CHAPTER 4

Statistical Modelling of Long-Term Wave Climates

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Abstract

The paper discusses the long-term statistical properties of ocean and coastal wave climates derived from the analysis of instrumental wave data. The aim of the work reported has been to determine the theoretical distributions, from those commonly used in analysis of wave data, which best describe the joint probability of significant wave height, Hs, and mean zero-upcrossing period, Tz. A method of modelling the wave climate in this manner has been developed utilizing parametric means of specification. The data base used in the study covers records from 18 sites around the British Isles.

Introduction

Mathematical models considered previously for this application have been examined as part of the study. Ochi (1978) proposed the use of a bi-variate (2-dimensional) Log-Normal distribution for the joint probability density function (pdf) denoted by p(Hs,Tz). This can be defined in terms of the first and second order statistical moments of the marginal pdfs of both Hs and Tz and a measure of the correlation between these wave properties. Appendix 1 presents a summary of the relevant mathematics for this and other probability distributions discussed herein.

Whilst the fit shown by Ochi for the Log-Normal distribution, applied to various data sets, was good for the bulk of the probability mass the tails, in particular that of Hs, were not well matched beyond a cumulative probability of about 0.99. It is this region which is often of interest in engineering design and a need for improved modelling of extremal (Hs,Tz) sea states has recently been identified by U.K. Department of Energy (1986).

As part of a research programme aimed at wave climate synthesis by the National Maritime Institute (NMI, but now British Maritime Technology), a development of Ochi's modelling was proposed by Fang and Hogben (1982). This involved the inclusion of a measure of the skewness in a term modifying the Log-Normal form of the marginal distribution of Hs.

The joint pdf, p(Hs,Tz), can be expressed as a product of the marginal pdf of Hs and the conditional pdf of Tz (given Hs), i.e.

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$$p(Hs,Tz) = p(Tz/Hs).p(Hs) \quad (1)$$

Houmb and Overvik (1976) utilized the form of this equation for the description of wave climate off the coast of Norway. For both the marginal distribution of Hs and the conditional distribution of Tz, they employed a 2 parameter Weibull distribution (equivalent to the 3 parameter Weibull but with A=0, see Appendix 1). Parameters B and C in p(Tz/Hs) were then specified as functions of Hs, following regression for their sites under study.

More recently Haver (1985) has proposed a similar form of modelling to that of Houmb and Overvik but with the conditional distribution of Tp (the spectral peak period, replacing Tz) fitted to a Log-Normal distribution. The marginal distribution of Hs is described in the lower region by the Log-Normal distribution also but for the upper tail ($Hs > \alpha$, where α is a chosen threshold) a 2 parameter Weibull distribution is employed.

This latter model has not been investigated herein but the other methods described above have been applied together with several modified approaches. These include; (i) an extension of Houmb and Overvik's model by the use of the 3 parameter distribution for both marginal and conditional probability distributions, (ii) use of a mixed Weibull and Log-Normal model with p(Hs) described by the former and p(Tz/Hs) by the latter, (iii) direct specification of p(Hs,Tz) in terms of a 2-dimensional Weibull distribution. For the latter, the derivation given by Kimura (1981) for application to short-term wave climates has been utilized. Relevant equations are contained in Appendix 1.

The study has been conducted in 3 parts. Firstly, the goodness of fit of the different models to the data was investigated, the models being fitted using statistical moments of the various data samples. This provided a shortlist of preferred probability distributions. Secondly, empirical relationships between the significant wave height, Hs, and both the conditional statistics of Tz and the parameters of the different models were investigated. In this way it was hoped to improve the means of description of the Tz domain of the wave climate. By considering the degrees of exposure and wind field statistics of the different sites trends were observed and, ultimately, regression equations linking Tz statistics to the marginal statistics of Hs have been established for 2 groups of sites, loosely termed 'oceanic' and 'coastal'. The third and final part of the study has been to use the regression equations for specification of the parameters of the different models and to again assess their respective merits.

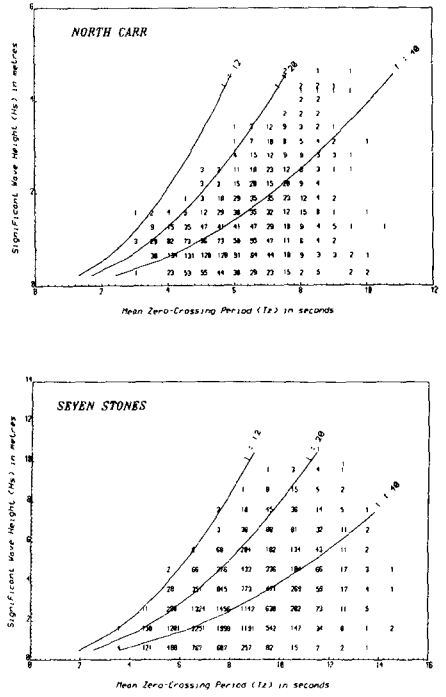
Data

The data base used in the study consisted of scatter diagrams (bi-variate histograms of the 3-hourly observation of Hs and Tz) covering periods of measurement ranging from 1 to 7 years. An example of such a diagram is given in Fig. 1. Locations of the various sites are indicated on the map in Fig. 2.

The data set is far from ideal since for few sites do recordings

exceed one or two years duration. In consequence, the effects of variability in annual wave climate will potentially cloud any 'between site' trends which may be present. Whilst these inadequacies of the data will have an unquantifiable effect on the results developed, this should not be so large as to undermine the value of the study.

