

## CHAPTER NINETY SEVEN

### A MODEL FOR OFFSHORE SEDIMENT TRANSPORT

M.J.F. Stive\* and J.A. Battjes\*\*

#### ABSTRACT

Observation of the two-dimensional breaking of random waves on a beach suggests that under conditions of active surf an important mechanism in the process of offshore sediment transport is the transport by the undertow or return flow, induced by the breaking of waves. It is found that a model incorporating this mechanism exclusively is able to describe the local sediment transport and the resulting bottom variation of a beach under random wave attack to a first approximation. A laboratory verification is made based on measurements of both the dynamics of the water motion and the bottom profile. Finally, a realistic equilibrium state is shown to result from the model.

#### 1. INTRODUCTION

The particular role of a nearly two-dimensional wave motion in the movement of sediment normal to the shore is poorly understood. It is generally assumed that a number of interaction mechanisms between this wave motion and the sediment motion contribute to the formation of the beach profile, also in the three-dimensional topographies that occur on a natural coast. Full account of all mechanisms can be taken if a description of both the horizontal velocity field,  $u(x,z,t)$ , and the sediment concentration field,  $c(x,z,t)$ , in space and time is available, so that the net cross-shore sediment transport,  $\bar{q}(x)$ , may be calculated from

$$\bar{q}(x) = \int_d u(x,z,t) \cdot c(x,z,t) dz \quad (1)$$

where the integration is performed over the instantaneous depth  $d$  and the overbar indicates time averaging. From the cross-shore variation of  $\bar{q}(x)$  the bottom changes may be derived.

Visual and experimental observation of random waves on a two-dimensional beach indicate that one of the more important mechanisms under active surf conditions may be the transport of sediment by the time mean, seawards directed flow near the bottom induced by the breaking of waves. It is shown that this mechanism is so dominant that a model incorporating this mechanism alone describes the bottom variations in the surf zone to a satisfactory, first approximation. This paper describes the properties of this one-mechanism model and its verification based on laboratory measurements both of the wave

---

\* Delft Hydraulics Laboratory, Delft, the Netherlands

\*\* Delft University of Technology, Delft, the Netherlands

motion and of the bottom profile. Furthermore, the equilibrium consequences of the model formulation are investigated.

In the elaboration of the present model simple, available formulations for the model elements have been used. Improved formulations can easily be incorporated without affecting the principle of the model. Extension of the model with other transport mechanisms is a logical step towards a more complete cross-shore sediment transport model. In this respect transport due to the asymmetry of the wave motion should be considered first.

## 2. MODEL FORMULATION

In principle the net cross-shore sediment transport may be calculated from Eq. (1). However, insufficient knowledge of the velocity and concentration field forces us to rely on a simplified form of Eq. (1), using the following observations and assumptions:

- (a) The sediment load,  $s$  ( $= \int_d c dz$ ), is mainly contained in the region near the bottom and as such locally determined.
- (b) The horizontal velocity field,  $u$ , is depth-uniform in the region near the bottom.

With the introduction of mean and (wave-induced) fluctuating components in the sediment load and the near-bottom horizontal velocity, i.e.  $s = \bar{s} + s'$  and  $u = \bar{u} + u'$ , the net sediment transport may be calculated as

$$\bar{q} = \overline{u \cdot s} = \overline{u \cdot \bar{s}} + \overline{u' \cdot s'} \quad (2)$$

Although this result reduces the problem to one which is potentially solvable on basis of our present knowledge we found it useful to reduce the problem further using the following assumptions:

- (c) The contribution due to the correlation between the fluctuating sediment load and horizontal velocity is small compared to that due to the mean (return) flow, i.e.  $\overline{u' \cdot s'} \ll \overline{u \cdot \bar{s}}$ .
- (d) The seawards directed, near-bottom mean flow velocity induced by a breaking wave is large compared to the mean flow velocity induced by a non-breaking wave of the same height, so that in a random wave field  $\bar{u}_{br} \gg \bar{u}_{nonbr}$ .

Based on these assumptions result (2) is simplified further to

$$\bar{q} = \bar{u}_{br} \cdot \bar{s} \quad (3)$$

In this approximation the net sediment transport in the surf zone is directed offshore. Simple, first approximations to the return flow,  $\bar{u}_{br}$ , and the mean sediment load,  $\bar{s}$ , may be as follows.

### the mean return flow in random breaking waves

It is assumed that in a random wave field breaking on a beach the majority of the breaking waves has a quasi-steady depth-similar flow field as described by Stive and Wind (1982) for breaking, periodic waves. Based on the dimensionless flow field presented there and adopting the observation that the flow profile is rather uniform over

the lower depths the return flow velocity in a periodic, breaking wave field is simply modelled as:

$$\bar{u}_{br,periodic} = 1/8 (g/d)^{1/2} H_b \quad (4)$$

where  $g$  is the acceleration of gravity,  $d$  the water depth and  $H_b$  the breaking wave height. The net mass flux below the level of the wave troughs becomes

$$M = 1/8 \rho (g/d)^{1/2} H_b d_t \quad (5)$$

where  $d_t$  is the water depth up to the trough level. This result corresponds closely to the net mass flux result above the level of the wave troughs for a steady, linear wave train on a horizontal bottom:

$$M = E/c \quad (6)$$

where  $E = 1/8 \rho g H^2$  is the wave energy density and  $c$  denotes the wave speed. After introducing the shallow water approximation to  $c$ :

