat.

Initial attempts found such a model numerically time-consuming, and impractical for engineering application due to the extensive input required. Therefore, a method is developed which combines the effects of the current and mean water level fluctuations, and is then included stochastically in the random breaking wave model.

Equivalent Depth The "equivalent water depth" is defined for a wave with absolute frequency ω and wave number k riding on a current as the water depth the wave would have to encounter with no current while still retaining the same wave number, i.e.

$$\omega^2 = \left[(gk \tanh kh)^{1/2} + kU\right]^2 = gk \tanh kh_e \tag{3}$$

where h_e is the equivalent water depth. A probability density function (pdf) for h_e in regards to surf beat can be developed. We first assume that the water particle velocity and mean water level associated with the standing long wave are normally distributed and uncorrelated at any location in the surf zone:

$$\mathrm{pdf}(U,h) = \frac{1}{2\pi\sigma_u\sigma_h} \exp\left\{-\frac{1}{2}\left[\left(\frac{U}{\sigma_u}\right)^2 + \left(\frac{h-h'}{\sigma_h}\right)^2\right]\right\} \tag{4}$$

where h' is the mean mean-water level, σ_u and σ_h, are variances of the current and water level respectively.

The conditional pdf of k and h, given ω, is derived by solving the dispersion relation (2) for U and invoking a standard transformation of random variables. The joint conditional pdf of h_e and h given ω (or k_o) is then found in a similar manner by applying an approximate solution to the dispersion relation (3) given by Nielsen (1984):

$$kh_e = \sqrt{k_o h_e}\left[1 + \frac{1}{6} k_o h_e + \frac{11}{360} (k_o h_e)^2\right] \tag{5}$$

where k_o is the deepwater wave number in the absence of currents. This joint pdf is given by:

$$\mathrm{pdf}(h_e,h/k_o) = \frac{1}{2\pi\sigma_u\sigma_h} \left|\sqrt{k_o}\left(-\frac{1}{2} h_e^{-3/2} + \frac{1}{12} k_o h_e^{-1/2} + \frac{11}{240} k_o^2 h_e^{1/2}\right)\right|$$

$$\cdot\left|(gk \tanh kh)^{1/2} - \frac{gk^2h\,\mathrm{sech}^2 kh + gk \tanh kh}{2(gk \tanh kh)^{1/2}} - \sqrt{gk_o}\right| \cdot \frac{1}{k^2}$$

$$\cdot\exp \frac{-1}{2}\left[\frac{gk_o + gk \tanh kh - 2\sqrt{gk_o}\,(gk \tanh kh)^{1/2}}{k^2\sigma_u^2} + \left(\frac{h-h'}{\sigma_h}\right)^2\right] \tag{6}$$

where k is given by (5) as noted. A marginal pdf for h_e is then determined by integrating with respect to h over physically realistic limits ($0 \rightarrow 2h'$) and normalizing. In the model this is accomplished numerically using Simpson's Rule. Thus, with estimates of σ_u and σ_h, the pdf (h_e) is determined at any location across the nearshore region. An example is shown in Figure 5, where h_e has been non-dimensionalized by h'.

Stochastic Treatment of Breaking Waves on Surf Beat To include surf beat in the random breaking wave model described previously, the pdf (h_e) can be utilized to generate "equivalent profiles". In the model, the marginal probability density function of equivalent water depth at each location in the surf zone is subdivided into 10 bins of equal area, also shown in Figure 5, and the average equivalent depth of each bin is assigned to a corresponding equivalent profile. Guza and Thornton (1985) provide information from which estimates of the required variances, σ_u and σ_h, can be extracted for the NSTS experiment previously described, and several equivalent profiles are shown in dimensionless form in Figure 6.

