CHAPTER ONE HUNDRED TEN

GRADING EFFECTS IN CONCENTRATION MEASUREMENTS

by:
J. van de Graaff *) &
J.A. Roelvink *)

ABSTRACT

In analysing measured sediment concentration distributions the effect of the non-uniformity of the bottom material is frequently not taken fully into account. Examples of calculations are given to clarify the grading effect on the parameters that determine the concentration distribution (e.g. diffusion coefficient distribution). Results of a preliminary test series indicate that the particle size affects the diffusion coefficient involved. \[ \varepsilon(\text{sediment}) = \beta \varepsilon(\text{fluid}); \beta \text{ size dependent}. \]

1 INTRODUCTION

The CERC formula is an example of an over-all description of the longshore sediment transport rate. A limited range of problems can be solved with the help of this formula since it only describes sediment transports due to wave-driven currents. In more general coastal engineering problems not only wave-driven currents but also currents like tidal currents, ocean currents and wind-driven currents play an important role. To be able to tackle the latter type of problems, one needs a transport formula to calculate the transport as a function of boundary conditions like water depth (h), wave height (H), wave period (T), current velocity (v) and bottom particle size (D50). The sediment transport rate for currents more or less perpendicular to the orbital plane of the waves can be simply described as:

*) Senior Scientific Officer
Delft University of Technology, The Netherlands

*) Student Assistant
Delft University of Technology, The Netherlands
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\[ S = \int_{0}^{h} v(z) \ast c(z) \, dz \quad (1.1) \]

where:
- \( S \) : sediment transport rate
- \( v(z) \) : current velocity (function of \( z \))
- \( z \) : height above the bottom
- \( c(z) \) : time averaged sediment concentration
- \( h \) : water depth

As far as the authors know, only the so-called Bijker formula (Bijker 1971; Van de Graaff and Van Overeem 1979) describes the transport in a physical sense according to Eq. (1.1) and can be used generally in coastal engineering problems. However, although a ready made formula is available, tests have been carried out all over the world in measuring the time average sediment concentration distribution over the water column under wave and current conditions. (Among many others: MacDonald 1977; Nielsen 1979; Bosman 1982.) Such experiments are carried out since it is felt that the concentration distribution as proposed by Bijker is not valid under all conditions.

By analysing concentration measurements one tries to find the underlying 'mechanism' which holds the particles in suspension and causes the measured distribution. Since the suspension mechanism is most likely diameter dependent, it is necessary to take fully into account the grading, the non-uniformity, of the bottom particles.

2 SUSPENSION MECHANISM AND 'THEORETICAL' DISTRIBUTIONS

Fig. 1 shows an example of a so-called time and bed averaged sediment concentration distribution over the water depth due to (random) waves running over a sloped beach profile. The samples have been taken by suction in a direction perpendicular to that of wave propagation. The 'time and bed averaged' procedure has been developed by Bosman 1982; it means that the suction time is long with regard to the wave period \((\approx 100 \, T)\) and that the suction pipe's intake nozzle is moved to and fro by a carriage over a few ripple lengths \((\approx 5 \, \lambda)\). By this procedure local effects (just above a ripple top quite different concentrations do exist than do above a ripple trough) are smoothed out.

In the example of Fig. 1, as well as in the other examples of this paper, the several measuring points have been measured simultaneously.

A distribution like in Fig. 1 is typical for random- as well as for regular waves. The fact that close to the (mostly) rippled bed high concentration rates are found, is caused by the scraping effect of the orbital motion of the waves over the bed.
Why and how the particles can reach the higher levels in the vertical, is not clear yet. The vertical orbital motion in the waves plays a convective role probably.

Whatever the mechanism may be, in this paper a simple diffusion mechanism is assumed.

Assuming a steady state condition, in general the concentration distribution over the depth can be described by the expression:

\[ w \cdot c(z) + \varepsilon(z) \cdot \frac{dc(z)}{dz} = 0 \]  \hspace{1cm} (2.1)

where:
- \( w \) : fall velocity
- \( c(z) \) : average concentration at level \( z \) above the bed
- \( \varepsilon(z) \) : eddy diffusion coefficient
- \( z \) : vertical upward directed ordinate; the bottom being \( z=0 \)

Assuming uniform material (\( w = \text{constant} \)) the distribution of \( \varepsilon(z) \) over the water depth defines the shape of the concentration distribution line over the water depth. The bottom concentration fixes the actual position of the concentration profile shape.
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As a final aim, the concentration distribution's characterizing parameters ($\epsilon(z)$ distribution) and the bottom concentration should be related to the boundary conditions. In this paper, however, a more modest aim will be pursued: analysing the effects of non-uniform material on the apparent $(z)$ distribution.

