CHAPTER ONE HUNDRED FIFTY FOUR

OSCILLATORY BOUNDARY LAYER FLOW OVER RIPPLED BEDS

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ABSTRACT

Characteristics of the oscillatory boundary layer flow above rippled beds were investigated through experiments and numerical calculations. Experiments were conducted in an oscillatory flow tunnel. Velocities above symmetric and asymmetric ripples were measured with split-hot-film sensors under conditions of both sinusoidal and asymmetric oscillations. The stress field in the boundary layer was evaluated based on the distributions of the measured velocity and Reynolds stress. Relations between vortex formation and turbulence were examined, and effects of the asymmetry of oscillatory main flow and of ripple form on the velocity field were discussed. Numerical calculations were carried out by integrating the Navier-Stokes equations with an implicit finite difference scheme. Formation of a lee vortex above ripples was simulated in the calculations. The bottom shear stress and the energy dissipation rate were estimated based on the results of the experiments and calculations.

INTRODUCTION

In order to understand the mechanisms of sediment movement as well as wave energy dissipation, it is important to investigate the characteristics of oscillatory boundary layer flow above rippled beds. The oscillatory flow over ripples is complex, since large vortices are formed in the leeside of the ripples and turbulence is produced intermittently. Quantitative understanding of the velocity and the stress field in the boundary layer is essential because of its relevance to various coastal processes such as sand transport, wave damping and mass transport.

Ripples take various shapes depending on the wave condition and the property of bed materials. It has been reported that the asymmetry of

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both the ripple shape and the time history of velocity near the bottom exerts a strong influence on the direction and the amount of net sediment transport. In order to accurately estimate the net sediment transport rate, extensive investigations under the conditions of asymmetric oscillations are required.

A number of theoretical and experimental studies have recently been made on the velocity distribution in the oscillatory boundary layer over sand ripples. Some of them are briefly reviewed in the following. Sleath (1974) calculated the flow field by explicit numerical integration of the Navier-Stokes equations and suggested that the flow might remain laminar up to considerably high Reynolds numbers. Longuet-Higgins (1981) applied the inviscid discrete vortex method to the simulation of oscillatory flow over steep ripples. He estimated the value of the drag coefficient of a ripple from the momentum balance of the simulated flow and found the estimated value to be in good agreement with experiments. Toit and Sleath (1982) measured distributions of the horizontal velocity component over rippled beds by using a laser-doppler-anemometer and compared them with those given by available theories. Detailed measurements of the velocity field have been performed by Sawamoto et al. (1982) and by the present authors (1982), with the aids of improvement of measuring technique. However, since the oscillatory boundary layer over ripples involves the complex motion of unstationary inhomogeneous turbulence, a quantitative analysis is not yet satisfactorily accomplished.

The first objective of the present study is to understand the characteristics of the velocity and stress field in the boundary layer through experiments. The second objective is to develop a numerical model to calculate such complicated boundary layer flow with a good accuracy. The third objective is to evaluate the bottom shear stress and the energy dissipation rate based on the measured and calculated results.

EXPERIMENTAL FACILITIES, CONDITIONS AND PROCEDURE

Experiments were conducted in an oscillatory flow tunnel, which consists of a loop of closed conduits and a hydraulically-driven piston as illustrated in Fig. 1. The test section is 2m long, 24cm wide and

![Fig. 1 Plan view of the oscillatory flow tunnel.](image)
31 cm in height. The motion of the piston is controlled through an electro-hydraulic servo-system, so that main flows of arbitrary velocity variation can be produced by inputting appropriate voltage signals. The period of flow oscillation is variable from 0.5 s to 10.0 s, and the maximum amplitude is 70 cm at the test section. This facility enables us to make experiments under such realistic conditions as asymmetric back-and-forth motion and irregular oscillation with prototype scales. Preliminary tests were carried out in order to determine the typical size and shape of sand ripples. Well-sorted sand with mean diameter of 0.2 mm was used as the bed material. Two-dimensional ripples with sharp crests and rounded troughs were observed for sinusoidal oscillations. The shape of these ripples was found to be well approximated by often adopted following equations.