$$M = E/c \approx 1/8 \rho (g/d)^{1/2} H^2 \quad (7)$$

In random waves on a beach the fraction of waves breaking at a point ( $Q_b$ ) varies with position. Battjes and Janssen (1978) have presented an implicit expression for  $Q_b$  as a function of the ratio of the rms wave height ( $H_{rms}$ ) to a local breaking height, which in turn is primarily depth-controlled. A simple, explicit approximation qualitatively close to this relation and quantitatively well in accordance with laboratory observations is:

$$\tilde{Q}_b = 20(H_{rms}/d)^5 \quad (8)$$

The return flow velocity in a random, breaking wave field is simply modelled here as

$$\bar{u}_{br,random} = \bar{u}_{br,periodic} \cdot \tilde{Q}_b \quad (9)$$

#### the mean sediment load in random breaking waves

A reliable, predictive model for the mean sediment concentration or even for the mean sediment load due to random, breaking waves is not available yet. For the present purposes a prediction method is derived based on the sediment concentration measurements and theoretical analyses presented by Nielsen et al (1978, see also Nielsen, 1979) for non-breaking waves and Bosman (1982) for breaking waves. The method is broadly described below and details are given in an Appendix.

Laboratory measurements of time- and bed-averaged concentration profiles under breaking, random waves conducted by Bosman (1982) indicate that (a) in the near-bottom layer the upward decay of the concentration is exponential and (b) the sediment load is mainly confined to and determined by the bottom layer, so that

$$\bar{s} = C_0 \cdot \lambda_1 \quad (10)$$

where  $C_0$  is a reference concentration at the bottom and  $\lambda_1^{-1}$  is the relative concentration gradient in the bottom layer. The bottom reference concentration is found to be nearly linearly proportional to a Shield's type parameter  $\theta'$  (see Appendix Eq. 23...25). Adoption of a diffusion-type model with a constant turbulent viscosity,  $\epsilon_1$ , and sediment fall velocity,  $w_1$ , for the bottom layer region leads to

$$\lambda_1 = \epsilon_1 / w_1 \quad (11)$$

which indicates that  $\lambda_1$  closely corresponds to a characteristic turbulent length scale. In fact  $\lambda_1$  is found to be linearly proportional to the ripple height in the ripple regime and to the wave boundary layer thickness in the sheetflow regime. Since the bottom layer is only weakly influenced by surface breaking, Nielsen's (1979) formulations for the bottom reference concentration and the viscosity coefficient were used for the situation of random, breaking waves, i.e. on basis of Bosman's measurements Nielsen's parameterizations of  $C_0$  en  $\epsilon_1$  were quantitatively adapted. The resulting prediction method for the mean sediment load covers the region of initiation of motion to sheetflow conditions for median grain diameters of say 100  $\mu\text{m}$  to 500  $\mu\text{m}$ . However, the parameterizations were only checked in a very limited region, so the results should be applied with caution. It is stressed that there is an urgent need to conduct and analyse sediment concentration measurements under surf conditions close to reality.

#### practical calculations

With the undertow and the mean sediment load known as functions of the local hydrodynamic conditions and the sediment properties the offshore sediment transport can be calculated. In practical computations the sediment properties may be assumed constant. The hydrodynamic conditions, however, are a function of the horizontal distance from the shore and the offshore wave parameters. These conditions can be derived from numerical calculations with the wave height decay model presented by Battjes and Janssen (1978). This may be regarded as the first discrete step in a practical calculation procedure, illustrated in Fig. 1. The rms wave height,  $H_{\text{rms}}$ , the breaking wave height,  $H_b$ , and the fraction of breaking waves,  $\bar{Q}_b$ , follow directly from the decay model. The local kinematics are calculated in the second step, where the rms orbital velocity,  $u_{\text{rms}}$ , is derived from linear theory and  $\bar{u}_{\text{br}}$  from Eq. (9). In the third step the sediment transport is calculated according to Eq. (3) where a proportionality constant,  $b$ , is introduced which should be of order unity if the model is right. It is noted that the sediment transport is locally determined only. Finally, the bottom changes are calculated through application of the mass balance equation for the sediment. This procedure may be repeated for the new beach profile.

