

CHAPTER TWO HUNDRED FOURTEEN

COMPARISON OF TURBULENT LATERAL MIXING MODELS

William G. McDougal and Robert T. Hudspeth¹

ABSTRACT

A variety of different lateral mixing models have been employed in the equation of motion for determining wave-induced longshore currents. The equation of motion is cast into a general form which enables a comparison of the various models. Analytic solutions for longshore currents are developed for seven different mixing models. A nonplanar beach profile is employed because it has been found to be representative of many beaches and it allows for a distinction between offshore and depth scaling of eddies. The seven different mixing models include models which vary monotonically with horizontal distance offshore and models which change form at the breaker line. Numerical results indicate that the longshore current profile is rather insensitive to the form of the mixing model for nonplanar beach profiles.

INTRODUCTION

Horizontal mixing due to turbulence has a significant influence on the longshore current profile. The equation of motion describing the longshore velocity is not forced seaward of the breaker line if energy flux is conserved. In the absence of lateral turbulent mixing, a discontinuity is developed in the longshore current at the breaker line for simple periodic waves. This physically unrealistic solution is improved by the inclusion of lateral turbulent mixing. The lateral diffusion of momentum flux couples the solutions across the breaker line and eliminates the velocity discontinuity.

There is a general concensus that lateral mixing should be included in the equation of motion. However, there is little agreement on the exact mathematical form of the mixing term and several different models may be found in the literature.

One of the earliest models is that of Inman, et al. (1971) in which an empirical relationship was determined from field studies of dye dispersion. Two physical discussions were presented to justify the model, a mixing length approach and a friction velocity based on the radiation stresses. For a given wave and beach, the model predicted a constant eddy viscosity. Bowen (1969) had assumed a constant eddy viscosity for estimating longshore currents and provided an estimate of the eddy viscosity based on laboratory measurements. Thornton (1970)

¹Ocean Engineering Program, Department of Civil Engineering, Oregon State University, Corvallis, Oregon 97331.

proposed a model based on the product of the water particle velocity and excursion length to develop a mean Reynolds stress due to wave motion. The magnitude of the eddy viscosity increased out to the breaker line, but decreased seaward of the breaker line. Longuet-Higgins (1970b) suggested that the scaling for the eddies is proportional to the horizontal distance offshore and the shallow water wave celerity. This eddy viscosity increases monotonically with distance offshore. Jones (1975) developed an eddy viscosity by analogy with mass transport and Fickian flux. The model provided reasonable estimates for laboratory data, but was in very poor agreement with the field measurements of Inman, et al. (1971). Kraus and Sasaki (1979) following Madsen, et al. (1978) used a model which was proportional to the maximum wave orbital velocity at the bottom. Battjes (1975) provided an energy dissipation derivation that led to an eddy viscosity similar to the Longuet-Higgins model for planar beaches. However, this similarity was only for planar beaches because the Battjes model was slope-dependent. This model has been extended by Battjes (1983) to include the diffusive transport of turbulence and a solution for the model was presented by Visser (1984).

In order to evaluate the effects of these various mathematical models for mixing, a general expression for the equation of motion is developed which allows examination of the different mixing models. Analytical solutions for longshore current on a nonplanar beach profile are developed for a variety of mixing models. Numerical results indicate that the longshore current profile is rather insensitive to the choice of any of these individual lateral mixing models. This insensitivity has also been noted by several other investigators. Wu (personal communication) employing a two-dimensional nonlinear numerical model observed little change in the longshore current profile for very different eddy viscosity models. Kraus, et al. (1980) examined a variety of mixing models and again noted a lack of sensitivity. The method employed to compare different mixing models in this paper is similar to the technique used by Kraus, et al. (1980).