\[ z_b = \frac{\eta}{2} \sin \left( \frac{2\pi}{\lambda} \right) \]  

(1)

\[ z_t = \frac{\eta}{2} \cos \left( \frac{2\pi}{\lambda} \right) \]  

(2)

where \( \eta \) is the wave height of a ripple, \( \lambda \) the wavelength, \( X \) an auxiliary parameter, and \( (z_b, z_t) \) represents the surface of ripples. Asymmetric ripples were observed for asymmetric oscillations as illustrated in Fig. 2. We selected several shapes of self-generated sand ripples as the representative geometry. Artificial ripples of the selected ripple geometry, both symmetric and asymmetric, were made of cement mortar for the following experiments.

The artificial ripples were installed on the bottom of the test section, and velocities were measured with split-hot-film sensors at about 100 measuring points close to the bed. Measuring points were arranged systematically in a vertical plane parallel to the main flow. The grid size was 1 cm in the horizontal direction and 0.08 cm to 2.0 cm in the vertical direction.

The split-hot-film sensor consists of two platinum films on a single quartz fiber as shown in Fig. 3. The output from the two films provides a set of data of the magnitude of the velocity component normal to the axis of the fiber, \( U_y \), and of the component normal to the split plane, \( U_{\text{sin} \theta} \). The magnitude of the velocity component parallel to the split plane is calculated from these two values though its sign cannot be determined.

\[ U_0 \]  

Fig. 2 Asymmetric oscillation and asymmetric ripples.
Fig. 3 Schematic diagram of a split-hot-film sensor.

Conditions of the present experiments are listed in Table 1, together with those of the previous experiments (cases H1 and H3) performed by Hamamoto et al. (1982). The period of oscillation is 4s and the wavelength of ripples is 12cm in all the cases. Cases 1 to 6 and H1 are for symmetric ripples under sinusoidal oscillation, and H3 is the case of asymmetric ripples under asymmetric oscillation. Prior to the velocity measurements, flow visualization was made in order to grasp characteristic features of the boundary layer flow of each case. It was found that cases 1 and 2 corresponded to the laminar flow condition. In cases 3 and 4, turbulence was detected in the leeside of a ripple and these cases were considered to be in transition. The flow in cases 5, 6, H1 and H3 was regarded turbulent through a full period.

Table 1 Experimental conditions.

<table>
<thead>
<tr>
<th>Case</th>
<th>T(s)</th>
<th>U0(cm/s)</th>
<th>d0(cm)</th>
<th>λ(cm)</th>
<th>η(cm)</th>
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<tr>
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<td>4</td>
<td>19.2</td>
<td>24.4</td>
<td>12</td>
<td>1.2</td>
</tr>
<tr>
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<td>28.3</td>
<td>36.0</td>
<td>12</td>
<td>2.0</td>
</tr>
<tr>
<td>H3</td>
<td>4</td>
<td>38.5*</td>
<td>36.0</td>
<td>12**</td>
<td>2.0</td>
</tr>
</tbody>
</table>

* ) asymmetric oscillation
** ) asymmetric ripple
T : period
U0 : maximum velocity of the main flow
d0 : orbital diameter of the main flow
The velocity was decomposed into the mean velocity and turbulent fluctuations. It involves some uncertainty to define turbulence in unstationary flow, for the reason that there exist a number of fluctuating components of various frequency in the turbulent boundary layer. We assumed that the fluctuating components with higher frequency than a critical value might be attributed to turbulence. In order to determine the appropriate cut-off frequency, spectrum of the velocity record was calculated, and it was found that the power spectrum density showed an abrupt increase at 5Hz when the flow became turbulent. The higher frequency component than 5Hz of the velocity record was therefore defined as turbulent fluctuation. Equi-phase mean velocity components \( u, w \) and products of turbulent fluctuations \( u'^2, w'^2, u'w' \) were calculated by averaging equi-phase data over 30 periods.