FIGURE 1 BI-VARIATE HISTOGRAM, OR SCATTER DIAGRAM, OF H_s AND T_z



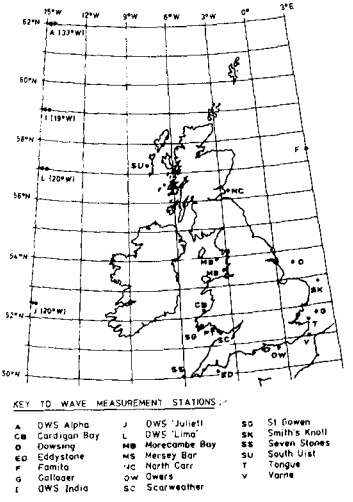


FIGURE 2 LOCATION MAP SHOWING SOURCES OF WAVE DATA

Analysis of Scatter Diagrams Forming the Data Base

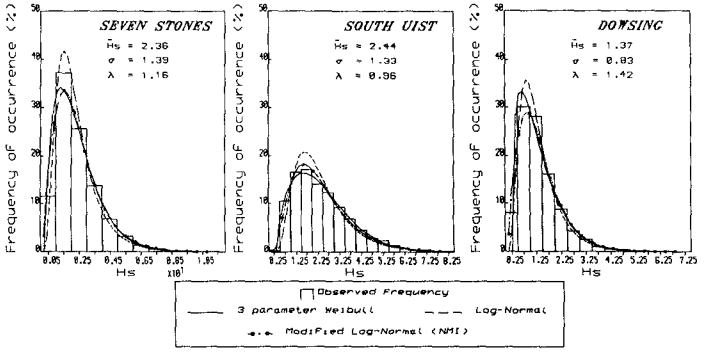
The statistical properties of Hs and Tz computed from scatter diagrams for the 18 sites are listed in Table 1. These parameters (mean, standard deviation and skewness of the marginal distributions and the correlation coefficient) can be used to define each of the theoretical probability distributions referred to above (and in Appendix 1). The Weibull distribution is most conveniently expressed in terms of parameters A,B,C, each being related to the statistical moments, and these are also included in Table 1.

Marginal Distribution of Hs: The marginal histograms of Hs from the data sets have been compared against the (3 parameter) Weibull, Log-Normal and Modified Log-Normal (NMI) distributions. Fully objective 'method of moments' fitting has been used in each case. Examples, in Fig. 3, of the observed frequency histograms with theoretical distributions superimposed show each of the above to

TABLE 1 STATISTICAL PROPERTIES OF SCATTER DIAGRAMS FORMING DATA BASE

Site	Marginal Hs			Marginal Tz			ρ_{Hs-Tz}	Marginal Hs			Marginal Tz		
	$\bar{H}s$	σ_{Hs}	λ_{Hs}	$\bar{T}z$	σ_{Tz}	λ_{Tz}		A_w	B_w	C_w	A_w	B_w	C_w
Seven Stones	2.36	1.39	1.16	7.72	1.52	0.45	0.489	0.40	2.15	1.43	4.39	3.75	2.32
South Uist	2.44	1.33	0.96	6.33	1.42	0.20	0.706	0.37	2.31	1.60	2.53	4.27	2.90
W S Alpha	3.06	1.67	1.08	8.31	1.26	0.23	0.592	0.62	2.71	1.49	5.03	3.68	2.81
W. S. India	3.36	2.07	1.44	9.43	1.29	0.52	0.617	0.80	2.75	1.24	6.75	3.03	2.19
W. S. Juliett	3.38	2.04	1.32	9.54	1.52	0.49	0.639	0.72	2.88	1.31	6.33	3.62	2.23
Famita	2.81	1.51	1.06	7.40	1.28	0.51	0.596	0.57	2.48	1.51	4.73	3.01	2.19
Galloper LV.	1.36	0.81	1.38	4.66	0.91	0.41	0.390	0.33	1.11	1.28	2.62	2.30	2.39
Tongue LV.	0.93	0.40	0.75	4.83	0.86	0.67	-0.021	0.23	0.79	1.84	3.23	1.81	1.95
Eddystone LH.	1.17	0.81	1.59	4.83	1.12	0.74	0.646	0.23	0.99	1.16	2.84	2.24	1.84
Varne LV.	1.24	0.76	1.43	5.62	0.84	0.75	0.064	0.28	1.02	1.25	4.13	1.68	1.84
Scarweather Bk.	1.17	0.87	0.94	7.00	1.86	0.55	0.299	-.21	1.54	1.62	3.23	4.26	2.14
St. Gowan LV.	2.01	1.27	1.10	6.68	1.42	0.49	0.342	0.17	2.04	1.48	3.69	3.37	2.22
Dowsing	1.37	0.83	1.42	5.17	1.17	0.42	0.557	0.34	1.11	1.26	2.56	2.95	2.37
North Carr LV.	1.11	0.76	1.51	5.79	1.30	0.33	0.489	0.20	0.97	1.20	2.68	3.50	2.56
Smith's Knoll LV	1.09	0.67	1.31	6.40	1.04	-.15	0.523	0.21	0.96	1.32	2.40	4.39	4.36
Mersey Bar LV.	1.23	0.71	1.76	4.97	0.91	0.67	0.624	0.46	0.79	1.08	3.27	1.92	1.95
Morecambe Bay LV	1.07	0.77	1.34	3.76	1.26	0.72	0.679	-0.08	1.07	1.30	1.48	2.56	1.88
Cardigan Bay	1.01	0.69	1.19	5.89	1.47	1.25	-0.053	0.05	1.05	1.41	3.91	2.16	1.36