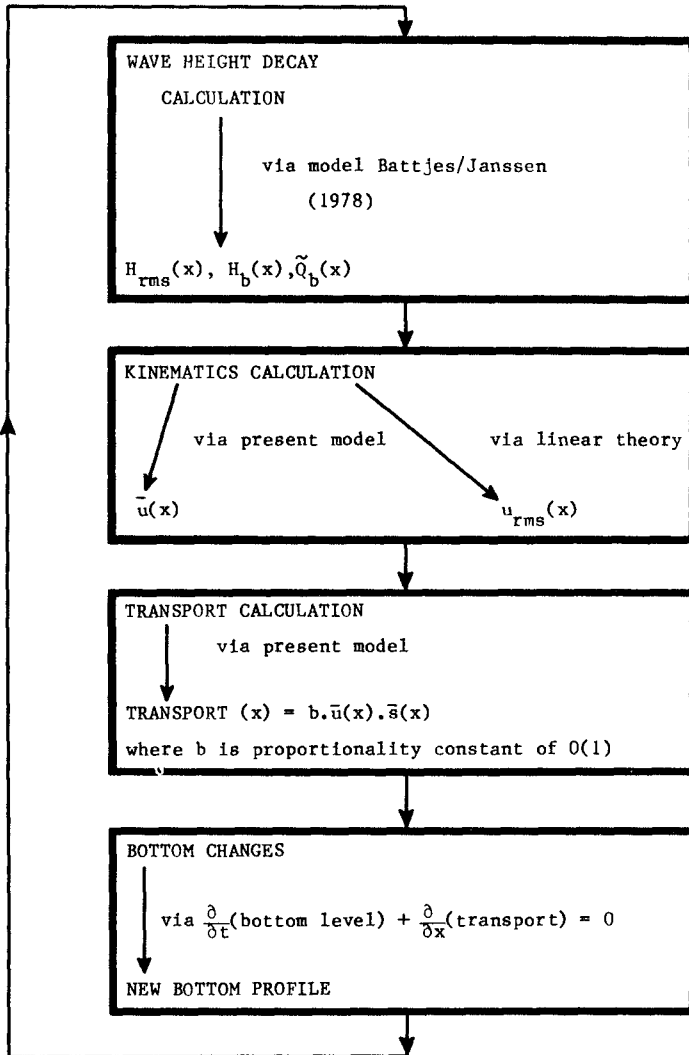


Fig. 1 Computation procedure

In the numerical evaluation of the above procedure a second order Runge-Kutta algorithm is used in the wave decay model and a modified Lax scheme in the bottom change calculations.

As a boundary condition on the waterline the present formulation yields  $\bar{q} = 0$ . To simulate the smoothing effect of swash motion on the sediment transport near the waterline  $\bar{q}(x)$  was damped starting from a depth of approximately half the initial wave height in proportion to the mean water depth.

### 3. MODEL VERIFICATION

A laboratory measurement programme aimed at verification of the present model has not yet been initiated. Therefore we rely on available measurement results of which some examples are presented here.

Firstly, model calculations of the wave kinematics on a beach of nearly constant slope (1:40) are compared with measurements in a large scale wave facility (see Fig. 2). The ability of the model to predict the wave height decay is extensively discussed by Battjes and Stive (1984). Here we are interested in the prediction of the return flow and the rms orbital velocity, which were measured 0.2 m above the bottom. The prediction appears to be sufficiently close for our purposes. It is noted that the trend of some theoretical overprediction of  $u_{rms}$  in the outer breaking regime towards negligible overprediction in the actual surf zone is similar to that found and partly explained by Van Heteren and Stive (1984).

Secondly, calculations of bottom changes are compared with measurements on three different beach profiles. The first case concerns a barred profile of relatively fine sediment in a small scale flume. The second case concerns a parabolic profile of somewhat coarser sediment in a small scale flume. The third case concerns the earlier mentioned nearly 1:40 profile of medium sized sediment in a large scale flume. Some characteristic parameters of these three cases are collected in Table 1, where the mean water depth is denoted by  $h$ , the rms wave height by  $H_{rmsr}$ , and the peak frequency by  $f_p$  with the subscript  $r$  denoting reference value in the horizontal part of the flume. In case 2 the initial wave conditions were varied to simulate a wave climate. This variation was accounted for in the computations.

case	source	profile	grain diameter ( $\mu\text{m}$ )	$h_r$ (m)	$H_{rmsr}$ (m)	$f_{pr}$ (Hz)
1	Van Overeem (1983)	barred	100	0.80	0.21	0.39
2	Boer (1984)	parabolic	145	0.80	0.11-0.19	0.40-0.53
3	Stive (1984)	plane	225	4.19	1.00	0.19

Table 1 Characteristic parameters of laboratory experiments

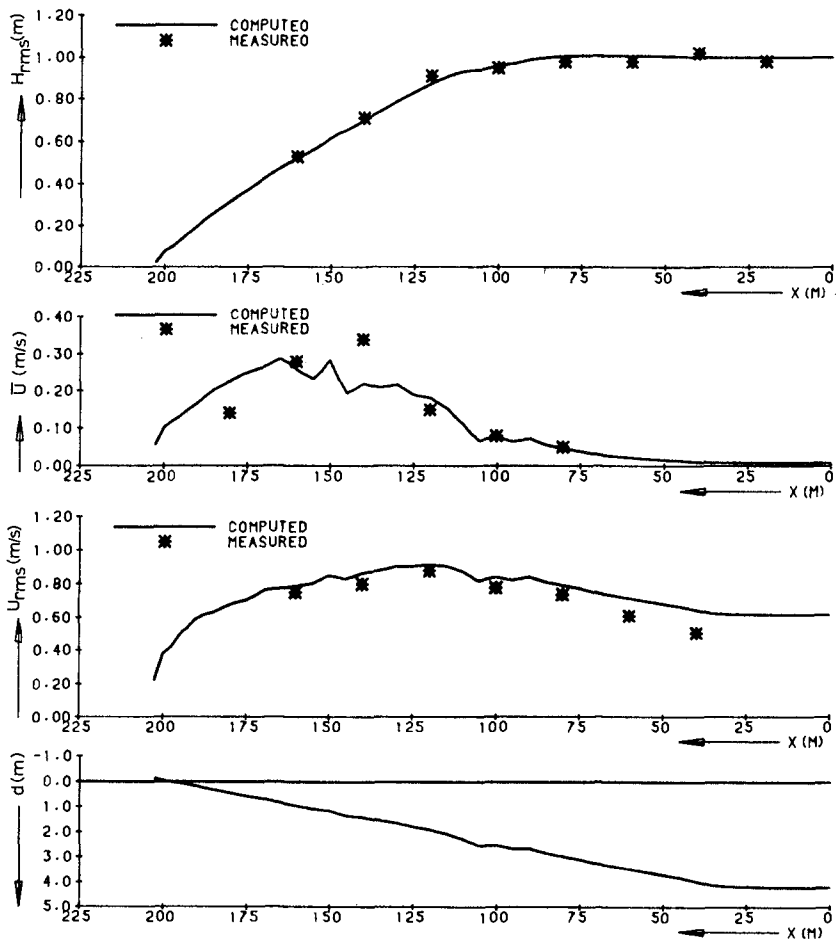


Fig. 2 Measured and computed wave height ( $H_{rms}$ ), return flow velocity ( $\bar{U}$ ) and rms orbital velocity ( $U_{rms}$ ) on a large scale, laboratory beach