In the following, discussions are concentrated mainly on case 6 because of space limitation. As stated before, the flow in case 6 was
turbulent through a full period. Coherent vortices began to be formed in the leeside of ripples when the velocity of the main flow reached its maximum. These vortices continued to develop until they were ejected after the flow reversal. The ejected vortices were transported over ripples during the next half period for a distance of twice the wavelength of a ripple. Figure 4 shows the time history of mean velocity components $U$, $W$ and products of turbulent fluctuation $\overline{u'^2}$, $\overline{w'^2}$, $\overline{u'w'}$ measured at points about 0.3 cm above a ripple crest and a trough. Values of $\overline{u'^2}$ and $\overline{w'^2}$ show two peaks in each half period, which corresponds to the passage of two vortices created in the leeside of the nearest and the neighboring ripples. Values of $\overline{u'w'}$ above a crest show a strong peak according to the passage of the first vortex but they do not show a peak with the passage of the second vortex. It follows that turbulence maintains a coherent structure just after the vortex ejection and that, as the vortex moves upward, turbulence decays and diffuses to show locally isotropic features.

The stress fields were evaluated by the integration of the two-dimensional momentum equations.

$$\rho \left( \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} (U^2) + \frac{\partial}{\partial z} (UW) \right) = \frac{\partial P}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}$$  

(3)

$$\rho \left( \frac{\partial W}{\partial t} + \frac{\partial}{\partial x} (UW) + \frac{\partial}{\partial z} (W^2) \right) = \frac{\partial P}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}$$  

(4)

where $P$ is the equi-phase mean pressure, $\rho$ is the density of water, and $\sigma_{xx}$, $\tau_{xz}$ and $\sigma_{zz}$ are the components of the Reynolds stress tensor defined by

$$\sigma_{xx} = 2\mu \frac{\partial U}{\partial x} - \rho \overline{u'^2}$$

$$\tau_{xz} = \mu \left( \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} - \rho \overline{u'w'} \right)$$

$$\sigma_{zz} = 2\mu \frac{\partial W}{\partial z} - \rho \overline{w'^2}$$

in which $\mu$ is the molecular viscosity.

The equi-phase mean pressure $P$ was evaluated by integrating equation (4) with respect to $z$. The mean pressure thus evaluated is the sum of the two components as follows.

$$P = P_t + P_d$$  

(5)

$P_t$ represents the pressure due to the acceleration of the oscillatory main flow and is given by the following relation,

$$P_t = -\rho \int_{x_0}^x \frac{\partial U}{\partial t} dx$$  

(6)

where $x_0$ is a reference point of which pressure is set to be zero and $U_0$ is the horizontal velocity of the main flow. $P_d$ means the component originated from the presence of the wavy boundary and is responsible for drag and lift forces.

Figure 5 shows distributions of the equi-phase mean velocity vector, Reynolds stress and mean pressure ($P_t$) in a half period. The flow is seen to be locally accelerated above a ripple crest and...
Fig. 5 Distributions of the mean velocity vector, Reynolds stress, and mean pressure (case 6).
decelerated above a trough. The process of generation and ejection of a lee vortex is clearly visualized. It is seen that the Reynolds stress is strong in the region of a lee vortex, and that, as the vortex is ejected upward, the area of high turbulence moves with it and then diffuses. $\rho$ is lower in the vortex core and higher on the upstream side of a ripple crest where the separated flow reattaches to the boundary. Characteristic velocity and stress field is created in the oscillatory boundary layer over ripples through the formation of lee vortices and the local acceleration of the flow.