FIGURE 3 MARGINAL FREQUENCY HISTOGRAMS OF H_s
(note: percentage frequency $\equiv p(H_s)\Delta H_s \cdot 100$)



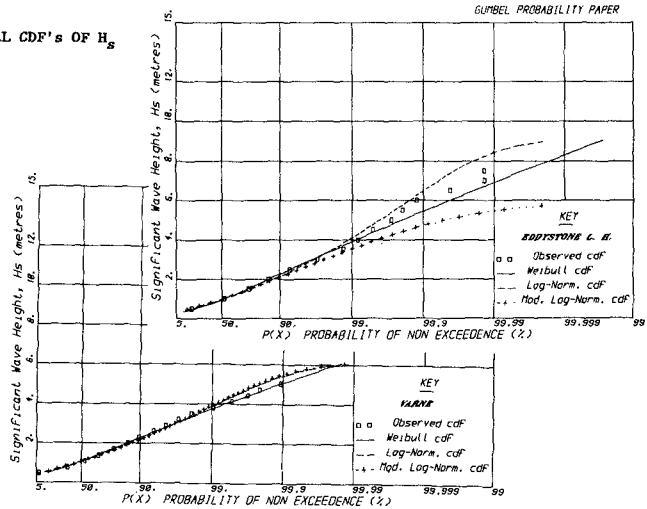
reasonably describe the main probability mass. An indication of relative performance, in terms of 'goodness-of-fit', can be obtained from an application of the χ^2 test. The χ^2 statistic is defined as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(Oi-Ei)^2}{Ei} \qquad (2)$$

where Oi and Ei are the observed and expected frequencies for class (i) of (k) classes. Because, generally, the number of classes did not conform to the requirements of the standard test, for the relevant sample sizes, resulting values of χ^2 cannot be related to appropriate 'levels of significance' in the usual way. Numerical values have, therefore, been used as only qualitative indicators of

goodness-of-fit. Utilizing this approach, and making allowance for the potential departures of the Weibull fit for the lowermost classes arising from the imposition of the location parameter A, this distribution produces the best fit. Indeed, this slight limitation associated with the Weibull distribution is restricted to a region (low Hs) of little practical significance (Salih, 1986).

FIGURE 4 MARCINAL CDF's OF H_s



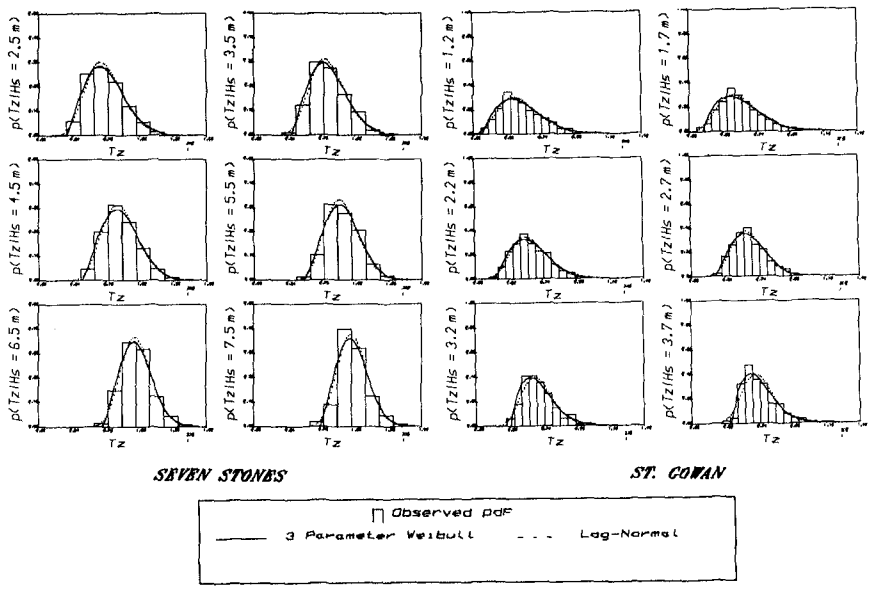
However, directing attention to the cumulative distributions of Hs the Weibull distribution is most clearly superior, as may be seen from Fig. 4. The NMI modification has not been found to improve the basic Log-Normal fit in the tails, a result which was borne out quantitatively from a Kolmogorov-Smirnov statistical test. Again, for the same reasons as those discussed above this goodness-of-fit test could not be applied rigorously. Nevertheless, the Weibull distribution performs most adequately and satisfied the test at a 5% significance level for a number of the data sets.

Marginal Distribution of Tz: The marginal distributions of Tz were treated in a similar way to that described for Hs above. In this case all three distributions showed a good fit to the data over the whole range of periods. Again, the Weibull distribution produced overall the lowest χ^2 values and generally showed the closest fit to the cdf. In view of the satisfactory behaviour of all distributions in the extreme tail the Kolmogorov-Smirnov test was not applied.