The comparison between measurements and computations is presented in Figs. 3...5. In case 1 the resultant profiles themselves are compared, while in the other cases it was judged more informative to compare the actual bottom changes. In general, the relative profile activity and distinct features, such as the offshore movement of a step and the formation of a bar, are predicted well by the computations. The actual quantitative changes are simulated reasonably (although sometimes somewhat shifted in space), if the proportionality constant  $b$  is made to vary between 0.25 and 1.0. The qualitative

predictions of details is surprisingly good at some locations. Apparently, the modelling of the sediment transport as function of the wave conditions is insufficiently accurate to yield a universal value for  $b$ . If, however, for prediction purposes a value  $b = 0.5$  is applied the bottom changes are at most overestimated or underestimated by a factor of two.

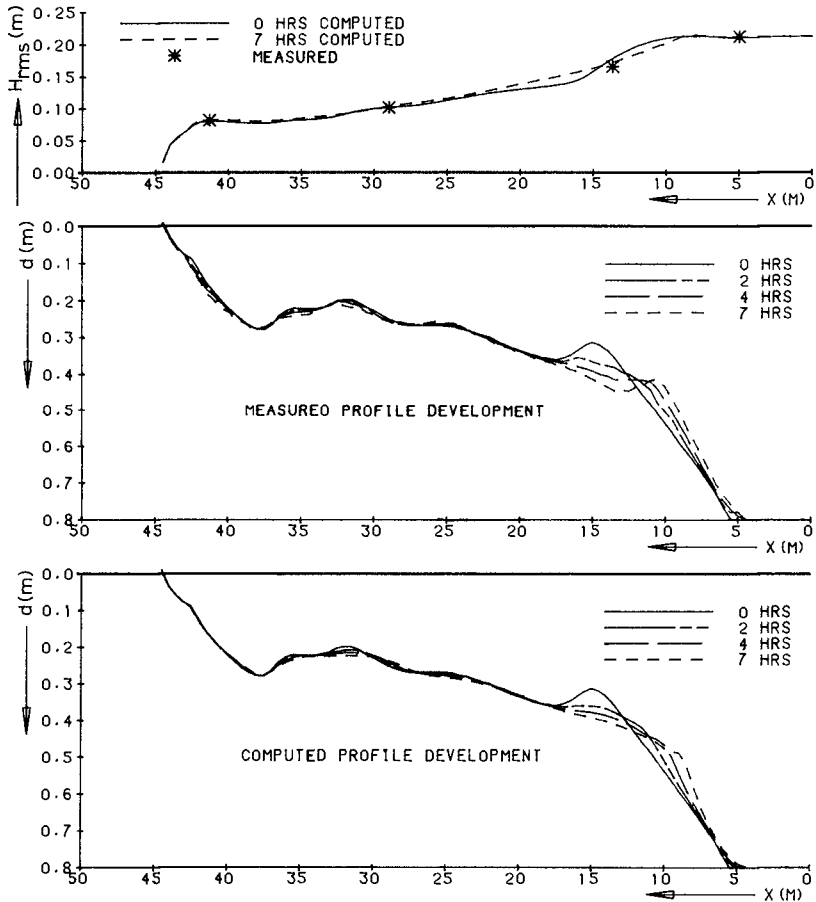


Fig. 3 Measured and computed ( $b = 0.25$ ) profile development on a small scale, laboratory beach: case 1

Finally, a preliminary comparison is presented of calculated and measured bottom changes of a beach acting as a foreshore of a protected dune (see Fig. 6). The dune revetment prohibits sediment transport, so that the sudden increase of sediment transport at the dune foot