Time average of the equi-phase mean velocity over one period is not zero in general. Figure 6 gives the residual current in an Eulerian frame. A pair of circulation cells is seen for the case of symmetric ripples and sinusoidal oscillation, whereas a unidirectional flow appears for the asymmetric condition. These patterns of steady streaming seem to be strongly influenced by the formation of lee vortices. Symmetric cells in case 6 are due to the formation of symmetric lee vortices, while the formation of asymmetric lee vortices is responsible for the asymmetric flow pattern in case $H_3$. Quantitative analysis of steady streaming is important because it plays an important role to determine the direction of sediment transport.

(a) symmetric condition (case 6).

(b) asymmetric condition (case $H_3$).

Fig. 6 Velocity distributions of residual current.
NUMERICAL MODELING

The boundary layer flow was calculated by numerically integrating the two-dimensional vorticity equations. The governing equations are the same with those used in the calculation made by Sleath (1974). In the present study, the finite difference scheme and the description of the boundary conditions were improved, so that we could calculate the flow at higher Reynolds numbers with a good accuracy. The governing equations were expressed in an orthogonal curvilinear coordinate system \((X,Z)\) in order to represent the wavy boundary, i.e. the shape of ripples. In this coordinate system, the two-dimensional vorticity equations are

\[
\frac{1}{J} \frac{\partial \chi}{\partial t} + \frac{\partial \psi}{\partial X} \frac{\partial \chi}{\partial Z} + \frac{\partial \psi}{\partial Z} \frac{\partial \chi}{\partial X} = \nu \frac{\partial^2 \chi}{\partial X^2} + \frac{\partial^2 \chi}{\partial Z^2} \quad \text{and} \quad J \frac{\partial^2 \psi}{\partial X \partial Z} = \chi
\]

in which \(\chi\) is the vorticity, \(\psi\) the stream function, \(\nu\) the kinematic viscosity, \(J\) the Jacobian of transformation and \(\nabla^2\) the Laplace operator given by \(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\). The relation between the coordinate system \((X,Z)\) and the Cartesian coordinate \((z,z)\) is defined as

\[
x = X - \sum \alpha_n e^{-2\pi i \lambda} \sin \frac{2\pi n}{\lambda} (X-\phi_n) \quad \text{and} \quad z = Z + \sum \alpha_n e^{-2\pi i \lambda} \cos \frac{2\pi n}{\lambda} (X-\phi_n)
\]

in which \(\alpha_n\) and \(\phi_n\) are coefficients to be determined so that the curve, \(Z = 0\), might give a good approximation of symmetric and asymmetric shape of wave-generated ripples.

The boundary conditions at the ripple surface and at the upper boundary are expressed by

\[
\frac{\partial \psi}{\partial Z} = \psi = 0 \quad \text{on} \ Z = 0
\]

\[
\psi = \psi_0 \delta \cos \omega t. \ \zeta = 0 \ \text{at} \ Z = \delta
\]

in which \(\delta\) is the height of the calculation domain, \(\psi_0\) the velocity amplitude of the main flow, and \(\omega (=2\pi/T)\) the angular frequency of the oscillation. Values of \(\psi\) and \(\zeta\) along the side boundaries are set to be periodic for the interval of a ripple wavelength.

The numerical integration was carried out by an implicit finite difference scheme, for the conditions of experimental cases 1 to 4 which corresponded to laminar flow or transition. Calculation was also conducted for the asymmetric oscillatory flow over asymmetric ripples, of which wavelength is 12cm and height is 1.7cm, with the value of stream function at the upper boundary given by

\[
\psi = \psi_0 \delta (0.8 \cos \omega t + 0.2 \cos 2\omega t)
\]
In order to examine the validity of the numerical model, the calculated and experimental results were compared. Figure 7 gives a comparison of the time history of horizontal velocity component at a point 0.3cm above a ripple crest. Agreement between the calculation and the experiment is fairly good.