Conditional Distribution of Tz: Fig. 5 shows an example of the conditional frequency histograms of Tz for various classes of Hs forming the scatter diagrams. Both Weibull and Log-Normal distributions follow the data closely. In addition, a 2 parameter Weibull distribution and predictions from the fitted 2-dimensional Log-Normal and Weibull distributions have been considered. χ^2 estimates have been made for all conditional distributions at each site and are tabulated in (Salih, 1986). These results showed the 3 parameter Weibull and Log-Normal distributions to provide the best

fits with near equal merit. The 2 parameter Weibull distribution adopted by Houmb and Overvik is unable to locate the central probability mass in many cases and produces large χ^2 errors. Of the two 2-dimensional distributions the Weibull form was found, in this test, to consistently produce the better fit over the Log-Normal.

FIGURE 5 CONDITIONAL FREQUENCY HISTOGRAMS OF T_Z



Joint Distribution of Hs and Tz: On the basis of the above findings contour maps overlaying the scatter diagrams were produced with the joint probability p(Hs,Tz) defined by:

(1) p(Hs)-Weibull, p(Tz/Hs)-Weibull; (W-W)
(2) p(Hs)-Weibull, p(Tz/Hs)-Log-Normal; (W-LN)
(3) p(Hs,Tz)-2-dimensional Log-Normal; (2DLN)
(4) p(Hs,Tz)-NMI modification to (3); (NMI)
(5) p(Hs,Tz)-2 dimensional Weibull; (2DW)

In order to establish the relative goodness-of-fits of these different approaches a 2-dimensional χ^2 computation was made. This was equivalent to the application of Eq. 2 to the conditional frequency histograms of Tz for each class of Hs in the scatter diagram with the resulting values summed to give an overall χ^2 estimate. Table 2 presents these findings which show that approach (1) above is best able to represent the characteristics of the data sets considered. Approach (2) is only marginally inferior. The Weibull distribution (5) again shows substantial improvement over the other 2-dimensional distributions.

TABLE 2
2-DIMENSIONAL χ^2 VALUES FOR COMPARISON OF SCATTER DIAGRAM OBSERVATIONS AGAINST VARIOUS THEORETICAL DISTRIBUTION FITS, USING METHOD OF MOMENTS

Site	W-W	W-LN	2DLN	NMI	2DW
Seven Stones	1243.5	643.9	2621.9	1937.6	1501.2
South Uist	17.5	36.9	97.5	98.3	101.0
W. S. Alpha	158.5	166.9	249.2	245.0	216.6
W. S. India	69.9	66.5	82.2	85.7	86.9
W. S. Juliett	30.2	29.3	88.9	131.1	42.2
Famita	313.1	281.2	714.6	690.1	517.2
Galloper LV.	155.9	170.4	696.0	528.1	240.9
Tongue LV.	51.4	139.8	266.4	402.4	196.2
Eddystone LH.	762.2	1110.6	2090.4	2105.1	1845.9
Varne LV.	71.1	71.5	146.3	135.6	180.1
Scarweather Bk.	187.7	237.4	1249.6	984.0	637.1
St. Gowan LV.	291.2	307.8	1813.2	1577.6	957.6
Dowsing	400.2	428.9	2390.6	1643.8	796.8
Smith's Knoll LV.	57.6	65.5	100.7	92.5	149.0
North Carr LV.	279.3	325.8	611.3	615.3	509.5
MERSEY BAR LV.	47.9	51.9	130.4	127.0	119.7
MORECAMBE BAY LV.	1075.7	936.1	2813.8	2747.3	1930.4
CARDIGAN BAY LV.	305.6	418.3	1065.0	997.3	877.9

Modelling the Joint Distribution of Hs and Tz

Having assessed the relative merits of the different probabilistic descriptors of p(Hs,Tz) it remains to develop means of modelling from the minimal amount of wave climate data that may be available. In this respect attention is focussed on the specification of the Tz domain, since marginal Hs statistics can be adequately established using existing methods. These include the various wave forecasting and hindcasting techniques from wind data, recently reviewed by Holtuijsen (1983) and the semi-empirical approach developed by British Maritime Technology in the U.K. and marketed as 'NMIMET' which uses both windspeed and visual waveheight archives (Andrews et. al., 1983).

A first step in this exercise is to establish empirical relationships to link the various statistical properties of Tz, in both the marginal and conditional domains, to those of Hs. From an initial investigation of this nature it was found that the data could be conveniently categorised into three groups, termed 'oceanic', 'coastal' and 'bays/estuarial' and the placements into these groups are indicated in Table 1. The statistics necessary for specification of the different probability models were then obtained for each group using 'least-squares' regression to the statistics of Hs. Due to the limited nature of the data bases, simple regression equations of the form;

$$Y = a.(\bar{Hs})^b.(\sigma_{(Hs)})^c.(\lambda_{(Hs)})^d \qquad (3)$$

were used where Y is any of the required Tz statistics and (a,b,c,d) are the regression coefficients.