EQUATION OF MOTION

The time- and depth-averaged longshore equation of motion for the nonplanar beach profile shown in Fig. 1 is given by [Longuet-Higgins (1970b)]

$$-\frac{d}{dx} S_{xy} + \tau_{by} + \frac{d}{dx} (\mu_e d \frac{d}{dx} v) = 0 \quad (1)$$

in which S_{xy} is the onshore-longshore component of radiation stress; τ_{by} is the bottom stress; μ_e is an eddy viscosity; d is the total depth (still-water-depth plus the wave-induced setup); and v is the longshore current. Implicit in this equation are the assumptions of steady state; of no longshore gradients, of slowly varying depth, of no surface stresses, of small viscous transport of momentum with respect to the turbulent transport, and of an eddy viscosity model. The eddy viscosity model may be represented by a general expression that will permit comparisons between various mathematical models that have been used to represent the turbulent stress.

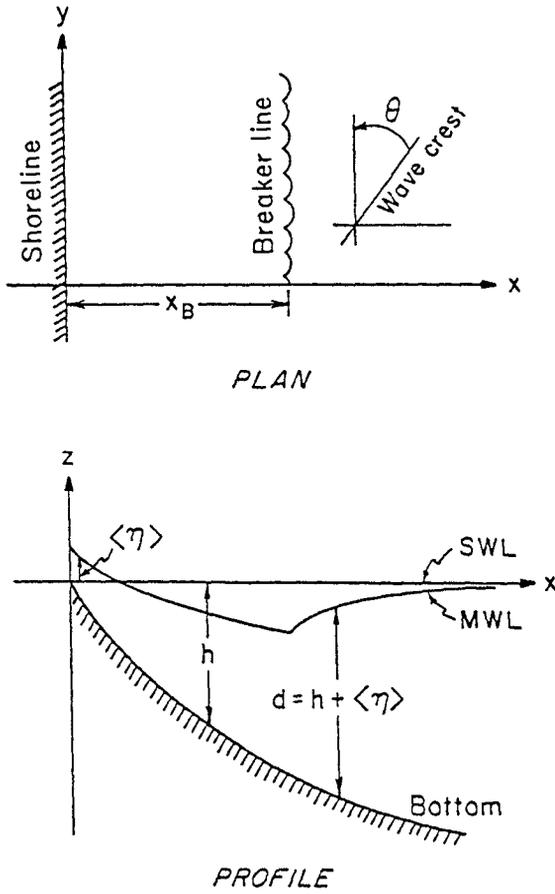


Figure 1. Definition Sketch

The divergence of the radiation stress is given by

$$\frac{d}{dx} S_{xy} = \begin{cases} -\frac{5}{16} \rho g \kappa^2 d \sqrt{d/d_B} \sin \theta_B \frac{d}{dx} d & ; x < x_B \\ 0 & ; x > x_B \end{cases} \quad (2a)$$

$$(2b)$$

in which ρ is the density of water; g is the acceleration due to gravity; κ is a breaker index; θ is the wave angle; and the subscript B denotes the value at the breaker line. It is assumed that wave energy flux is conserved seaward of the breaker line, that wave angles are small, that shallow-water wave conditions exist shoreward of the breaker line, and that the breaking wave height is proportional to the local water depth ($H = \kappa d$).

The bottom shear stress may be approximated using linear, shallow water wave theory and small-angle relationships [cf. Liu and Dalrymple (1978)]

$$\tau_{by} = -\frac{C_f}{\pi} \kappa \rho \sqrt{gd} v \quad (3)$$

in which C_f is an empirically determined friction coefficient of order 0.01 and π is a numerical constant.

Longuet-Higgins (1970b) assumed that the eddy viscosity coefficient is proportional to a velocity scale and a length scale was taken to be a linear function of the horizontal distance offshore. This particular choice for horizontal scale of the turbulent eddies is limited by the shoreline boundary inside the surf zone. However, for a planar beach this scaling gives the same offshore dependency as eddies which are limited by the depth. Therefore, it is of interest to examine a non-planar, concave-up beach profile where these two types of eddy scaling may be treated differently. An eddy viscosity model similar to that used by Longuet-Higgins (1970b) will be adopted, but the length scale of the eddies will be considered to be an arbitrary exponential function of the horizontal distance offshore. The following general expression allows comparison with other turbulence models by varying the exponent to represent the scale of the eddy size; i.e.,

$$\mu_e = N\rho (gd)^{1/2} \frac{x^p}{x_B^{p-1}} \quad (4)$$

where N is a numerical constant and p is a shape factor for scaling the eddy size.