Figure 8 illustrates the calculated velocity vector in a half period for the case 3. Salient features of the oscillatory flow over rippled bed are reproduced, such as the local acceleration above a ripple crest and the vortex formation in the lee side. The steady streaming was also evaluated from the calculated flow field and it was concluded that characteristic features observed in the experiments were well reproduced for both symmetric and asymmetric conditions. The present model can thus simulate the flow field in which a lee vortex is formed and transition takes place.

The flow structure in a Lagrangian frame is also of interest concerning the sediment transport. In order to study the movement of water particles, a numerical experiment was conducted by using the result of the calculation for the asymmetric condition. Water particles were placed at grid points of the numerical calculation and the movement of the particles which started at the phase of maximum velocity of the main flow was traced in one period. Figure 9 shows an example of trajectories of particles which started from grid points above the steeper flank of a ripple. Figure 10 gives the final displacement of all particles after one period. Some particles were trapped in a lee vortex formed on the steeper flank and were transported leftward and upward with the vortex ejection. There occurred no trapping to a vortex.
Fig. 8 Calculated flow field (case 3).
on the milder flank. The resultant net transport of particles near bottom became leftward. Ripples on natural beaches have the steeper flank to the onshore direction and the milder flank to the offshore direction. Although this experiment does not incorporate the effect of the settling of particles nor the interaction between the fluid and particles, the result appears to provide a suggestion on the mechanism of sediment transport to the offshore direction.

**EVALUATION OF THE BOTTOM SHEAR STRESS**

The evaluation of bottom shear stress is essential for the problem of wave damping and sediment transport. The bottom shear stress was estimated by using the results of experiments and numerical calculations. The bottom shear stress was decomposed into following two components.

\[
\tau_b = \tau_{bs} + \tau_{bd}
\]

where \( \tau_b \) is the total shear stress, \( \tau_{bs} \) is the shear stress due to skin friction and \( \tau_{bd} \) is that due to form drag. The skin friction is
important in the study of sediment transport, while the total shear stress becomes important in the estimation of wave damping.

(1) Bottom shear stress due to skin friction
The skin friction at the bottom was calculated from the results of numerical modeling by the following relation,

\[ \tau_{bs} = \mu \left( \frac{\partial}{\partial x} (f \frac{\partial \psi}{\partial z}) - \frac{\partial}{\partial x} (f \frac{\partial \psi}{\partial x}) \right) = \mu \xi_b \]

(15)

where \( \mu \) is the viscosity and \( \xi_b \) is the vorticity at the bottom. Figure 11 shows the time history of \( \tau_{bs} \) in case 3 at various points along a ripple surface. Chain lines with two dots indicate the amplitude of the bottom shear stress on a flat plate under the same flow condition. The amplitudes of \( \tau_{bs} \) at a ripple crest (solid line) and at a trough (dotted line) are about 1.6 and 0.7 times the amplitude for a flat plate respectively. These ratios are almost the same with the rates of the local acceleration of the flow. Time mean values of \( \tau_{bs} \) over one period are seen to be negative on a flank of a ripple (broken line and chain line with one dot), which means steady shear stress is exerting in the direction from a trough to a crest.

It is very difficult to estimate the value of bottom skin friction from experimental results, since there is no measurement of velocity in the immediate vicinity of the bottom boundary. We made an attempt to evaluate the bottom skin friction at a crest using the value of shear stress estimated at a measuring point closest to the bottom. The shear stress at the bottom surface was expressed by integrating equation (3) as

\[ \tau_{bs} = \mu \left( \frac{\partial}{\partial x} (f \frac{\partial \psi}{\partial z}) - \frac{\partial}{\partial x} (f \frac{\partial \psi}{\partial x}) \right) = \mu \xi_b \]

(15)