In the event, due to the small size of the 'bays' group this has not been treated by regression until such time that further data sets are incorporated into the data bank. Resulting regression equations are summarised in Appendix 2 but it must be appreciated that these would

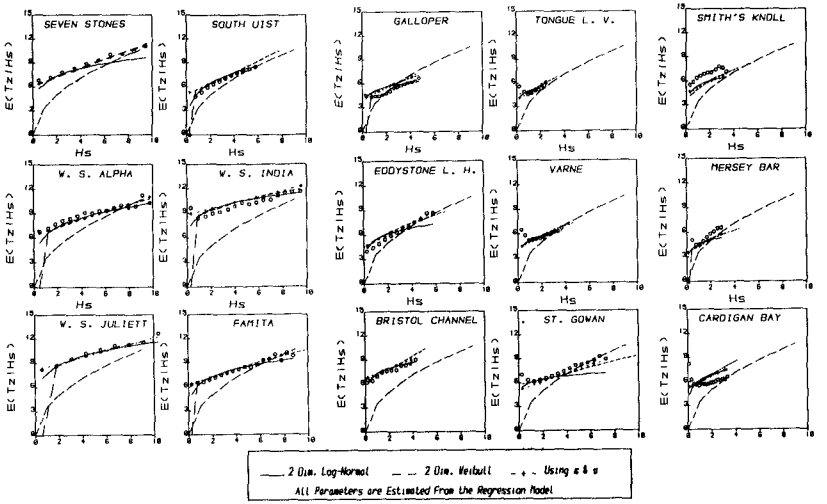
be continuously updated as new data is added. The regression equations produced mean relative errors of about 10% (3.8-17.9%) when compared with the various data sets in each group. The 'oceanic' group produced smaller errors than the 'coastal' group of sites. Dealing with each statistic in turn:-

Conditional Mean E{Tz/Hs}: Conditional mean values for each class of Hs for various sites are plotted in Fig. 6. With increasing Hs it was anticipated that the values of E{Tz/Hs} would become equivalent to the Tz values of the associated wind generated sea states, since the effects of swell would diminish. This was substantiated by the superimposition of such relationships based on the moments of Pierson-Moskowitz spectra onto the plots. In S.I. units this can be expressed as:

$$E\{Tz/Hs\} = 3.55.(Hs)^{0.5} \qquad (4)$$

Departures from this relationship for the lowest classes of Hs, therefore, give an indication of the presence of swell conditions at the different sites. A systematic variation from site to site in this respect is observed where the most exposed locations generally show the greatest departures. To model this behaviour a linear variation of E{Tz/Hs} with Hs has been assumed, defined by an intercept of the period axis κ and slope ν. Regression using the data sets for the two climate categories, as appropriate, produces the fits shown in Fig. 6. Also included here are the relationships output from the use of the relevant regression equations in the 2-dimensional models.

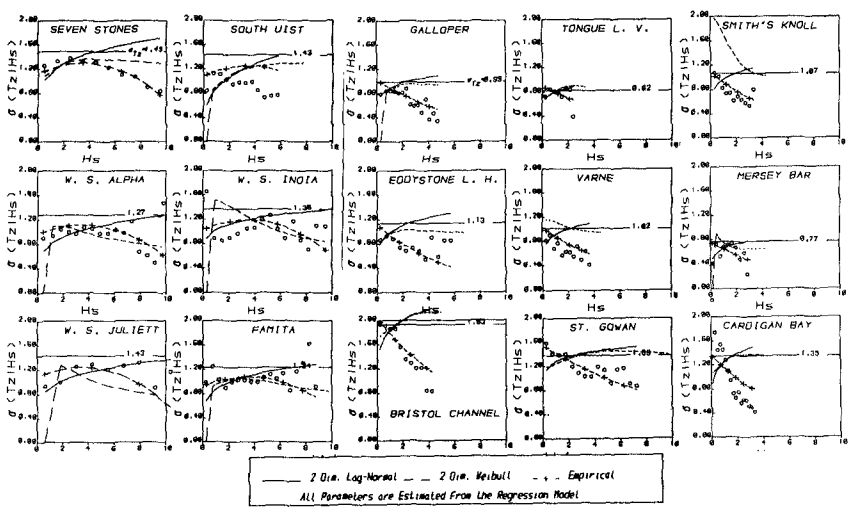
FIGURE 6 RELATIONSHIP BETWEEN CONDITIONAL MEAN VALUE OF T_Z, $E\{T_Z/H_S\}$, AND H_S



Marginal and Conditional Standard Deviations $\sigma_{(Tz)}$ and $\sigma_{(Tz/Hs)}$: Fig. 7 shows plots of the variation of $\sigma_{(Tz/Hs)}$ with Hs for sites from both groups used in the regression exercises. From the general trends

observed a quadratic variation was chosen for the 'oceanic' group and an exponential decay for the 'coastal' group. Regression equations for these are given in Appendix 2 and involve the marginal standard deviation $\sigma_{(Tz)}$ which is itself expressed in terms of a regression relationship. Resulting functions are shown on the graphs. Behaviour of the fitted 2-dimensional probability models are again included and in several instances these show serious divergence from the observed trends. This indicates the inflexibility of these theoretical functions to describe features of the scatter diagrams in close detail.

FIGURE 7 RELATIONSHIP BETWEEN CONDITIONAL STANDARD DEVIATION OF T_Z, $\sigma(T_Z/H_S)$, AND H_S



The various data sets taken individually show, in Fig. 7, significant departure from the regression functions illustrating the approximate nature of the modelling and pointing to a need for further data so that segregation into additional groupings may be made practicable.

Conditional Skewness $\lambda_{(Tz/Hs)}$ and Weibull Parameters $A_{(Tz/Hs)}$ and $A_{(Tz)}$: Definition of the 3 parameter Weibull distribution normally requires sample estimates of the first three statistical moments (see Appendix 1). However, the reliability of these estimates diminishes with the 'order' of the statistic for samples of limited size. In consequence, the computed values of skewness, λ, being a third order statistic, show much greater scatter than that shown by $E\{Tz/Hs\}$ and $\sigma_{(Tz/Hs)}$ which are determined from the first and second order moments respectively. This problem is particularly acute for the highest classes of Hs where the number of observations forming the samples are small. The scatter observed in plots of $\lambda_{(Tz/Hs)}$ against Hs (Salih, 1986) was such that no underlying trend could be detected and they are not presented here.