Introducing (2), (3), and (4) into (1) gives

$$\frac{N\pi}{C_f \kappa} x_B^{1-p} \frac{d}{dx} \left(x^p d^{3/2} \frac{d}{dx} v \right) - d^{1/2} v = \begin{cases} -\frac{5}{16} \frac{\pi}{C_f} g^{1/2} \kappa \frac{\sin \theta_B}{d_B^{1/2}} d^{3/2} \frac{d}{dx} d & ; x < x_B \\ 0 & ; x > x_B \end{cases} \quad (5a)$$

$$(5b)$$

Nondimensionalizing all length scales by the horizontal surf zone width, x_B , and the velocity scales by the no-mixing planar beach velocity at the breaker line, v_{BL} , given by Longuet-Higgins (1970a), the dimensionless longshore equation of motion (denoted by upper case letters) becomes

$$\frac{N\pi}{C_f \kappa} \frac{d}{dX} \left(X^p D^{3/2} \frac{dV}{dX} \right) - D^{1/2} V = \begin{cases} -\Delta D^{3/2} \frac{dD}{dX} & ; X < 1 \\ 0 & ; X > 1 \end{cases} \quad (6a)$$

$$(6b)$$

in which

$$X = x/x_b \quad (6c)$$

$$V = v/v_{BL} \quad (6d)$$

$$\Delta = \left(\frac{x_B}{d_B} \right)^2 \quad (6e)$$

$$v_{BL} = \frac{5}{16} \frac{\pi \kappa}{C_f} s v \sin \theta_B \sqrt{gd} \quad (6f)$$

and s is the planar beach slope including setup.

The boundary conditions which are to be imposed to determine the integration constants require that the velocities be bounded at both the shoreline ($X = 0$) and offshore ($X \sim \infty$) and that both the magnitude of and the gradient of velocity be continuous at the breaker line ($X = 1$).

BEACH PROFILE

Equations (6) are expressed in terms of an arbitrary total water depth, $D(X)$. However, this depth profile must increase monotonically with horizontal distance offshore. This restriction is due to the breaker index which relies on the so-called spilling breaker assumption in which the breaking wave height is considered to be proportional to the local water depth. This assumption is frequently employed in analytical longshore current models and is obviously a weak assumption. However, it is not the intent of this paper to improve on breaking wave models, but rather to cast the equation of motion into a somewhat standard form and to then examine the influence of various mathematical expressions for the lateral mixing models on the longshore current profile.

The monotonically increasing depth profile is required to avoid wave heights which increase over inshore troughs. An arbitrary, but monotonic, depth profile is given by

$$D = B X^q \quad (7)$$

in which q is an arbitrary shape factor that need not be an integer and B is analogous to slope. Equation (7) allows the equation of motion to be expressed in terms of a general depth profile according to

$$P \frac{d}{dX} \left[X^{(3/2 + q + p)} \frac{d}{dX} v \right] - X^{1/2 + q} v = \begin{cases} -q X^{5/2 + q - 1} & ; X < 1 \quad (8a) \\ 0 & ; X > 1 \quad (8b) \end{cases}$$

in which P is a mixing strength parameter defined by

$$P = \frac{N\pi}{C_f \kappa} B \quad (8c)$$

Equations (8) are general and may be applied to a variety of beach profiles which increase monotonically in depth offshore. For $p=q=1$, (8) reduce to the planar beach equation of Longuet-Higgins (1970b). For $p=1$, $q=1/2$ (8) reduce to the Bruun profile evaluated by McDougal and Hudspeth (1983a). Rather complex beach profiles may be modeled piecewise using (8) along with appropriate matching of boundary conditions at the ends of each piecewise segment. This approach was employed by McDougal and Hudspeth (1984a) to model a composite beach having a non-planar, concave-up beach profile in the offshore segment with a planar beach face on the nearshore segment.