Because of the currents, the incipient breaking condition of Moore (1982) is not applicable. A Miche-type wave steepness condition is therefore adopted to determine incipient breaking

$$\frac{H_b}{L} = 0.124 \tanh kh_e \qquad (7)$$

where H_b is the wave height at incipient breaking and L the wave length. Note that when kh_e is small, (7) reduces to

$$H_b = 0.78\ h_e \qquad (8)$$

In the numerical model, once incipient breaking is attained, a slightly modified form of (1) is invoked to describe wave decay

$$\frac{\partial[H^2(Cg_e/\sigma)]}{\partial x} = \frac{-K}{h'}\frac{Cg}{\sigma}\left[H^2 - \Gamma^2 h'^2\right] \qquad (9)$$

where the equivalent water depth is used to calculate Cg_e on the L.H.S., and the average depth (h') used on the R.H.S. If the wave height is less than the local stable wave ($\Gamma h'$) but is greater than the incipient condition (7), the wave is simply "trimmed" to equal H_b.

Each representative wave from the original joint histogram of wave height and period is transformed across its 10 equivalent profiles and the results at each location averaged. Thus an average behavior for each wave from the original offshore histogram is calculated for given surf beat conditions.

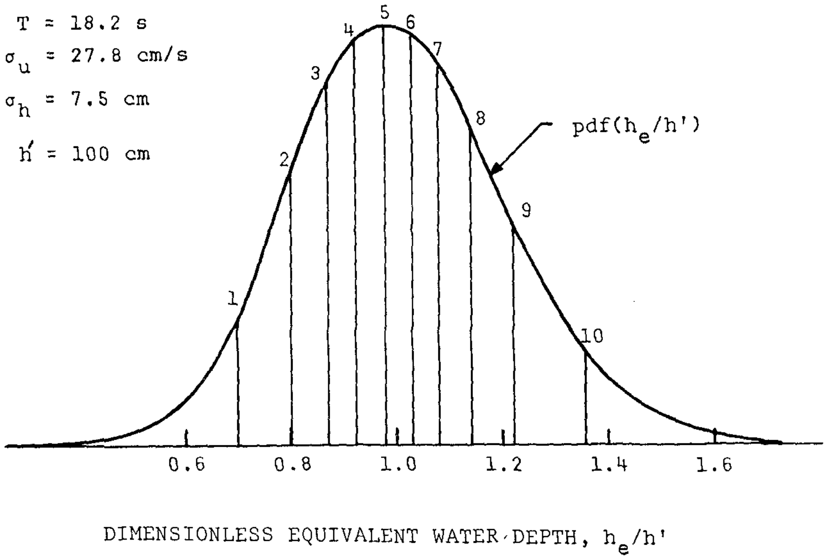

Figure 5. Example of marginal probability density function for equivalent water depth.

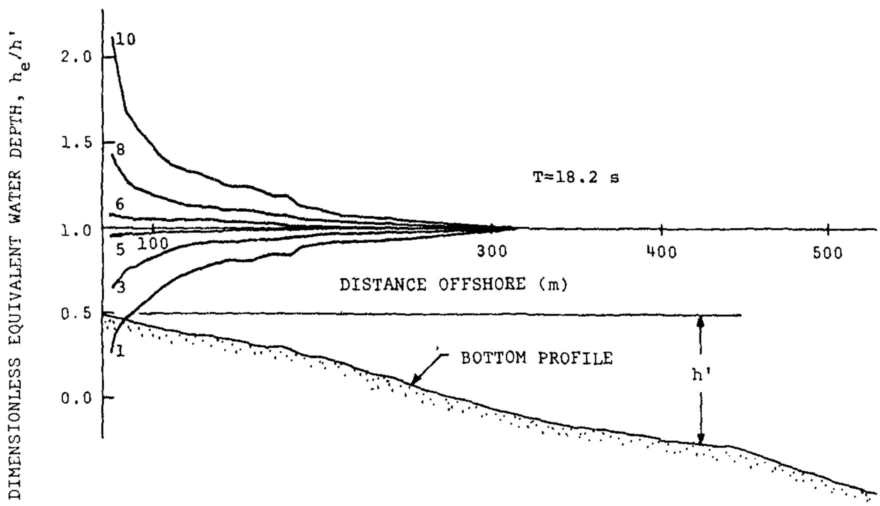

Figure 6. Example set of dimensionless "equivalent depth profiles" for NSTS data (for peak period T = 18.2 s).