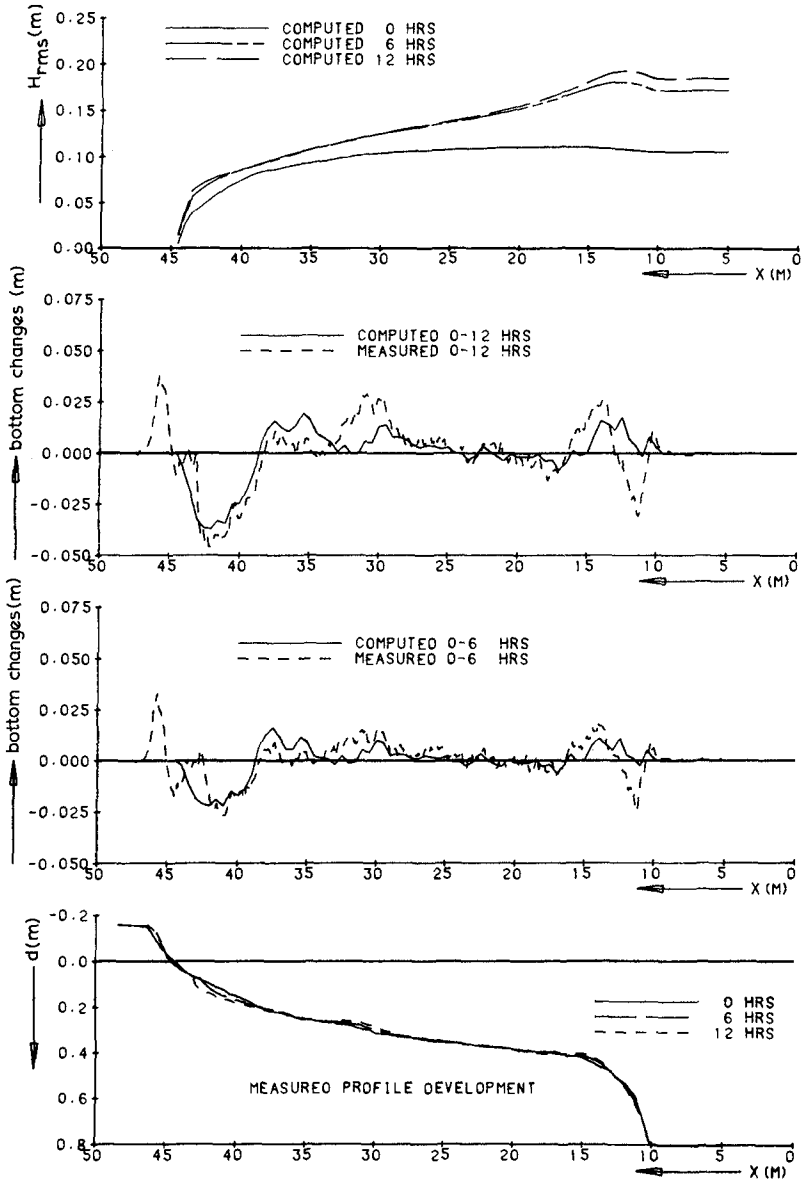


Fig. 4 Measured and computed ( $b = 1.0$ ) bottom changes on a small scale, laboratory beach: case 2

causes an erosion hole. The hydraulic conditions during the experiments simulated the conditions of a design storm surge, so that both the water level and the wave conditions varied in time. The variation of the conditions was taken into account in the calculations. It appears that the erosion at the foot is well predicted applying a value of  $b = 0.5$ . This indicates that the presently modelled return flow mechanism plays an important role in the dune erosion process.

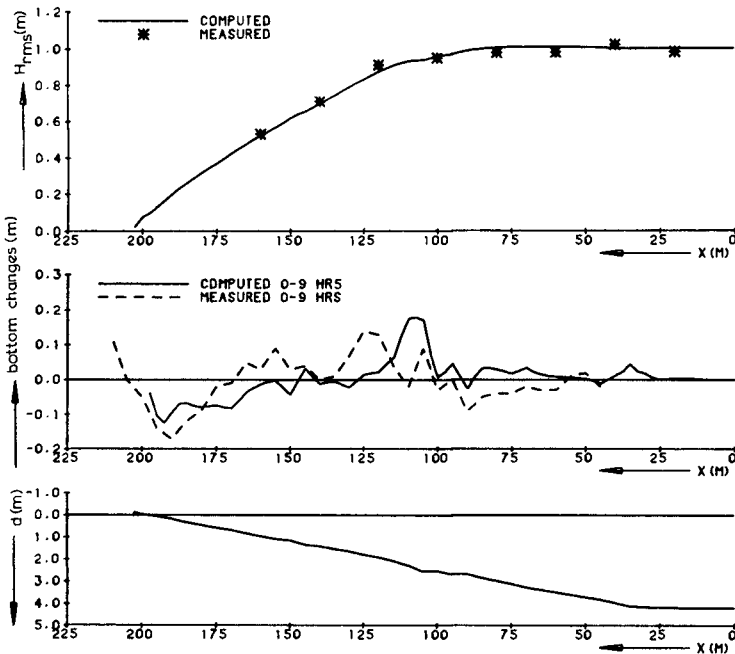


Fig. 5 Measured and computed ( $b = 0.25$ ) bottom changes on a large scale, laboratory beach: case 3

4. EQUILIBRIUM STATE

An important aspect in the verification of the present model is the reality of the profile equilibrium state which results from the model formulations. A state of equilibrium is said to be reached when gradients in the sediment transport are absent so that the profile shape is stable. This definition implies that a two-dimensional surf zone is assumed to have a sediment source on the shoreside and a sediment sink at the seaside. The consequences with respect to natural beaches are described below.