Natural beaches tend to be concave-up rather than planar. A beach profile in which the still-water-depth is proportional to the horizontal

distance offshore raised to the 2/3 power has been deduced from physical arguments by Bruun (1954), Dean (1977), and Bowen (1978). In an examination of a total of 502 beach profiles along the Atlantic and Gulf coasts of the United States, Dean (1977) estimated a relationship between the coefficient of proportionality and the grain size. The 2/3 power profile has also been observed along the Danish coast by Bruun (1954).

McDougal and Hudspeth (1983) determined the setup/setdown on this 2/3 type beach profile and found the total depth (still-water-depth plus wave-induced setup) to be well approximated by a total water depth profile that is proportional to the horizontal distance offshore raised to the 1/2 power. This 1/2 power total depth profile will be employed in this study. This profile enables a distinction between the length scale associated with the offshore distance and local depth length scale in the eddy viscosity.

EDDY VISCOSITY MODELS

Analytical solutions are developed for seven different eddy viscosity models for the 1/2 power beach profile. Accordingly, $q = 1/2$ in (8) for all values of p used below. The relevant parameters for comparing the models are summarized in Table 1. Solutions to (8) have been given by Hildebrand (1976). Longshore current profiles are plotted for each model in Figures 2 and 3 for mixing strengths of 0.1 and 1.0, respectively.

Linear Scale: $p = 1$

The eddy viscosity model employed by Longuet-Higgins (1970b) assumed that the velocity and length scales were proportional to the shallow-water wave celerity and the horizontal distance offshore. This corresponds to a value of $p=1$ in (8). Solutions to the equation of motion satisfying the boundary conditions are given in McDougal and Hudspeth (1984b).

Depth Scale: $p = 1/2$

Battjes (1975) has suggested that a more appropriate length scale for eddies is the depth rather than the horizontal distance offshore. For an eddy length scale proportional to the depth this yields $p = 1/2$ for a nonplanar, $X^{1/2}$ concave-up profile.

Maximum Scale: $p = 3/2$

It may be shown by the method of Frobenius (Hildebrand, 1976) that the maximum value p may obtain is 3/2.

Constant Scale: $p = -1/4$

Bowen (1969) assumed and Inman, et al. (1970) observed that the eddy viscosity was a constant independent of the horizontal distance offshore. For an $X^{1/2}$ beach profile, a constant eddy viscosity given by (4) requires that $p = -1/4$.

Table 1. Summary of Mixing Models for Nonplanar Beaches (q=1/2)

Model	p	Mixing Strength (P)
Linear Scale	1	$\frac{N\pi}{C_f \kappa} B$
Depth Scale	1/2	$\frac{N\pi}{C_f \kappa} B$
Maximum Scale	3/2	$\frac{N\pi}{C_f \kappa} B$
Constant Scale	-1/4	$\frac{N\pi}{C_f \kappa} B$
Wave Kinematic Scale	1/4 ; X < 1 -7/4 ; X > 1	$\frac{\pi \kappa}{8 C_f} \frac{B^2}{\omega} \left(\frac{g}{d}\right)^{1/2}$
Modified Linear Scale	1 ; X < 1 1/2 ; X > 1	$\frac{N\pi}{C_f \kappa} \beta$
Energy Dissipation Scale	1/3 ; X < 1 (4r ₂ +3)/12 ; X > 1	$\frac{N\pi}{C_f} \left(\frac{5}{32} \kappa^2 B^7\right)^{1/3}$