Results The model including surf beat effects was run for the NSTS experiment and is compared to the data and the model results without surf beat in Figure 7. Note that surf beat did cause decay in H_{rms} to begin sooner and with greater intensity, but not to the anticipated degree. It appears that only unrealistically energetic surf beat could yield reasonable comparison of the model in its present form to this data set. Although there are some compromises encountered when adapting the decay expression (1) for regular waves on steady currents to random waves on surf beat, it is highly unlikely that they could account for the remaining discrepancy between model and data. We are now left to explore the possibility that the manner in which the data was filtered and analyzed could be responsible for the unexpected behavior.

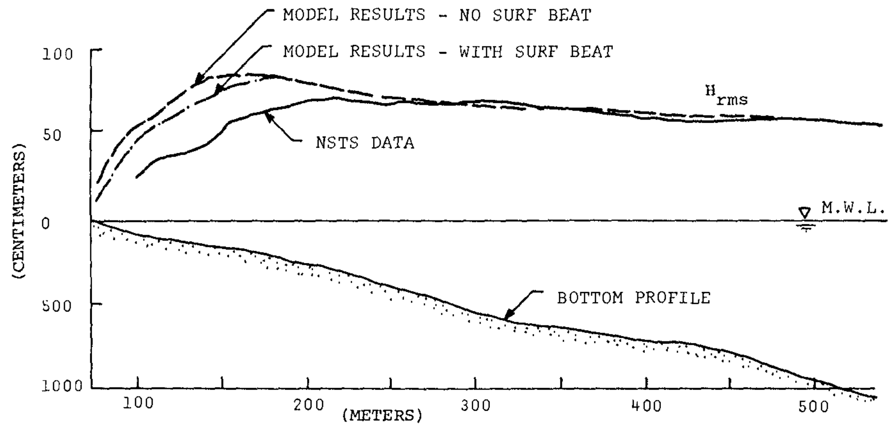

Figure 7. Comparison of H_{rms} transformation between model, with and without surf beat, to field data of Thornton and Guza (1983).

FILTERING-INDUCED "CLIPPING" OF WAVE HEIGHT

Application of the zero-up-crossing technique for analyzing wave records often involves first low-pass filtering the signal to remove higher frequency oscillations from the free surface. This high frequency "noise" increases the number of waves counted in a record by the up-crossing technique, and results in lower calculated values for statistically representative waves, such as H_{rms}. The original data for NSTS-Torrey Pines was taken at 64 Hz, block averaged which reduced the sampling rate to 8 Hz, deglitched, then low-pass-filtered "to substantially reduce energy between 0.5 and 1 Hz", and finally output to tape at 2 Hz (Gable, 1979). Thornton and Guza (1983) treated these time series further by Fourier Transforming the filtered record and zeroing the amplitude coefficients above 0.5 Hz (for gages in shallower water). The time series was then reconstructed and wave heights and periods determined using the up-crossing technique. However, if the original waves are either very peaked (as they are when approaching the break point) or have sudden discontinuities (as is the case at the front face of a wave at incipient breaking, or the face of a bore), filtering in the above manner can significantly "clip" the wave height. As noted, wave conditions during NSTS were characterized by long-period, low deepwater steepness swell, which become very peaked in shallow water. For the peak frequency of 0.055 Hz, significant deepwater height of 79 cm, and local water depth equal 279 cm (i.e. where the data begin decay), and assuming Stream Function Theory is valid (Dean, 1974), the free surface before and after filtering above 0.5 Hz (the ninth harmonic in this instance) is shown in Figure 8. Note that the wave height was clipped by almost 25%.