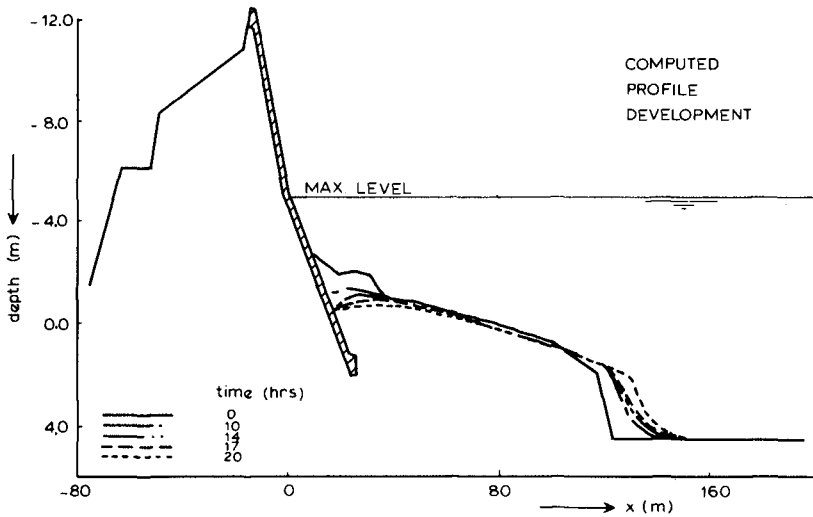
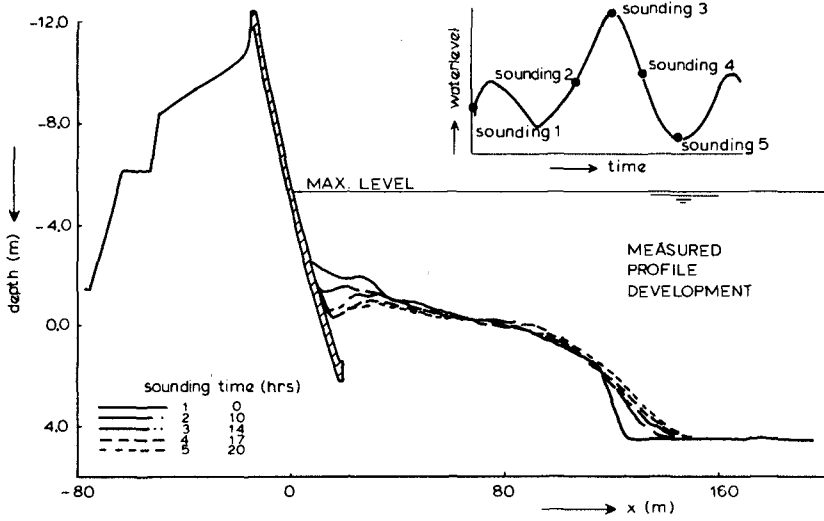


Fig. 6 Measured and computed ( $b = 0.5$ ) profile development on a beach at the foot of a protected dune

In order to perform our analysis it is necessary to introduce simplified relations for the model parameters. Firstly, the shallow water approximations to the breaking wave height,  $H_b$ , the orbital velocity,  $u_{rms}$ , and the orbital amplitude,  $a_{rms}$ , are adopted, i.e.

$$\begin{aligned} H_b &= \gamma' d & (\gamma' = \text{constant}) \\ u_{rms} &= \frac{1}{2} H_{rms} (g/d)^{\frac{1}{2}} & (12) \\ a_{rms} &= \frac{1}{2} H_{rms} (g/d)^{\frac{1}{2}} (2\pi f_p)^{-1} \end{aligned}$$

Secondly, in the  $\theta'$ -range relevant to surf zone conditions ( $\theta' \approx 1.0 - 1.5$ ) the mean sediment load,  $\bar{s}$ , is approximately proportional to the orbital amplitude,  $a_{rms}$ , and to the Shields parameter,  $\theta'$ , (see Figs. 8 and 9 of the Appendix), i.e.

$$\bar{s} :: a_{rms} \theta' \quad (13)$$

where  $\theta'$  is defined by Eq. (24). Inserting the results in Eq. (3) yields the following proportionality relation for the offshore sediment transport,  $\bar{q}$ :

$$\bar{q} :: H_{rms}^6 d^{-6} f_p^{-1} D^{-1} \quad (14)$$

where  $D$  is a characteristic grain diameter. For the transport to be constant it is a necessary condition that

$$H_{rms} :: d^{3/4} f_p^{1/8} D^{1/8} \quad (15)$$

The relation between wave energy and water depth may further be elaborated by use of the energy balance equation, i.e.

$$\frac{d}{dx} (E.c_g) + \text{Diss} = 0 \quad (16)$$

where the dissipation term due to wave breaking following Battjes and Janssen (1978) is proportional to:

$$\text{Diss} :: Q_b f_p H_b^2 \quad (17)$$

An approximation to Battjes and Janssen's implicit equation for  $Q_b$  yields

$$Q_b :: (H_{rms}/d)^6 \quad (18)$$

where it is noted that this result differs from Eq. (8) (used in the derivation of Eq. (14)), since here an accurate approximation is sought to  $Q_b$  in the wave decay model.

Using the results (17) and (18) and again adopting shallow water approximations, the energy balance equation gives

$$\frac{d}{dx} (H_{rms}^2 d^{\frac{1}{2}}) :: H_{rms}^6 f_p d^{-4} \quad (19)$$

Combining the governing Equations (15) and (19) and integration yields

$$d \propto \hat{x}^{2/3} f_p D^{1/3} \quad (20)$$

where  $\hat{x}$  is the horizontal distance from the water line. With respect to the dependence of  $d$  on  $\hat{x}$  this result is well in accordance with empirical findings first noticed by Bruun (1954). Also the other dependencies are found to be in at least qualitative accordance with observations (see e.g. Dean, 1984, whose results indicate a quantitative similar grain diameter dependence). These results lend support to the present model.

## 5. DISCUSSION AND CONCLUSION

The present offshore transport model incorporates only one of the mechanisms responsible for cross-shore sediment movement that is expected to exist on beaches under nearly two-dimensional wave attack, i.e. sediment transport due to breaking wave induced return flow. The model is applied to some laboratory beaches exposed to random breaking wave action. Realistic, first order predictions of the bottom changes are obtained. These laboratory findings indicate that in the active surf zone the return flow mechanism dominates others mechanisms. The predictions may be improved by including other mechanisms, e.g. such as due to the asymmetry of the wave motion.

