CHAPTER NINETY THREE

Stability of Multiple Inlets

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Abstract

A computational framework is presented to evaluate the stability of two inlets connecting the same bay to the ocean. It is assumed that both inlets are scoured in alluvial material. The main ingredients in the stability analysis are the closure surface and the equilibrium stress surface. The closure surface of an inlet is defined as the relation between the tidal maximum of the bottom stress and the cross-sectional areas of both inlets. The equilibrium stress surface of an inlet is the relation between the tidal maximum of the bottom stress and the cross-sectional area of that inlet at the time the inlet is in equilibrium with its hydraulic environment. The method is applied to Pass Cavallo and the entrance to the Matagorda Shipping channel further referred to as Matagorda Inlet. Both inlets connect Matagorda bay to the Gulf of Mexico.

Introduction

Over a period of decades, many inlets exhibit a change in morphology. These changes are often caused by a gradual increase in length resulting from the deposition of sediment at the ocean-and bay side of the inlet. This in turn leads to a gradual decrease in conveyance and cross-sectional area. Superimposed on this trend are sudden changes in inlet dimensions associated with storms, changes in bay surface area (diking) and the opening or closure of companion inlets. The inlet will respond to these sudden changes by either closing or returning to the old or a new equilibrium morphology. The time for the inlet to adapt depends on whether the inlet is in a shoaling or scouring mode. When in a shoaling mode, the response time depends on the supply of sediment, i.e. the littoral drift, and could vary from weeks to years. On the contrary when in a scouring mode the response time will be short, i.e. less than a few weeks.

If as a result of a sudden change in inlet dimensions or hydraulic conditions the inlet tends to close, the inlet is said to be unstable. If instead the inlet returns to the original or a new equilibrium morphology, the inlet is said to be stable. The

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extent to which the inlet dimensions or the hydraulic environment can be altered before the inlet tends to close determines the degree of stability of the inlet.

In general stability analysis of inlets deal with the response of an inlet to 1) sudden changes in its dimensions resulting from storms, 2) a reduction in bay surface area and 3) the opening of a companion inlet, either by dredging or extreme events. In this paper, emphasis is on the response of an inlet to the opening of a companion inlet.

The stability analysis as presented in this paper basically deals with the conveyance of an inlet. Therefore the analysis does not yield a definite answer with regard to the navigability of the inlet. For this, more detailed information on cross-sectional areas and depths along the entire channel axis is needed.

Inlet Hydraulics; Closure Curve and-Surface

In analysing the stability, inlets are schematised to prismatic channels. The channel has a cross-sectional area $A$, (usually taken equal to the cross-sectional area of the gorge), a hydraulic radius $R$, a length $L$ and a friction coefficient $F$. The sum of exit and entrance losses are represented by a coefficient $m$. A coefficient $u$ is introduced to account for contraction of the flow. In terms of the foregoing parameters the cross-sectionally averaged velocity, $u$, in the inlet follows from

$$u = K' \sqrt{2g \left| n_0 - n_b \right|}$$

with

$$K' = \mu \sqrt{\frac{R}{mR + 2FL}}$$

In Eq. (1) $n_0$ is the ocean tide and $n_b$ is the bay tide. For a given head difference $|n_0 - n_b|$, inlets with the same $K'$ factor have the same current speed.

The bottom shear stress, $\tau$, in the inlet is related to the cross-sectionally averaged velocity by

$$\tau = \rho Fu |u|$$

in which $\rho$ is the density of water. In this paper, the friction factor $F$ is related to the hydraulic radius and bottom roughness $k$ by the Colebrook-White expression

$$F = \frac{12R^{-2}}{18 \log \frac{18 \log}{k}}$$

When considering a relatively short and deep bay connected to the ocean by more than one inlet the condition of continuity may be expressed as
\[ \Sigma Q_i = A_b \frac{d\eta_b}{dt} \]  

(4)

in which \( Q \) is the rate of flow in the inlet, \( A_b \) is the bay surface area and the summation is over the number of inlets.

For a given ocean tide, bay surface area and shape of the cross-section and using Eqs. (1)-(4) it can be shown that for a single inlet bay system the tidal maximum of the bottom shear stress \( \tau \), is a strong function of \( A \) and a weak function of \( L, k \) and \( m \).

\[ \tau = \tau(A; L, k, m) \]  

(5)

Here it is assumed that the hydraulic radius \( R \) is uniquely determined by the cross-sectional area \( A \). The general shape of the closure curve \( \tau = \tau(A) \) is indicated in Fig. 1. For small values of \( A \), \( \tau \approx A \) and for large values of \( A \), \( \tau \approx A^{-2} \).