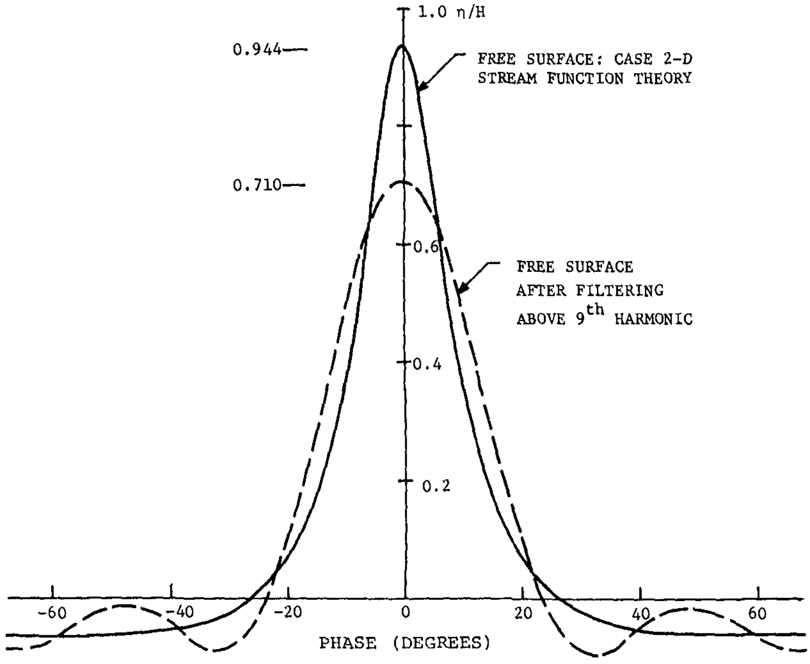

Figure 8. Wave height "clipping" due to low-pass-filtering of peaked waves. Conditions are analogous to NSTS data.

It seems the clipping artifice was induced more by the original filtering of the raw data, as one of these records from the inner surf zone was analyzed with and without the additional filtering of Thornton and Guza, and only a 6% drop in H_{rms} was found (Thornton - personal communication). This artifice is also believed by the authors to be responsible for the apparent agreement between the Rayleigh pdf and histograms of wave height in the surf zone as reported by Thornton and Guza (1983). Filtering the waves makes them appear more sinusoidal and narrow-banded, so assuming a Gaussian sea becomes, artificially, more valid. This would also appear to be why

Thornton and Guza found $H_{rms} = \sqrt{8m_o}$ to be relatively valid in the surf zone.

MEAN WAVE STEEPNESS "EFFECTS" ON RANDOM WAVE TRANSFORMATION

A few previous investigations have noted an apparent dependence of random wave decay on mean wave steepness, e.g. Battjes and Stive (1985). These models increase the decay in wave height for low deepwater steepness waves by varying empirical coefficients. The authors believe this requirement may be an artifice of assuming a Gaussian sea in the surf zone, i.e. that $H_{rms} = \sqrt{8m_o}$. A low steepness wave becomes peaked in shallow water and even though it may have the same actual height as a higher steepness wave, it contains less energy. (Steepness was found by the authors (1984,85) to have little effect on wave height decay after breaking is initiated.) Therefore, if energy is used to calculate H_{rms} rather than the actual free surface displacement between trough and crest, a lower value for wave height is produced. As a result, for the transformation of H_{rms} as defined by $\sqrt{8m_o}$, the heights of the breaking waves in the model must be artificially suppressed to obtain good fit if the measured waves were of low steepness.

CONCLUSIONS AND RECOMMENDATIONS

1) The model for random wave transformation in the nearshore region and surf zone described herein (without surf beat) appears valid in comparison to the field data of Hotta and Mizuguchi (1980), in regards to both statistically representative waves and the probability density function of wave height. However, it does not compare well to the field data of Thornton and Guza (1983). The differences are believed to be due to a "clipping" artifice induced by low-pass-filtering during the original treatment of the raw data.

2) To accurately represent surf beat in any nearshore wave model, depth and current variations should be included.

3) The stochastic model which utilizes an equivalent depth approach to represent surf beat shows that random wave transformation in the surf zone can be noticeably affected by surf beat and is characterized by an increase in breaker decay.

4) In nature, surf beat effects may be significant, but are probably limited to the inner surf zone where the surf beat is strongest.

5) In laboratory experiments, surf beat effects might be unrealistically amplified if long wave energy is trapped in the tank.

6) High frequency filtering can artificially reduce wave height, especially for waves of low deepwater steepness.

7) $H_{rms} = \sqrt{8m_o}$ is not valid in the surf zone. This is believed to be the major reason for the "observed" dependence of decay on wave steepness found in previous studies.

8) It is recommended that in data analysis, low pass filtering be used only to obtain wave periods and that the original record be used to obtain wave heights.

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