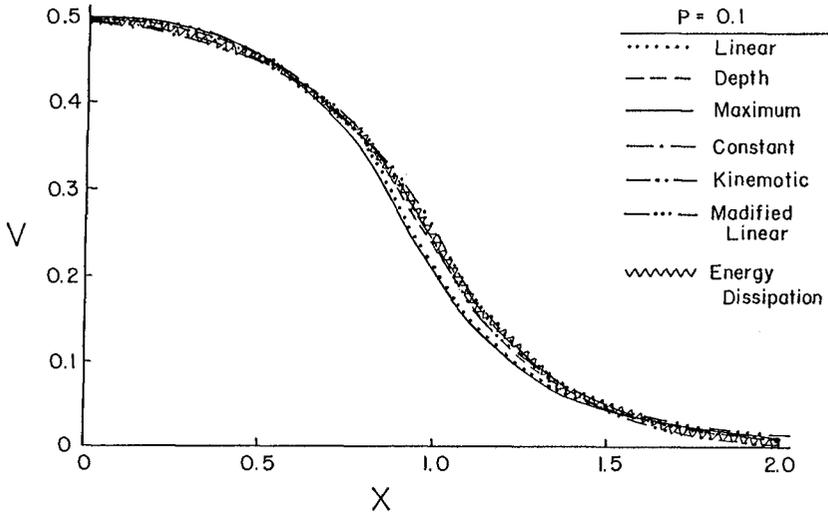


Figure 2. Comparison of Longshore Current Profiles (P = 0.1, q = 0.5)

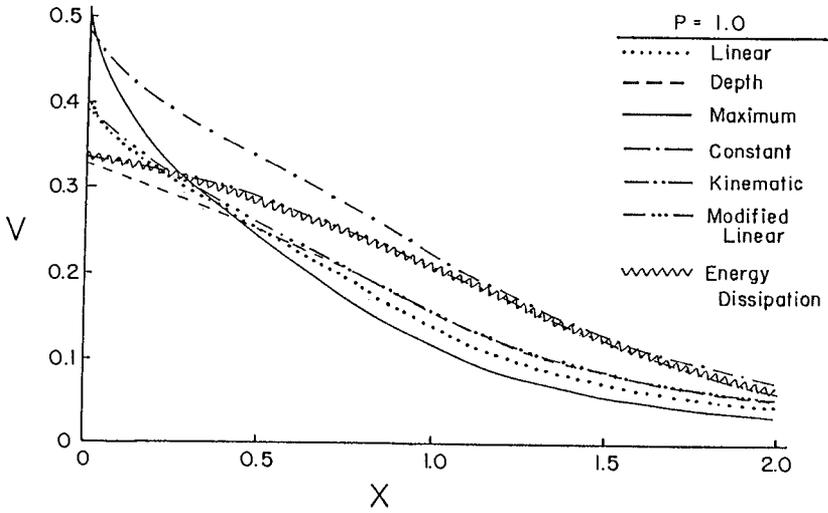


Figure 3. Comparison of Longshore Current Profiles ($P = 1.0, q = 0.5$)

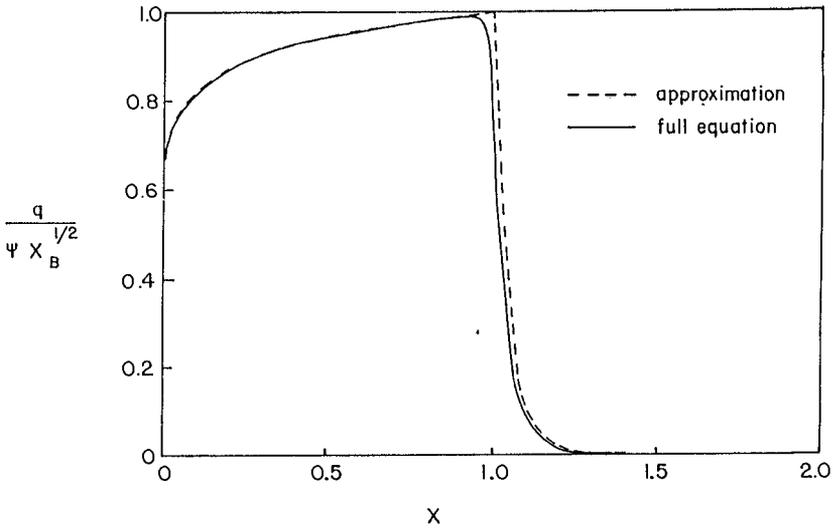


Figure 4. Comparison of Actual and Approximate Velocity Scales for the Energy Dissipation Model