For a two inlet bay system

\[ \tau_{I} = \tau_{II}(A_{I}, A_{II}; L_{I}, F_{I}, m_{I}, L_{II}, F_{II}, m_{II}) \]  

(6)

\[ \tau_{II} = \tau_{I}(A_{I}, A_{II}; L_{I}, F_{I}, m_{I}, L_{II}, F_{II}, m_{II}) \]  

(7)

The subscripts I and II refer to the respective inlets. The general shape of the closure surface \( \tau_{II} = \tau_{II}(A_{I}, A_{II}) \) is indicated in Fig. 2.

In practice, difficulties in calculating the closure curves and closure surfaces might be encountered because of the uncertainties in the values of the inlet lengths, the bottom roughness and the energy loss coefficients.
The Equilibrium Bottom Shear Stress; Equilibrium Shear Stress Curve and Surface.

It is postulated that inlets in alluvial material will adjust their dimensions until a bottom shear stress $\tau_{eq}$ is reached, Bruun [1978]. In general $\tau_{eq}$, referred to as the equilibrium shear stress, will be a function of the littoral drift, wave climate and the dimensions of the inlet cross-section.

In principle for a given coast, and assuming that the littoral drift and wave climate are relatively uniform, the relation between equilibrium shear stress and inlet dimensions can be derived from the tidal prism-cross-sectional area relationships given by among others O'Brien [1969] and Jarrett [1976]. Unfortunately the correlation functions used to derive these relationships lack a physical basis and especially for the region of interest, i.e. the Gulf of Mexico coast, show considerable scatter. Therefore in the application to Pass Cavallo and Matagorda Inlet rather than making use of the tidal prism-area relationships, the value of $\tau_{eq}$ will be determined from the observation that in 1959 Pass Cavallo and in 1971-1972 Matagorda Inlet was in equilibrium with the hydraulic environment.

In a qualitative sense the shape of the equilibrium shear stress curve (pertaining to a one-inlet bay system) and the equilibrium shear stress surface (pertaining to a two-inlet bay system) are presented in respectively Figs. 1 and 2.
Stability of a Single-Inlet Bay System

Referring to Fig. 1, a condition for the existence of an equilibrium flow area is that the closure curve and equilibrium stress curve intersect. The cross-sectional area $A_1$, represents an unstable inlet. The cross-sectional area $A_2$ pertains to a stable equilibrium flow area. The equilibrium interval of $A_2$ extends from the value of $A_1$ to infinity, Escoffier [1940].

Stability of a Two-Inlet Bay System

To delineate the stability of a two inlet-bay system, for each of the inlets the intersection of its closure surface and equilibrium stress surface is projected in the $A_I, A_{II}$ plane; see Fig. 3. These curves are referred to as equilibrium flow curves. The equilibrium flow curve for inlet I is the loci of the set of values $(A_I, A_{II})$ for which $T = Te$ and similarly for inlet II. The enhanced part of the equilibrium flow curves represent stable equilibrium flow areas. Therefore a condition for the simultaneous existence of stable equilibrium flow areas for both inlets is that the enhanced parts of the equilibrium flow curves intersect. In that case the cross-hatched area represents the equilibrium interval. If the actual values of the inlet cross-sections $(A_I, A_{II})$ are outside the equilibrium interval, one or both inlets will close. For details on the stability of a two-inlet bay system, see van de Kreeke [1985].

![Equilibrium flow curves for Inlets I and II](Image)

Application to Pass Cavallo and Matagorda Inlet, Texas

The data used in this section is derived from Ward [1982], [1983] and Bruun, [1983]. Until 1963 Pass Cavallo was the only inlet connecting Matagorda Bay and the Gulf of Mexico. A second inlet was added in 1963; this inlet constitutes the entrance to the Matagorda shipping channel. For the location of the inlets and a plan view of the bay, see Fig. 4.
Matagorda Bay is a shallow bay with an average depth of 3 m. The Bay surface area is 1100 km$^2$. As a result of the shallow depth and the relatively large horizontal dimensions, the time it takes for the tidal wave to traverse the Bay is on the order of a few hours. Consequently in a strict sense Eq.(4) does not apply and to properly calculate the closure curve and closure surfaces, the propagation of the tide in the bay would have to be accounted for. Because this goes beyond the scope of this study, an approximate solution is sought in which the bay surface area is allowed to fluctuate uniformly. To arrive at realistic current speeds and tidal prisms a reduced bay surface area of 350 km$^2$ is introduced.

Tides off Matagorda Bay are mixed. Maximum current speeds occur during spring tide conditions at which diurnal tides are prevalent. The spring tidal range is approximately 1 m.