As an alternative approach to the full specification of the 3 parameter Weibull distribution by 'method of moments', an empirical

method was considered for definition of parameter A. Computation of remaining parameters B and C is then achieved from the first two moment estimates which are also necessary for fitting to the other probability distributions under consideration here.

Parameter $A_{(Tz/Hs)}$ represents a lower limit on the value of wave periods in the scatter diagram. If the notion of a maximum 'sea state steepness', S, is taken to be the ratio of Hs to a deep water wave length expressed in terms of this minimum wave period, it can be expressed as:

$$S = 2\pi.Hs/\{g.(A_{(Tz/Hs)})^2\} \quad (5)$$

Various steepness curves defined in this way were superimposed on the scatter diagrams, see Fig. 1, and a value of 1/12 was found to provide an upper envelope of the observations consistently for the majority of data sets. This empirical relationship has thus been incorporated into the method of modelling for the 3 parameter Weibull distribution.

A similar situation arises when employing the 2-dimensional (3 parameter) Weibull distribution since in this case an estimate of $A_{(Tz)}$, the lower limiting value of Tz from the marginal data, can render unnecessary the estimation of the sample skewnesses. In this case values of 4.0 and 3.0 have been adopted intuitively from observation of the scatter diagrams for the 'oceanic' and 'coastal' groups respectively.

Linear Correlation Coefficients $\rho_{(Hs-Tz)}$ and $\rho_{(\log(Hs)-\log(Tz))}$: These statistics are required in the specification of the 2-dimensional Weibull and Log-Normal distributions respectively. Sample estimates of the former are included in Table 1 and regression equations are given in Appendix 2.

Synthesis of Scatter Diagrams using the Regression Equations:

Test against Original Data Base: Applying the regression equations to each of the probability models provides a means of testing each approach on equal terms. The data input in these circumstances are the relevant statistics of Hs and the site exposure group, which leads to the appropriate regression equations.

Table 3 lists the 2-dimensional χ^2 values per unit observation for all sites with the exception of the 'bays' group. Overall, approach 1 (W-W) produces the minimum departures from the observed data as measured by this test whilst approach 2 (W-LN) produces the best fit in more cases. Both of these 'marginal/conditional distribution' approaches to the joint probability modelling are better than the 2-dimensional theoretical models although the Weibull form is only slightly inferior. However, although the χ^2 values do not show great disparity between any of the models, the 2-dimensional versions are significantly less able to closely represent the distributions of Tz for the higher Hs classes. This can be seen from the frequency histograms in Fig. 8 and the scatter diagrams in Fig. 9. It is immediately apparent that in the predictive mode departures from observed histograms are substantially greater than those associated

Site	W-W	W-LN	2DLN	NMI	2DW
Seven Stones	0.1278	0.1007	0.2434	0.1685	0.1185
South Uist	0.1887	0.1521	0.1836	0.1677	0.1707
W. S. Alpha	0.2286	0.2650	0.2182	0.2367	0.2165
W. S. India	0.4156	0.2564	0.1932	0.2145	0.3446
W. S. Juliett	0.0902	0.0826	0.0956	0.1228	0.0612
Famita	0.1234	0.0790	0.1289	0.1126	0.1417
Galloper LV.	0.2957	0.4904	0.7661	0.7191	0.6938
Tongue LV.	0.1657	0.1555	0.2935	0.2999	0.1817
Eddystone LN.	0.7114	0.7520	0.9474	0.9803	0.9677
Varne LV.	0.5037	0.5766	0.4047	0.4004	0.5050
Scarweather Bk.	0.2294	0.2561	0.4353	0.3550	0.2574
St. Gowan LV.	0.2109	0.2087	0.2709	0.2197	0.1486
Downing	0.1164	0.1092	0.3071	0.2877	0.1852
Smith's Knoll LV.	0.7682	0.8912	0.7454	0.7398	0.5673
North Carr LV.	0.2503	0.2816	0.2199	0.2726	0.2302
No of 'Firsts'	3	5	2	1	4
$\Sigma \chi^2$:	4.442	4.657	5.453	5.297	4.790

TABLE 3
DIMENSIONAL χ^2-VALUES, PER UNIT OBSERVATION RESULTING FROM APPLICATION OF REGRESSION MODELS TO ORIGINAL DATA BASE

TABLE 4
2-DIMENSIONAL χ^2-VALUES, PER UNIT OBSERVATION RESULTING FROM APPLICATION OF REGRESSION MODELS TO INDEPENDENT DATA

Site	W-W	W-LN	2DLN	NMI	2DW
W. S. Lima	0.1077	0.0790	0.0895	0.0808	0.1687
Owers LV.	0.4213	0.4307	0.2561	0.2523	0.2681
Cardigan Bay	0.4937	0.4842	0.5496	0.5264	0.5421

with the earlier analysis phase.

Test against Independent Data: Data sets from W.S. Lima, Owers light vessel, and Cardigan Bay (considered earlier but not used in the regression), see Fig. 2, have been compared against those synthesised by the models, Table 4 shows resulting normalised 2-dimensional χ^2 values. With such a small set of test data results cannot be conclusive but, contrary to the above findings, in these tests at least, the 2-dimensional models perform with comparable accuracy to the 'marginal/conditional' models. Further testing is clearly necessary.