Fig. 11 Calculated history of local bottom shear stresses (case 3).
where $z'$ is the vertical distance from the bed, $z$, the height of the point closest to the bed and $\tau_{\infty}$, the shear stress estimated at $z'=z$. The integral was evaluated by means of the trapezoidal rule, where the values of $\partial U/\partial t, \partial (U^2)/\partial x, \partial (U W)/\partial z$ and $\partial \sigma_{xy}/\partial x$ were assumed to be zero at the bottom and to change linearly within the domain of integration. The value of $\partial P/\partial x$ was assumed to be constant in this domain. Figure 12 shows the time history of the bottom shear stress at a crest evaluated through the experiment (indicated by O) and the numerical model (solid line). It is found that the bottom shear stress estimated through experiments shows a strong peak when a vortex passes. Scatter of plotted data implies the need to improve the validity of the assumptions and the accuracy of the pressure.

(2) Energy dissipation rate
Consideration of the energy budget of the mean flow leads to the following equation.

$$D = \rho \frac{dU_0}{dt} \frac{1}{\lambda} \frac{\partial}{\partial t} \int \frac{1}{2} \rho (U^2 + W^2) d\lambda$$

where $D$ is the energy dissipation rate averaged over one wavelength of a
Fig. 13 Time history of the energy dissipation rate (case 6).

ripple and \( \psi_0 \) is the value of stream function at the upper edge of the boundary layer.

Figure 13 gives the time history of \( D \) in case 6 evaluated from experimental data. The energy dissipation rate is large positive when the main flow is decelerated \( (0<\omega t<\pi/2) \), and becomes negative when accelerated \( (\pi/2<\omega t<\pi) \). This means that during the stage of the deceleration of the main flow the energy of the mean flow is dissipated through the generation of turbulence, and that a part of turbulence energy is transferred back to the mean flow during a certain period of the acceleration, which may correspond to the phenomenon of reverse transition.

The time mean value of \( D \) is conventionally related to the velocity of the main flow by

\[
\bar{D} = \frac{1}{2} \rho f_s U_0 \frac{U_0}{U_0}
\]

where \( f_s \) is the energy dissipation factor. Evaluation of the energy dissipation factor through experiments was also performed by Bagnold (1946) and Carstens et al. (1969). However, since their experimental conditions are quite different from those of the present experiments, we cannot compare our results with them directly. Values of \( f_s \) estimated through the present experiments and calculations are given in Fig. 14 together with values of friction factor given by Jonsson (1966). The quantity \( k \) represents the value of equivalent roughness and \( U_0 \) is the orbital diameter of the main flow. Values of the energy dissipation factor are of the order of 0.05 and independent of the Reynolds number in the domain of the present study. The energy dissipation factor generally differs from the friction factor because there is a phase shift between the main flow velocity and the bottom shear stress. In the turbulent flow, however, this phase shift is so small that the
values of \( \lambda \) are regarded almost the same with those of the friction factor. By comparing the values of the energy dissipation factor and the friction factor in rough turbulent region of Fig. 14, we can estimate the values of equivalent roughness to be of the same order with the wave height of a ripple. Since there are only three data in rough turbulent region in the present study, more experiments under conditions of oscillatory flow are required to draw a generalized conclusion. It should also be noted that the energy dissipation due to sediment movement has to be taken into account under movable bed condition.

**CONCLUDING REMARKS**

It was concluded through experiments that lee vortices exert a strong influence on the velocity and stress fields of the oscillatory boundary layer above a rippled bed. The relation between the turbulence production and lee vortex evolution was quantitatively discussed based on the results of experiments under both symmetric and asymmetric conditions. The distribution of the residual current velocity was also presented. Using the numerical model developed, the boundary layer flow was accurately simulated up to considerably high Reynolds numbers. The bottom shear stress and the energy dissipation rate were evaluated through the results of experiments and numerical calculations. The authors intend to continue the work on the numerical modeling to make it possible to calculate the flow at higher Reynolds numbers based on the distribution of the measured Reynolds stress.
REFERENCES