Thornton Model Scale: $p = 1/4, - 7/4$

Thornton (1970) proposed a model which relates the eddy viscosity to the absolute value of the time-averaged product between the water particle excursion due to the wave motion, l' , and the water particle velocity fluctuation due to the waves, u' . This model is given by

$$\mu_e = \rho \langle |u' l'| \rangle \quad (9)$$

where $\langle \cdot \rangle$ is the temporal averaging operator over the wave period.

Evoking the both small wave angle assumption and linear wave theory at the bottom, (9) may be written as

$$\mu_e = \frac{\rho \omega H^2}{8 \sinh^2 kd} \quad (10)$$

in which k is the wave number and ω is the radian wave frequency. Inside the breaker line shallow water assumptions are made and the eddy viscosity is given by

$$\mu_e = \rho g \frac{\kappa^2}{8\omega} d \quad (11)$$

Substitution of this eddy viscosity into (1) instead of using (4) results in a different mixing strength parameter, P_T , given by

$$P_T = \frac{\pi \kappa B^2}{8 C_f \omega} \left(\frac{g}{d} \right)^{1/2} \quad (12)$$

and the resulting equation of motion is equivalent to $p = 1/4$.

Seaward of the breaker line the wave-induced turbulence is much less. The eddy viscosity model proposed by Thornton (1970) changes form at the breaker line. The waves are no longer breaking and the shallow water assumption is inappropriate. In this relatively deeper region, the eddy viscosity decays approximately hyperbolically with increasing distance from shore. Assuming deep water conditions, using the concave beach profile and requiring μ_e to be continuous at the breaker line, the offshore eddy viscosity is approximately given by

$$\mu_e = \rho g \frac{\kappa^2}{4\omega} d_B \left(\frac{d}{d_B} \right)^{3/2} \quad (13)$$

The mixing strength parameter is the same as for $X < 1$ given by (16) and the resulting equation of motion is equivalent to $p = -7/4$.

Modified Linear Scale: $p = 1, 1/2$

Kraus and Sasaki (1979) developed a model which is applicable in the offshore employing results from Madsen, et al. (1978). Requiring continuity of the eddy viscosity at the breaker line, this model may be approximated as

$$\mu_e = \begin{cases} N\rho \times \sqrt{gd} & ; X < X_B & (14a) \\ N\rho \times \sqrt{gd} \frac{d_B}{d} & ; X > X_B & (14b) \end{cases}$$

which is the same as the Longuet-Higgins model shoreward of the breaker line. The modification in the offshore region reduces the rate at which the eddy viscosity increases, but it still increases monotonically. The resulting mixing strength parameter is given by (8c) and the equation of motion corresponds to $p = 1$ for $X < 1$ and $p = 1/2$ for $X > 1$.

Energy Dissipation Scale: $p = 1/3, (4r_2 + 3)/12$

Battjes (1975) estimated the horizontal turbulent momentum exchange from an examination of the turbulent energy dissipation. The model turns out to be similar in form to an eddy viscosity model, but this assumption was not made a priori. However, this similarity was only for planar beaches because the model was slope-dependent. This model was not applicable seaward of the breaker line. Citing results from dye studies in the surf zone, Battjes (1975) observed that the turbulence is primarily contained within the surf zone, but that there must be some production of turbulence seaward of the breakers to avoid a velocity discontinuity at the breaker line. By including the diffusive transport of the turbulent momentum, Battjes (1983) extended the range of application of the turbulence model to include the entire nearshore region. A solution to the equation proposed by Battjes (1983) for the balance of turbulent energy was presented by Visser (1984).