FIGURE 8 CONDITIONAL FREQUENCY HISTOGRAMS OF T_z COMPARED AGAINST REGRESSION MODEL OUTPUTS

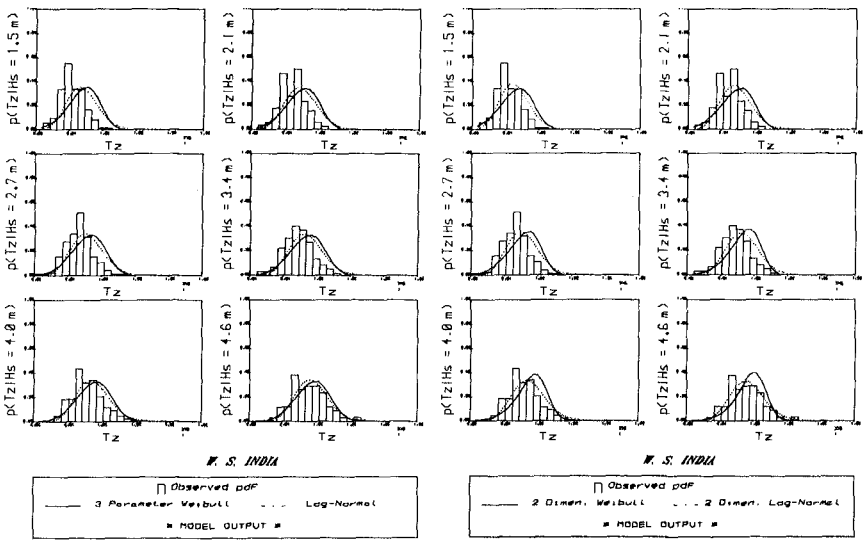
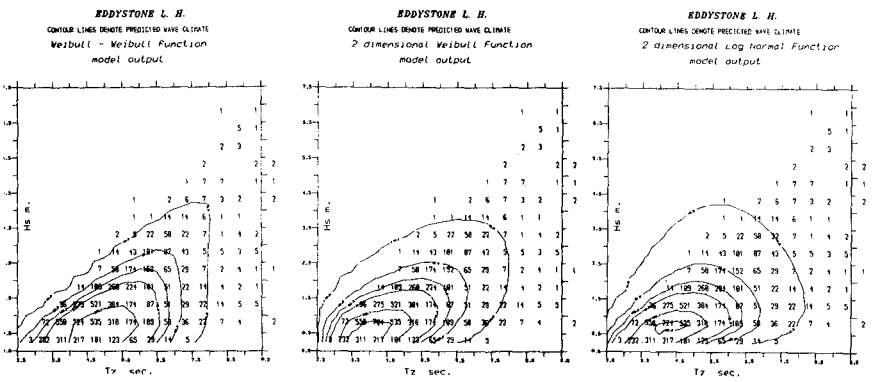


FIGURE 9 CONTOUR PLOTS FROM REGRESSION MODELS SUPERIMPOSED OVER OBSERVED SCATTER DIAGRAMS



Conclusions

(1) From an analysis of scatter diagrams from 18 sites around the British Isles it has been found that the 3 parameter Weibull distribution provides a better fit than the Log-Normal distribution to the marginal properties of Hs and Tz whilst both functions describe with near equal merit the conditional behaviour of p(Tz/Hs).

(2) Regression equations linking the statistics of the various probability distributions considered to the statistical moments of Hs have been established on the basis of two categories of site exposure, loosely termed 'oceanic' and 'coastal'. Further segregation of site classification is not practicable until the data base is extended.

(3) Links between site category, exposure conditions (such as 'effective fetch' and wind field strengths) and Hs statistics are currently under investigation although preliminary findings have proved inconclusive.

(4) Synthesis of wave climates when tested against the data sets available have shown the relative accuracy of the different approaches to follow the rank order:-

1) p(Hs)-Weibull, p(Tz/Hs)-Weibull
2) p(Hs)-Weibull, p(Tz/Hs)-Log-Normal
3) p(Hs,Tz)-2-dimensional Weibull
4) p(Hs,Tz)-NMI modified 2-dimensional Log-Normal
5) p(Hs,Tz)-2-dimensional Log-Normal

(5) Limited tests of the regression based models to independent data have not clearly substantiated the above and, in consequence, further validification is necessary.

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Appendix 1 : Probability Distributions

The probability density function (pdf) of a random variable x is denoted by p(x) and the cumulative distribution function by P(X), where,

$$P(X) = \text{Prob}(x \leqslant X) = \int_{-\infty}^{X} p(x)\, dx \qquad (I.1)$$