In the literature some basic $\epsilon(z)$ distributions have been suggested all leading, in case of uniform bottom material, to typical concentration distributions. Three of those $\epsilon(z)$ distributions will be mentioned, according to their names as found in literature.

a) Coleman type
b) Rouse type
c) Bhattacharya type

In Table 1 the characteristics of the $\epsilon(z)$ distributions are given together with the resulting concentration distributions.

<table>
<thead>
<tr>
<th>$\epsilon(z)$ distr.</th>
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<tr>
<td>Coleman</td>
<td>constant</td>
<td>$\epsilon(z)=\text{const.}=\epsilon$</td>
<td>$\epsilon(z)=\epsilon_0 \exp\left(-\frac{z}{\epsilon_0}\right)$</td>
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</table><p>ight)$ |
| Rouse                | parabolic     | $\epsilon(z)=\epsilon_{\text{max}} \frac{z}{h}$ | $\epsilon(z)=\epsilon_0 \left(1-\frac{z}{h}\right)$ | $\epsilon(z)=\epsilon_{\text{max}} \left(1-\frac{z}{h}\right)$ |
| Bhattacharya         | triangular    | $\epsilon(z)=\frac{\epsilon_{\text{max}}}{h}$ | $\epsilon(z)=\epsilon_0 \left(1-\frac{z}{h}\right)$ | $\epsilon(z)=\epsilon_{\text{max}} \left(1-\frac{z}{h}\right)$ |</p>

Table 1. $\epsilon_h$: bottom concentration ($z=0$)

The $\epsilon(z)$ distributions as given in Table 1 hold for sediment. In river flow sediment studies, a distinction is made frequently between $\epsilon_s$(sediment) and $\epsilon_f$(fluid). In general a simple direct relation between $\epsilon_s$ and $\epsilon_f$ is assumed:

$$\epsilon_s = \beta \epsilon_f$$

(2.2)

where: $\beta$ : a proportionality factor.

A widespread discussion is going on concerning the actual $\beta$-value; values $<1$ as well as $>1$ have been mentioned. The case $\beta>1$ seems to get the greater support at the moment. (See for instance Van Rijn 1982). Furthermore it seems most likely that $\beta$ is particle size dependent. For coarser material larger $\beta$ values can be expected.

In a unidirectional flow the relation between $\epsilon_f$ and $\epsilon_s$ seems clear. It is felt that the turbulence in the masses of water is also the working agent in holding the particles in suspension (micro-scale). Under wave action a still less understood situation exists. Most likely the micro-scale turbulence is less important than the orbital motion induced processes (macro-scale). However, as
mentioned already, in the wave action case also a simple diffusion process is assumed. With respect to $\beta$ values a similar approach is chosen as for unidirectional flow, whatever the real meaning of a $\beta$ value may be under wave action.
In section 5 some results of preliminary tests are reported concerning probable $\beta$ values.

3 RELEVANCY OF GRADING EFFECTS

Trying to fit the real concentration measurements of figures like Fig. 1 with the distributions as predicted by the standard cases of Table 1, leads to large differences. In reality none of the given types holds for most of the measured concentration distributions. A more appropriate $c(z)$ distribution should be found and is therefore one of the ultimate aims of concentration analysis studies. The 'theoretical' $c(z)$ distributions of Table 1 can, however, be used fruitfully to clarify the effect of the grading of the bottom material. Because of its simple behaviour a Coleman type $c(z)$ distribution is chosen in the next example.

Starting from an arbitrary bottom concentration, the resulting concentration distribution over the water depth has been calculated for 5 different graded bottom materials. Fig. 2 gives the particle size distributions involved; standard log-normal distributions have been chosen.

![Fig. 2 Particle Size Distributions](image)
In the calculation it is assumed that the particles in suspension close to the bed (bottom concentration) have the same particle size distribution as the original bottom material. Furthermore that amount of sand is assumed to exist of k fractions, all equal in weight. In each fraction only particles between narrow limits are present in conformity with the particle size distribution.

If k is large enough, the particle size distribution within a certain fraction can be assumed to be constant. For k=10 the characteristic particle size $D(i)$ of the i-th fraction ($1 < i < 10$) becomes:

$$D(i) = D(10 \times i - 5)$$  \hspace{1cm} (3.1)

The fall velocity $w(i)$ belonging to the particle size $D(i)$ can be computed with:

$$w(i) = \frac{1}{10} \left( 0.476 \log^2 D(i) + 2.180 \log D(i) + 3.226 \right)$$  \hspace{1cm} (3.2)

Formula (3.2) holds for salt water with a temperature of 5° C.