Natural beaches are only confronted with an active surf zone during a relatively small period of time. Moreover, the position of the surf zone varies with the tidal level. Obviously a realistic prediction of the beach profile response under general, natural conditions requires the inclusion of more sediment transport mechanisms than the return flow mechanism alone. If, however, one is interested in the relatively large response of beaches under storm conditions (when the surf zone is extended significantly offshore) the present model is expected to perform satisfactorily. This is confirmed by the fact that the equilibrium profile shape following from the model is in good accordance with field observations.

## ACKNOWLEDGEMENT

This work was done as a part of the TOW Study Programme for Coastal Sediment Transport in the Netherlands. The Programme is financed by the Dutch Government and jointly carried out by the Ministry of Transport and Public Works (Rijkswaterstaat), Delft University of Technology and Delft Hydraulics Laboratory. Thanks are due to J. v. Overeem for making available the results presented in Fig. 6.

APPENDIX: The Mean Sediment Load

As shown by Bosman (1982) sediment concentration distributions in random, breaking waves may be satisfactorily described by a "double layer" or "double first order" model for a rather wide variety of laboratory situations, such as horizontal and sloping bottom, no and net currents, non-breaking and breaking random waves. Based on a diffusion type description for each layer the model leads to an exponential concentration distribution as follows (see also Fig. 7):

$$C(z) = \begin{cases} C_0 e^{-z/\lambda_1} & \text{for } z < A \\ C(A) e^{-(z-A)/\lambda_1} = C_0 e^{-A/\lambda_1} e^{-(z-A)/\lambda_2} & \text{for } z > A \end{cases} \quad (21)$$

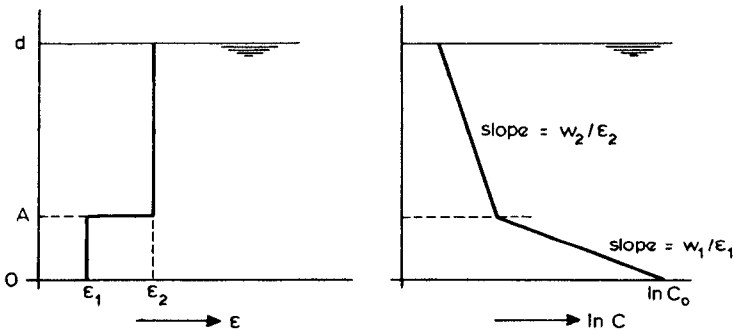


Fig. 7 "Double first order" model for the sediment concentration distribution in random, breaking waves (after Bosman, 1982)

where A is the bottom layer thickness. Each layer has a constant turbulent viscosity,  $\epsilon$ , and a fall velocity, w, for the sediment in that layer such that  $\lambda_1 \equiv \epsilon_1/w_1$  and  $\lambda_2 \equiv \epsilon_2/w_2$  (based on diffusion). Bosman's measurements show that the upward exponential decay of the concentration is so strong that the total sediment load,  $\bar{s}$ , is mainly confined to and determined by the bottom layer as follows:

$$\bar{s} = C_0 \cdot \lambda_1 \quad (22)$$

where  $C_0$  is the bottom concentration and  $\lambda_1^{-1}$  is the relative concentration gradient in the bottom layer.

These laboratory flume measurements only cover a limited range of conditions, for instance only one median grain diameter was used. To generalize the results we follow the work of Nielsen (1979). Nielsen gives no explicit formulation for the sediment concentrations or the total load due to random waves and certainly not due to random, breaking waves. He does present semi-empirical expressions for the ripple height,  $\xi$ , and the bottom reference concentration,  $C_0$ , due to random waves. With respect to the effects of breaking we adopt the observation that the sediment concentrations near the bottom and the

geometry of the bottom due to breaking waves do not differ essentially from those due to non-breaking waves. This implies that we may rely on formulations for non-breaking, random waves.

#### bottom reference concentration

As in steady flow the bottom reference concentration is assumed to be determined by the Shields parameter, i.e. the dimensionless bed shearstress, and Nielsen suggests also for random waves:

$$C_o = K(\theta' - 0.05) \frac{2}{\pi} \arccos(0.05/\theta')^{\frac{1}{2}} \quad (23)$$

where K is a constant and the Shields parameter  $\theta'$  is defined as

$$\theta' = \frac{f_w u_{rms}^2}{\Delta g D} \quad (24)$$

where  $f_w$  is a friction factor,  $u_{rms}$  is the rms orbital velocity,  $\Delta$  is the relative sediment density ( $\rho_s/\rho_w - 1$ ) and D is a characteristic grain diameter. The friction factor is given as (Swart, 1974):

$$f_w = \exp[5.2(2.5 D/a)^{0.2} - 6.0] \quad (25)$$

where a is the orbital amplitude. The "arccos" term in Eq. (23) accounts for the fraction of time that the critical stress value of 0.05 is exceeded. Confrontation of expression (23) with Bosman's bed-averaged measurements confirms Nielsen's suggestion. The constant K has a best value of 0.028, the same value as found at the ripple crests under periodic waves.