The various forms of probability distribution considered herein are as follows:-

(i) Weibull Distribution

$$p(x) = \frac{(x-A)^{C-1}}{B^C} .C.\exp\left\{-\left[\frac{x-A}{B}\right]^C\right\} \qquad (I.2)$$

where A, B and C are constants, representing location, scale and shape parameters of the distribution. These parameters are related to the statistical moments of x as follows:-

$$\text{Mean}, \quad \bar{x} = E\{x\} = A + B\Gamma(1+1/C)$$
$$\text{Variance}, \quad \sigma^2_x = E\{(x-\bar{x})^2\} = B^2[\Gamma(1+2/C) - \Gamma^2(1+1/C)]$$
$$\text{Skewness}, \quad \lambda_x = \frac{[\Gamma(1+3/C) - 3\Gamma(1+2/C)\,\Gamma(1+1/C) + 2\Gamma^3(1+1/C)]}{[\Gamma(1+2/C) - \Gamma^2(1+1/C)]^{3/2}} \qquad (I.3)$$

where Γ is the Gamma function

(ii) Log-Normal Distribution

$$p(x) = \frac{1}{\sqrt{2\pi}\, xb} \cdot \exp\{-\frac{1}{2b^2}(\log(x) - a)^2\} \qquad (I.4)$$

where parameters a and b are the mean and standard deviation of log(x) and are related to the moments of x as follows:-

$$\bar{x} = \exp\{a + b^2/2\}$$
$$\sigma^2_x = \exp\{2a + b^2\}[\exp\{b^2\} - 1] \qquad (I.5)$$

In the context of the paper, x can take the form of either Hs or Tz in the above equations. However, in the remaining 2-dimensional distributions it is convenient to make direct reference to the wave parameters.

(iii) 2-Dimensional Log-Normal Distribution

$$p(Hs,Tz) = \frac{1}{Hs.Tz} \cdot \frac{1}{2\pi\sqrt{1-\rho^2_{ht}}\,\sigma_h\sigma_t} \cdot \exp\{\frac{-R}{2(1-\rho^2_{ht})}\}$$

$$\text{and } R = [\frac{(h-\bar{h})^2}{\sigma_h^2} - 2\rho_{ht}\frac{(h-\bar{h})}{\sigma_h} \cdot \frac{(t-\bar{t})}{\sigma_t} + \frac{(t-\bar{t}^2)}{\sigma_t^2}] \qquad (I.6)$$

where $h = \log(Hs)$; $t = \log(Tz)$; $(\bar{h},\sigma_h)$ and $(\bar{t},\sigma_t)$ are the equivalents of (a,b) in Eq. (I.5), representing the fit parameters for the respective marginal distributions; and ρ_{ht} is the linear correlation coefficient, $\rho_{ht} = E\{(h-\bar{h}).(t-\bar{t})\}/\sigma_h\sigma_t$ (I.7)

(iv) 2-Dimensional Weibull Distribution

$$p(Hs_N,Tz_N) = \frac{m\,n}{4\beta} Hs_N^{m-1}\, Ts_N^{n-1} \exp\{-\frac{1}{2\beta}(\Phi_2 Hs_N^m + \Phi_1 Tz_N^n)\}.I_o(Hs_N^{m/2}.Tz_N^{n/2}.\gamma/\beta) \qquad (I.8)$$

where I_o is the Modified Bessel function of zero order; γ is a correlation parameter linked to the linear correlation coefficient (of the form of Eq. (I.7)) as follows:

$$\rho = \frac{\Gamma(\frac{m+1}{m})\,\Gamma(\frac{n+1}{n})\,[F(-\frac{1}{m}; -\frac{1}{n}; 1; \frac{\gamma^2}{\Phi_1\Phi_2}) - 1]}{\sqrt{[\Gamma(\frac{m+2}{m}) - \Gamma^2(\frac{m+1}{m})]\,[\Gamma(\frac{n+2}{n}) - \Gamma^2(\frac{n+1}{n})]}}\ ;\ \beta = \Phi_1\,\Phi_2 - \gamma^2 \qquad (I.9)$$

F is the Hypergeometric function; $m \equiv C_{Hs}$, $n \equiv C_{Tz}$, $\Phi_1 \equiv \frac{1}{2}(B_{Hs})^m$,

$\Phi_2 \equiv \frac{1}{2}(B_{Tz})^n$, $Hs_N \equiv (Hs - A_{Hs})$ and $Tz_N \equiv (Tz - A_{Tz})$.

Parameters A, B and C are the Weibull parameters from the marginal distributions of Hs and Tz as appropriate.

Appendix 2 : Regression Equations

In the following (O) represents the 'oceanic' group of data sites and (C) represents the 'coastal' group. Using the definition of Eq. (3), most statistics have been expressed in the form,

$$Y = a\ (H_s)^b\ (\sigma_{(Hs)})^c\ (\lambda_{(Hs)})^d$$

Y		a	b	c	d
$\bar{T}_z$	(O)	4.482	0.51	-0.11	0.70
	(C)	8.758	-0.67	0.80	-0.41
$\sigma_{(Tz)}$	(O)	11.453	-3.55	3.69	-1.40
	(C)	2.984	-1.59	1.75	-0.75
$\rho_{(Hs-Tz)}$	(O)	2.329	-2.88	4.23	-2.71
	(C)	0.348	-0.28	0.14	1.26
$\rho_{(logHs-logTz)}$	(O)	3.353	-3.73	5.48	-3.68
	(C)	0.429	-1.20	0.30	1.03
κ	(O)	2.872	0.85	-0.44	1.12
	(C)	6.510	-0.42	0.68	-0.54
ν	(O)	3.484	-2.72	2.24	-1.59
	(C)	0.326	1.05	-1.11	0.82
* μ	(C)	3.0	-1.40	1.70	-0.88
ν	(C)	-0.172	-1.10	0.305	0.531

* for 'coastal' sites: $\sigma_{(Tz/Hs)} = \mu \exp(-\nu H_s)$
  for 'oceanic' sites: $\sigma_{(Tz/Hs)} = (0.75 - 0.01 H_s + 0.07 H_s^2)\ \sigma_{(Tz)}$