Assuming a constant $\varepsilon(z)$ value $\varepsilon$ over the water depth, the concentration distribution over the depth of each fraction can be computed with:

$$c(z,i) = c(b,i) \exp \left( \frac{-w(i) z}{\varepsilon} \right)$$  \hspace{1cm} (3.3)

For each fraction the same $\varepsilon$ value is assumed (thus in this calculation $\beta=1$ for each particle size).

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![Concentration Distribution](image-url)
The ultimate concentration \( c(z) \) at height \( z \) above the bottom yields:

\[
    c(z) = \sum c(z,i)
\]

(3.4)

Fig. 3 shows the result for \( \varepsilon = 0.002 \, \text{m}^2/\text{s}, \, k = 10 \) and the diameter distributions of Fig. 2, all with a D50 of 150 \( \mu \text{m} \). Depending on the rate of grading, characterized by the D90/D10 ratio, a variety of concentration curves can be found. For normal beach material the D90/D10 ratio ranges from 2 to 3.

From the calculated contributions of the fractions to the ultimate concentration at level \( z \) above the bottom, the apparent size distribution at that level can be computed. Fig. 4 gives the D50 distribution for some rates of grading.

![Fig. 4 D50 Distribution over Water Depth](image)

A quick, but rough, idea of the order of magnitude of the mistake made when one does not take into account the non-uniformity of the bottom material in analysing measured concentration distributions, can be found as follows.

Assume one has actual measuring points as indicated in Fig. 3 (see on line (2) ). Assume moreover that one does not take into account the grading effect and one is focussed on finding the constant \( \varepsilon \) value that explains the position of the measured points as well as possible. So one fits a straight line through the measuring points in Fig. 3. From the slope of that line the apparent (but wrong) \( \varepsilon \) value can be found. The ratio \( \varepsilon(\text{apparent})/\varepsilon(\text{actual}) \) is then a measure of the mistake made. In Fig. 5 some results are given. To find \( \varepsilon(\text{apparent}) \) not a complete fit calculation has been
made, but quite simply, only the real concentrations $c(0.85)$ and $c(0.15)$ have been taken into account [$c(0.85)$ means concentration at elevation $z=0.85 \, h$]. From Fig. 5 it can be seen that, depending on the rate of grading and on the actual $e$ value a variety of mistakes can be found. It seems clear that it is absolutely necessary to take grading effects into full account in analysing measured concentration distributions.

4 ACTUAL CONCENTRATION MEASUREMENTS

In Fig. 6 a measured concentration distribution is given as found during a test in a wave flume of the Fluid Mechanics Laboratory of the Delft University of Technology. The boundary conditions are as follows:

- horizontal bed with natural ripples
- water depth $h = 0.30 \, m$
- wave conditions: $H = 0.10 \, m; \, T = 1.7 \, s$
- $D_{50} = 97 \, \mu m; \, D_{10} = 75 \, \mu m; \, D_{90} = 126 \, \mu m$

The measured points have been approximated according to the procedure as briefly described in Appendix I. Starting with the (extrapolated) bottom concentration and taking into account 20 fractions the $e(z)$ distribution which describes the approximating curve precisely, can be computed. Fig. 7 presents the resulting $e(z)$ curve.
Fig. 6 Concentration Distribution Movable Bed

Fig. 7 \( \varepsilon(z) \) Distribution
(In the same figure the result is given when one does not take into account the non-uniformity of the bottom material; again large differences can be found.) The size distribution of the samples caught at the various heights above the bottom has been analysed with the help of the Delft University Settling Tube (see for details Slot and Geldof 1984) of the Fluid Mechanics Laboratory. Also the distribution according to calculations has been determined. Fig. 8 gives the comparison between measurements and calculations. Although some differences are found, the conclusion is that the general trend is rather good fit.

![Fig. 8 D10 - D50 - D90 Distribution](image)

In the calculations as described in this section, β=1 has been assumed. The suggestion of size dependent β values being greater than 1 as briefly discussed in section 2, may be an explanation of the fact that, especially at greater heights above the bed, the D90 values are underestimated by the calculations and the D10 values slightly overestimated.

In order to gain some insight into possible β values a preliminary test has been carried out in one of the flumes of the Delft Hydraulics Laboratory.