$$\mu_e = M \rho \kappa d q \tag{15a}$$

in which

$$q = \begin{cases} [A_1 x^{r_1} + \frac{5}{16} \kappa^2 g^{3/2} d^{3/2} \frac{d}{dx} d]^{1/3} & ; x < x_p & (15b) \\ [A_2(x-a)^{r_2}]^{1/3} & ; x > x_p & (15c) \end{cases}$$

and M is an empirical coefficient of order 1; r_1 , r_2 , and a are wave field parameters; and A_1 and A_2 are integration constants which can be

determined by requiring continuity of q and the gradient of q at the plunge point, x_p . The production of turbulence begins at the location of actual breaking, x_p , not the location of the initiation of breaking, x_{B1} and this effect was included in the Visser solution. However, this effect was not included in any of the other models. To allow direct comparison of the models it will be assumed that $x_p = x_B$.

When the nonplanar beach profile is employed in Eqs. (15) and the wave field parameters are evaluated, the resulting forms for the eddy viscosity are not compatible with Eq. (4). Therefore, the following approximations are used

$$q = \begin{cases} \psi x^{1/12} = \psi x_B^{1/12} X^{1/12} & ; X < 1 & (16a) \\ \psi x_B^{1/12} \left(\frac{x}{x_B}\right)^{1/3} r_2 = \psi x_B^{1/12} X^{1/3} r_2 & ; X > 1 & (16b) \end{cases}$$

where

$$\psi = \left(\frac{5}{32} \kappa^2 g^{3/2} \beta^{5/2}\right)^{1/3} \quad (16c)$$

These approximations are compared with the full equation for a nonplanar profile and with $x_p = x_B$ in Figure 4. Upon substitution into the equation of motion, a different mixing strength parameter, P_B , results.

$$P_B = \frac{M\pi}{c_f} \left(\frac{5\kappa^2}{16} B^7\right)^{1/3} \quad (17)$$

The resulting eddy scale of (4) requires that $p = 1/3$ and $p = (4r_2+3)/12$ for $x < 1$ and $x > 1$, respectively. The solution is sensitive to the breaking wave height. Larger wave heights yield velocity profiles which appear to have lower lateral turbulent mixing strengths. The numerical results shown in Figures 2 and 3 are for small wave height conditions.

DISCUSSION

Solutions to the longshore equation of motion have been developed for seven different lateral mixing models on a nonplanar beach profile which are summarized in Table 1. Longshore velocity profiles obtained for these solutions are shown in Figures 2 and 3 for each mixing model for mixing strengths of $P = 0.1$ and $P = 1.0$, respectively. There are relatively small differences between these velocity profiles, and these differences decrease as the mixing strength decreases due to the obvious requirement that all of the models must converge for $P = 0$.

The differences between the equations listed in Table 1 for the mixing strength parameters defined by Battjes and by Thornton compared to the other four models appear to be significant. However, the dif-

ferences in the numerical values for the longshore current computed by each of these models are rather small and most of the models have similar dependencies on the various parameters. Each of the models exhibit similar dependencies on mixing for nonplanar beaches in that steeper slopes yield greater mixing strengths. Increasing the bottom roughness results in decreasing the strength of lateral mixing in each model. All of the models listed in Table 1, except the wave kinematic and the energy dissipation, are inversely related to the breaker index. This implies that if the spilling wave longshore current models are applied to plunging or surging breakers the mixing should be reduced. The wave kinematic model is linearly dependent on the wave period such that longer waves will be more mixed than shorter waves. The energy dissipation model is also a function of the wave conditions with larger wave heights being less mixed.

These numerical results demonstrate that the longshore current profile is only weakly dependent on the analytical form of the lateral mixing model employed. This observation suggests that any reasonable analytical model for the lateral turbulent mixing may be selected for a particular application based solely on its computational efficiencies. However, the kinematic and energy dissipation models provide more physically realistic estimates of mixing. The velocity profiles are obviously sensitive to the strength of mixing. Komar (1975) suggests that values for the mixing strength should generally be less than 0.4.