#### ripple height

Semi-empirically Nielsen derived the following expression for the ripple height due to random waves:

$$\xi/a = 21(\psi)^{-1.85} \quad \text{for } \psi > 10 \quad (26)$$

$$\xi/a = 0.3 \quad \text{for } \psi \leq 10$$

$$\text{where } \psi = \frac{u_{rms}^2}{\Delta g D}$$

#### turbulent viscosity coefficient

Semi-empirically Nielsen derived for periodic waves that the viscosity coefficient made dimensionless by the length scale ripple height,  $\xi$ , or boundary layer thickness,  $0.4 \delta$ , and the velocity scale  $gT$  is a complicated function of the parameter  $a\omega/w_1$ , so

$$\epsilon_1 / [(\xi + 0.4\delta) gT] = f(a\omega/w_1) \quad (27)$$

where  $\omega$  is the angular velocity and T the wave period.

In the length scale quantity  $(\xi+0.4\delta)$  the ripple height,  $\xi$ , dominates in situations with noticeable ripples and the boundary layer thickness,  $0.4\delta$ , dominates in the absence of ripples. The latter is defined as (Jonsson and Carlsen, 1976):

$$\delta/a = 0.072 (2.5D/a)^{0.25} \tag{28}$$

From a comparison with the measurements of Bosman we have found that also in the region  $aw/w_1 > 20$  the lower asymptote (Eq. 6.112) of the expression (Eq. 6.111) suggested by Nielsen (1979) for periodic waves coincides best with the random wave measurements. This asymptote is given as:

$$\varepsilon/[(\psi+0.4\delta) gT] = 0.35 \cdot 10^{-3} (aw/w_1)^{0.68} \tag{29}$$

results

The present prediction method is based on the above described findings. In summary, for the prediction of the mean sediment load,  $\bar{S}$ , expressions (23), (26), (28) and (29) are used. Logically Bosman's measurements correspond well with this formulation (see Fig. 8a). The sensitivity of the method for a variation in grain diameter is shown in Fig. 8b.

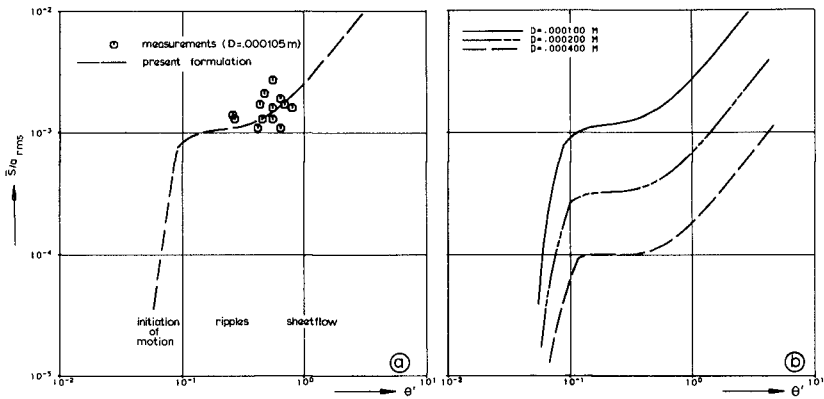


Fig. 8 Non-dimensional sediment load as a function of the Shields parameter for  $f_p = 0.5$  Hz



## REFERENCES

- Battjes, J.A. and J.P.F.M. Janssen, Energy loss and set-up due to breaking of random waves, in Proceedings 16th International Conference Coastal Engineering, pp. 569-587, ASCE, New York, 1978.
- Battjes, J.A. and M.J.F. Stive, Calibration and verification of a dissipation model for random breaking waves, in Proceedings 19th International Conference Coastal Engineering, ASCE, New York, 1984.
- Boer, S., Coastal protection scheme Lido di Ostia, Report M 2027, Delft Hydraulics Laboratory, 1984.
- Bosman, J.J., Sediment concentrations under waves and currents (in Dutch, publication in preparation), Report M 1875, Delft Hydraulics Laboratory, 1982.
- Bruun, P., Coast erosion and development of beach profile, Techn. Memo No. 44, CERC, 1954.
- Dean, R.G., Applications of equilibrium beach profile concepts, in Proceedings 19th International Conference Coastal Engineering, ASCE, New York, 1984.
- Jonsson, I.G. and N.A. Carlsen, Experimental and theoretical investigations in an oscillatory turbulent boundary layer, J. Hydraulic Research, Vol. 14, No. 1, pp. 45-60, 1976.
- Nielsen, P., Some basic concepts of wave sediment transport, Series paper No. 20, Institute of Hydrodyn. and Hydr. Eng., Techn. Univ. of Denmark, 1979.
- Nielsen, P., I.A. Svendsen and C. Staub, Onshore-offshore sediment movement on a beach, in Proceedings 16th International Conference Coastal Engineering, pp. 1475-1492, ASCE, New York, 1978.
- Stive, M.J.F., A scale comparison of waves breaking on a beach, accepted for publication in J. Coastal Eng., 1984.
- Stive, M.J.F., and H.G. Wind, A study of radiation stress and set-up in the nearshore region, J. Coastal Eng., 6: 1-25, 1982.
- Swart, D.H., Offshore sediment transport and equilibrium beach profiles, Publication No. 131, Delft Hydraulics Laboratory, 1974.
- Van Heteren, J. and M.J.F. Stive, Wave kinematics and wave directionality in the surf zone, in Proceedings 19th International Conference Coastal Engineering, ASCE, New York, 1984.
- Van Overeem, J., Morphologic behaviour of beach fill with underwater dam, Report M 1891, Delft Hydraulics Laboratory, 1983.