5 EFFECT OF GRAIN SIZE ON β-VALUES

The research on concentration distributions is one of the main topics of the 'Sediment Transport' working group of the Dutch Applied Research Program Rijkswaterstaat - Coastal Research (TOW). This TOW program is carried out in
close cooperation between the Public Works Department, the Delft Hydraulics Laboratory, the Delft University of Technology and the Royal Dutch Meteorological Institute. In the framework of the 'Sediment Transport' group a single series of tests has been carried out by the Delft Hydraulics Laboratory in order to get a first idea of the effect of grain size on β-values. The results have been analysed by the authors.

Normal movable bed tests are unsuitable to clarify the mechanism involved. The rate of 'diffusion coefficient production' in an actual case does not depend on the boundary conditions (to be kept constant) like water depth, wave height and wave period only, but also on the form (height and length) of the bed ripples. These bed ripples are, given the boundary conditions, highly diameter dependent. A constant rate of wave and bed induced diffusion activity, equal for several grain sizes, can only be achieved with a fixed rippled bed, covered with such a small amount of particles, that the original bed form is not disturbed. In the tests with a horizontal bottom, wooden fixed ripples have been applied; ripple height: 0.017 m; ripple length: 0.07 m. Seven tests have been carried out with nearly uniform sand; D50-values of 87, 112, 170, 192, 221, 280 and 346 μm respectively. Although uniform material was desired, it turned out from fall velocity tests in the settling tube, that some variations existed. The D90/D10 ratio actually ranged from 1.3 to 1.5. A single test was carried out with well-graded material: D50 = 217 μm; D90/D10 = 2.15.

In the tests one set of boundary conditions was maintained. Water depth h = 0.30 m; (regular) wave height H = 0.08 m; wave period T = 2 s. In each test a restricted amount of sand was thrown into the flume. Two to five series of simultaneously sucked samples in 4 points were gathered for each material tested. Fig. 9 shows an example of the vertical concentration distribution. Mark the quite similar distribution shape of the curves (1) and (2), despite the fact that for curve (1) the concentrations are approximately a factor 15 less than for curve (2). In case (1) less sand was thrown into the flume.

In a first analysing round, real uniformity was assumed for the 'uniform' material tests. Since in all cases nearly straight lines were found for the concentration distribution over the water depth (see Fig. 9), pointing to a Coleman-type distribution, constant ε values can be calculated with the help of the proper formula of Table I. Fig. 10a represents the results, showing a distinct dependency of the calculated ε values on the settling velocity. A nearly linear relationship is found. By extrapolation (w→0) a (fluid)-value can be calculated. For the tested materials β-values up to 1.8 can be found for w = 0.04 m/s (D50 ≈ 300 μm).
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Delft Hydraulics Laboratory test

\[ h = 0.30 \text{ m} \]
\[ H = 0.08 \text{ m} \]
\[ T = 2 \text{ s} \]
\[ D_{50} = 87 \mu \text{m} \]
fixed bed

Fig. 9 Concentration Distribution

10a. \( \varepsilon_{s} \) as a function of \( w \) 
(\( \beta = 1 \))

10b. Resulting \( \varepsilon_{\text{fluid}} \) 
(\( \beta = 1 + 23w \))

Fig. 10 Diffusion Coefficient vs. Fall Velocity
In a second analysing round, the real grading of the 'uniform' materials was taken into account. As a hypothesis a linear relationship (to be stated) between $w$ and $\beta$ was assumed. The fit procedure described in Appendix I was applied. Due to the grading and the applied fit procedure, a non-constant $\varepsilon$ distribution over the water depth is found this time. The variation over the water depth, however, is not very large. An average value over $0 < z < 0.09$ m has been calculated. By an iteration process the most appropriate $\varepsilon$ (fluid) value and the $\beta$-slope value $m$ were calculated. Fig. 10b shows the ultimate result.

$$\varepsilon_{\text{fluid}} = 0.000337 \text{ m}^2/\text{s}; \beta = 1 + 23.0 \times w$$

(5.1)

The relationship that was found to exist between $\beta$ and $w$ holds for the set of boundary conditions during the test series.
As long as a similar relationship has not been derived for other sets of boundary conditions, the $\beta - w$ relationship cannot be used generally. Tests with different boundary conditions, to be carried out in the Fluid Mechanics Laboratory of the Civil Engineering Department of the Delft University of Technology, are being prepared. Since a general $\beta - w$ relationship is not yet available, the calculations as presented in section 4 cannot be refined at this moment.