During a 1959 hydrographic survey the following pertinent inlet characteristics were observed, $A = 8000$ m$^2$, $R = 4$ m, $L = 2350$ m. An average value of the friction coefficient derived from observed velocity profiles is $F = 5 \times 10^{-3}$. Taking $\mu = 1$ and $m = 1$, it follows that for Pass Cavallo in 1959, $K' = 0.4$. Using the approximate solution presented by Keulegan [1951], Eqs. (1) and (4) with $A_b = 350$ km$^2$ yield $\bar{C} = 0.94$ m/sec and a tidal prism equal to $2.5 \times 10^8$ m$^3$. This is close to the observed values of respectively approximately 1 m/sec and $2.5 \times 10^8$ m$^3$. Furthermore it follows from Eq.(3), $k = 0.17$ m.
During the years preceding the 1959 survey the geomorphology of Pass Cavallo changed little and it seems reasonable to assume that at the time of the survey the inlet was in equilibrium with the hydraulic environment. It then follows from Eq. (2) with $F = 5 \times 10^{-3}$ and $u = 0.94$ m/sec that $\tau_{eq} = 4.4$ N/m$^2$. This value is in the middle of the range of equilibrium shear stress values, 3.5–5.5 N/m$^2$, suggested by Bruun [1978].

Making use of Eqs. (1)-(4) and the data listed in Table I, the closure curve for Pass Cavallo, Eq. (5), was calculated. The result is presented in Fig. 5. The relation between $R$ and $A$ used in the calculations, see Fig. 6, was established by assuming the inlet to have a parabolic shape and a constant width to maximum depth ratio equal to the value of 333 observed in the 1959 survey.

Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$L$</td>
<td>2350 m</td>
</tr>
<tr>
<td>$k$</td>
<td>0.17 m</td>
</tr>
<tr>
<td>$m$</td>
<td>1</td>
</tr>
<tr>
<td>$R = R(A)$</td>
<td></td>
</tr>
</tbody>
</table>

Values of pertinent parameter used in calculating the closure curve for Pass Cavallo prior to the opening of Matagorda Inlet.

$$L = 2350 \text{ m}$$
$$k = 0.17 \text{ m}$$
$$m = 1$$
$$R = R(A) \text{ see Fig. 6}$$
$$2H = 1 \text{ m}$$
$$T = 86,400 \text{ sec}$$
$$A_{b} = 3.5 \times 10^{8} \text{ m}^2$$
$$A_{c} = \frac{1}{2}\pi R^{2}$$

Fig. 5 Closure curve and equilibrium stress curves for Pass Cavallo prior to the opening of Matagorda Inlet.

Two models are used for the equilibrium stress curves. In the first model the equilibrium shear stress is assumed constant and equal to the value observed in 1959 i.e. $\tau_{eq} = 4.4$ N/m$^2$. In the second model the current speed, $u$, is assumed constant and equal to 0.94 m/sec. The equilibrium shear stress follows from Eq. (2) with $F$ given by Eq. (3). Equilibrium shear stress curves resulting from the two models are presented in Fig. 5. Note that in the second model $\tau_{eq}$ varies with $A$. 
It follows from the closure curves and equilibrium shear stress curves presented in Fig. 5 that for both equilibrium shear stress models Pass Cavallo is stable. However the degree of stability especially when using the second shear stress model is at best marginal.

In 1963 a second inlet, Matagorda Inlet, was added. The sides of this inlet are protected by rubble and therefore in the stability calculations the width of the inlet is assumed constant and equal to the present width of 300 m. Furthermore a parabolic cross-section is assumed. The resulting relation between R and A for Matagorda Inlet is presented in Fig. 6. Other values of pertinent parameters used in calculating the closure surfaces of Pass Cavallo and Matagorda Inlet, i.e. Eqs. (6) and (7), are presented in Table II.

From observations it follows that after the opening of Matagorda Inlet, Pass Cavallo entered a shoaling mode, whereas Matagorda Inlet continued to scour. Surveys carried out in 1971-1972 yielded $A_I = 7,500 \text{ m}^2$ and $A_{II} = 3,600 \text{ m}^2$. Using the data in Table II, it follows from Eqs. (1) and (4) that in 1971-1972 $U_I = 0.75 \text{ m/sec}$ and $U_{II} = 1.2 \text{ m/sec}$. These values agree with the observation that compared to the 1959 survey, current speeds in Pass Cavallo in 1971-1972 had considerably decreased and were well over 1 m/sec in Matagorda Inlet. The calculated values of