REFERENCES

- Battjes, J.A. (1975). Modeling of turbulence in the surf zone. Proc. Symp. on Modeling Techniques, 1050-1061.
- Battjes, J.A. (1983). Surf zone turbulence. Proc. XXth Congress of IAHR.
- Bowen, A.J. (1969). The generation of longshore currents on a plane beach. J. of Mar. Res., 27:206-215.
- Bowen, A.J. (1978). Simple models of nearshore sedimentation; beach profiles and longshore bars. Proc. Conf. Coastlines of Canada.
- Bruun, P. (1954). Coastal Erosion and Development of Beach Profiles, Beach Erosion Board Tech. Memo. No. 44, 82 pp.
- Dean, R.G. (1977). Equilibrium Beach Profiles; U.S. Atlantic and Gulf Coasts, University of Delaware Ocean Engr. Rept. No. 12, 45 pp.
- Hildebrand, F.B. (1962). Advanced Calculus for Applications, 2nd Edition, Prentice-Hall, Inc., New Jersey, 733 pp.
- Inman, D.L., R.J. Tait and C.E. Nordstrom (1971). Mixing in the surf zone. J. of Geophys. Res., 76:3493-3514.
- Jones, D.L. (1975). The Effect of Vertical Seawalls on Longshore Currents, Ph.D. Thesis, University of Florida, 189 pp.

- Komar, P.D. (1975). Longshore currents and sand transport on beaches. Proc. Civil Engr. in the Oceans/III, 1:333-354.
- Kraus, N.C. and T.O. Sasaki (1979). Influence of wave angle and lateral mixing on longshore current. Marine Sci. Comm., 5:91-126.
- Kraus, N.C., N. Mimura, and K. Horikawa (1980). Investigations of dispersion in and around the surf zone. Proc. 27th Conf. on Coast. Engr. in Jap., ISCE, pp. 173-177 (in Japanese).
- Liu, P.L-F and R.A. Dalrymple (1978). Bottom frictional stresses and longshore current due to wave with large angles of incident. J. of Mar. Res., 36:357-375.
- Longuet-Higgins, M.S. (1970a). Longshore currents generated by obliquely incident sea waves, 1. J. of Geophys. Res., 75:6778-6789.
- Longuet-Higgins, M.S. (1970b). Longshore currents generated by obliquely incident sea waves, 2. J. of Geophys. Res., 75:6790-6801.
- Madsen, O.S., D.W. Oslendorf, and A.S. Reyman (1978). A longshore current model. Proc. Coastal Zone 78, ASCE, 3:2332-2341.
- McDougal, W.G. and R.T. Hudspeth (1983). Wave setup/setdown and longshore currents on nonplanar beaches. Coast. Engr., 7:103-117.
- McDougal, W.G. and R.T. Hudspeth (1984a). Longshore sediment transport on Dean beach profiles. Proc. 19th Coast. Engr. Conf.
- McDougal, W.G. and R.T. Hudspeth (1984b). Influence of turbulent mixing on longshore currents. Ocean Engr., in review.
- Thornton, E.B. (1970). Variation of longshore current across the surf zone. Proc. 12th Coast. Engr. Conf., 291-308.
- Visser, P.J. (1984). Uniform longshore current measurements and calculations. Coast. Engr. Conf. Abstracts, pp. 212-213.

ACKNOWLEDGEMENTS

The authors wish to thank J.A. Battjes, P.J. Visser, N.C. Kraus, and C.S. Wu for their comments and the information which they generously provided.

This research was supported by the Oregon State University Sea Grant College Program, National Oceanic and Atmospheric Administration Office of Sea Grant, Department of Commerce, under Grant No. NA81AA-D-00086 (Project No. R/CE-13). The U.S. government is authorized to produce and distribute reprints for governmental purposes, notwithstanding any copyright notation that may appear hereon.