6 CONCLUSIONS

- The non-uniformity of the bottom material strongly affects the concentration distribution over the water depth under wave action.
- When the concentration distribution is described with the help of a diffusion equation, the diffusion coefficient seems to be particle-size dependent; $\varepsilon_{\text{sediment}} = \beta \varepsilon_{\text{fluid}}$, the $\beta$ coefficient being approximately a linear function of $w$.
- However, it is not clear yet whether a similar $\beta - w$ relationship holds for quite different boundary conditions or not. Further research will have to clarify that topic.
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Appendix I

FIT PROCEDURE MEASURED CONCENTRATION DISTRIBUTIONS

In behalf of the calculation procedures of this paper (e.g. $e$ - and D50- distributions over the water depth) one needs a mathematical description of the concentration distribution over the water depth. Mostly –compare Fig. 1– a restricted amount of measuring points is available. Besides that the measuring points show some scattering. When applying fit procedures with many degrees of freedom, unrealistically high bottom concentrations and strange curves in the upper section of the vertical can be found frequently.

This is unacceptable for the aims, stated in this paper, and when the bottom concentrations have to be related to the boundary conditions also.

The described fit procedure seems rather objective and holds true for tests on model scale at least.

From Fig. 1 ($z^*$ vs. ln $c(z)$) it can be seen that:

- highest concentrations occur near the bed
- a gradual decrease in concentration occurs towards higher levels above the bottom.

Looking at the gradients as a function of $z^*$ for arbitrary concentration distributions, one can schematize the gradients in curves according to Fig. A1. (In Fig. A1 a 'standard' square is assumed. The vertical axis ($z^*$) ranges from 0 to 1 and the horizontal axis ($d \ln(c)/dz^*$) also from 0 to 1. The actual gradients can be found by multiplying with a negative factor). It seems to be better —and easier— to impose some restrictions on the gradient curve than to do so on the original concentration curve.

Fig. A1

Standard Square
REQUIREMENTS OF GRADIENT CURVES IN STANDARD SQUARE

1) For $z^*=0$, the gradient is 1 and at its maximum. At the bottom the largest gradients are assumed to occur. So curves like curve (a) in Fig. A1 are not acceptable.

2) No negative gradients. A negative gradient in the standard square at a certain level above the bottom means that the concentration increases towards higher distances from the bottom. That seems quite unlikely; experiments never showed any clear evidence to that. Also zero-gradients are unlikely. In the standard square minimal $b$ values (see Fig. A2) of 0.025 are assumed. The value of 0.025 is subjective; however, slightly different values hardly affect the total result as it turned out after analysing a number of test cases.

3) No large gradients close to the bed in the standard square. In Fig. A1 curve (b) seems attractive. At the bed the largest gradient occurs; nevertheless close to the bed also rather large gradients occur. A distribution like curve (b) will never lead to unrealistically high bottom concentrations. However, from actual measurements it turned out that curves like (c) and (d) in Fig. A1 are more likely in some cases. However, with curve (d) extremely high bottom concentrations can be found. So it seems appropriate to make some restrictions. In Fig. A2 and A3 some possible filling-ups of the standard square are given. By assuming a maximum value of 8 for $p$ (see next section), very high bed concentrations can be avoided.
RESULTS

The curves in Fig. A2 can be described by:

\[ \frac{d(\ln(c))}{dz^*} = q \left[ b + (1-b) \left( \frac{(a-z^*)}{a} \right) \right]^p \]  \quad (A.1)

where:
- \( q \): (negative) scale factor
- \( b \): horizontal ordinate point \( A \) \((0.025 < b < 1)\)
- \( a \): vertical ordinate point \( A \)
  
  (in the procedure proposed \( a \) can be
  1 or 0.9 or 0.8 or 0.7; these values
  are rather subjective).
- \( p \): exponent of fit parabola for gradient
  curve \((p \text{ can be } 2; 4; 6 \text{ or } 8)\)

Integration of Eq. (A.1) yields:

\[ \ln(c) = q \left[ b z^* - (1-b) \left( \frac{a-z^*}{a} \right) \right] + d \left( \frac{(a-z^*)}{a} \right)^{(p+1)}/(p+1) + d \]  \quad (A.2)

where: \( d \) is the integration constant

In a computer program the procedure starts with \( a=1 \) and
\( p=8 \). The 'best' values of the parameters \( q \), \( b \) and \( d \) are
determined with a least squares method. If it turns out
that \( b < 0.025 \) or \( b > 1.0 \), then \( b \) is fixed at 0.025 or 1.0
respectively and the procedure is restarted. The
parameters \( q \) and \( d \) are determined now. In the next step
it is checked whether a better result is found with \( a=1 \)
and \( p=6 \), and so on. Next the procedure starts with \( a=0.9 \)
and \( p=8 \) again. As stated the least \( a \)-value is taken to be
0.7.

From test series it turned out that the fit procedure
proposed seems to be rather objective. (Compare Fig. 1)