![Figure 6: Relation between hydraulic radius and cross-sectional area for Pass Cavallo and Matagorda Inlet](image-url)
the maximum bottom shear stresses are \( \tau_1 = 2.8 \text{ N/m}^2 \) and \( \tau_{II} = 5 \text{ N/m}^2 \). Because of the relatively rapid response to scouring it may be assumed that during the 1971-1972 survey Matagorda Inlet was in equilibrium with its hydraulic environment and therefore \( \tau_{eq II} = \tau_{II} = 5 \text{ N/m}^2 \). This value is still within the range of equilibrium shear stress values suggested by Bruun [1978]. The values of the K' factor pertaining to the 1971-1972 conditions are \( K'_1 = 0.38 \) and \( K'_{II} = 0.65 \).

Table II

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pass Cavallo (I)</th>
<th>Matagorda Inlet (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>2350 m</td>
<td>2150 m</td>
</tr>
<tr>
<td>k</td>
<td>0.17 m</td>
<td>0.17 m</td>
</tr>
<tr>
<td>m</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>R</td>
<td>( R(A) ) see Fig. 6</td>
<td>( R(A) ) see Fig. 6</td>
</tr>
<tr>
<td>2H</td>
<td>1 m</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>86,400 sec</td>
<td></td>
</tr>
<tr>
<td>( A_p )</td>
<td>( 3.5 \times 10^8 \text{ m}^2 )</td>
<td></td>
</tr>
</tbody>
</table>

The two models used to construct the equilibrium shear stress surface for Pass Cavallo are the same as before. The equilibrium shear stress model for Matagorda Inlet corresponding to Model I of Pass Cavallo is \( \tau_{eq II} = 5 \text{ N/m}^2 \). The slightly higher value of the equilibrium shear stress for Matagorda Inlet is in agreement with the observation that jettied inlets require somewhat larger-tidal prism and thus larger bottom shear stress to maintain a certain cross-sectional area than unjettied inlets, Jarrett [1976]. The equilibrium shear stress model for Matagorda Inlet corresponding to model II of Pass Cavallo is \( \tau_{eq I} = \rho F (1.2)^2 \).

The equilibrium flow curves (i.e. the projection of the intersection of the closure surfaces and equilibrium shear stress surfaces) using model I are presented in Fig. 7. Similarly for Model II the equilibrium flow curves are presented in Fig. 8. In these figures, the subscript I refers to Pass Cavallo and the subscript II refers to the Matagorda Inlet.

Because the enhanced parts of the equilibrium flow curves do not intersect, a combination \((A_I, A_{II})\) for which both inlets are in equilibrium with the flow conditions does not exist. In 1971-1972, the cross-sectional area for Pass Cavallo and Matagorda Inlet were respectively \( A_I = 7,500 \text{ m}^2 \) and \( A_{II} = 3,600 \text{ m}^2 \). Obviously, at that time, Pass Cavallo was in a shoaling mode. Since then the cross-sectional area of Pass Cavallo has gradually decreased leading Matagorda Inlet into a scouring mode. It is expected that this trend will continue. Because of the relatively fast response of an inlet to scouring the values of \((A_I, A_{II})\) are expected to closely follow the enhanced part of the equilibrium flow curve for Matagorda Inlet.
In the absence of tropical storms and hurricanes, Pass Cavallo will ultimately close and the cross-sectional area of Matagorda Inlet will take on a value between 8,000 and 9,000 $m^2$.

**Fig. 7** Equilibrium flow curves for Pass Cavallo and Matagorda Inlet using equilibrium shear stress Model I.

**Fig. 8** Equilibrium flow curves for Pass Cavallo and Matagorda Inlet using equilibrium shear stress Model II.

**Concluding Remarks**

In applying the stability analysis, problems as to how to define the length of the inlet, how to estimate values of energy loss coefficients and bottom roughness and what to assume for the shape of the inlet will be encountered. In addition, if the inlet is in a shoaling or scouring mode, the question as to how the shape, the energy loss coefficients and the friction coefficient will change need to be addressed. Because the answers to many of these questions are not known, the proper evaluation of inlet stability requires calculating closure curves (surfaces) and equilibrium stress curves (surfaces) for a large range of inlet dimensions, energy loss coefficients, friction coefficients,
shapes of cross-sectional areas and closure scenarios.

Because of time and financial constraints the number of calculations in this study had to be limited. Also, the simplifying assumption of a uniformly fluctuating bay level is not justified. Therefore the final conclusion with regard to Pass Cavallo should be interpreted cautiously. However so far, all calculations point at a closure of the inlet barring the occurrence of tropical storms and hurricanes.

Acknowledgment

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References


