CHAPTER NINETY TWO

# EXPERIMENTAL VERIFICATION OF SIMILARITY CRITERIA FOR EQUILIBRIUM BEACH PROFILES

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## ABSTRACT

This paper presents an attempt towards the determination of the geometric distortion for beach mobile bed models. At first, a derivation of a scaling law for modelling equilibrium beach profiles is given, based on Valembois' (1961) work. Further, an experimental verification of some criteria presented in literature for the reproduction of beach profiles in coastal mobile bed models (e.g. Noda, 1971; Yalin, 1971 and Vellinga, 1982) is carried out, leading to a comparison with the criterion developed herewith.

Model experiments were undertaken in a two-dimensional wave flume with regular waves in an attempt to verify the validity of the four similarity criteria for the reproduction of beach profiles.

For fine sand as model material, the experimental results proved valid the modified criterion based on Valembois (1961), among all criteria used. The use of this similarity criterion showed that the beach profiles were correctly reproduced in the model.

The use of cellulose acetate as model material did not give good results, and none of the similarity criteria used were verified. The use of this lightweight material should be avoided for modelling beach processes.

Further testing with finer sand is recommended to analyse the influence of the dimensionless fall time parameter (Dean, 1973) and to confirm its use for the prediction of equilibrium beach profiles.


## INTRODUCTION

Wave sediment transport in the nearshore zone is an important factor in the achievement of engineering solutions

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to beach erosion problems. The determination of sediment transport rates along the shore and on-offshore is a subject of continuing research efforts all over the world. Usually, these studies are developed with the aid of Coastal mobile bed models. Of particular interest is the study of beach profiles, its formation due to wave action and its equilibrium form.

A beach profile is shaped depending on the interaction of the waves and the beach material. It has been reported in literature that the beach reaches an 'equilibrium profile', defined as a beach profile which maintains its geometrical form in time, see for instance, Eagleson et al (1963), Wiegel (1964), Nayak (1970), and Swart (1974).

A coastal mobile bed model is necessarily distorted. The actual value of this distortion is imposed by the model grain size, i.e. cannot be freely chosen. Bijker (1967) stated that the mobile bed model distortion should be equal to the ratio between the equilibrium beach slopes in the model and prototype. This was later endorsed by Fan and Le Méhauté (1969) and Kamphuis (1975).

Hence, it is common practice in littoral transport model investigations to select the model distortion by conducting preliminary flume tests to determine the equilibrium beach slope of different sand grain sizes under the action of the scaled down waves.

## SCALE DEFINITIONS

For the retangular coordinate system, the general model scale $n$ is given by the vertical scale value

$$n = n_z \tag{1}$$

where the scale $n$ is defined as the ratio of prototype value over the model value.

In an undistorted small scale hydraulic model all geometric scales are the same. In a geometrically distorted model, the two horizontal scales $n_x$ and $n_y$ are identical but differ from the vertical scale $n_z$. The geometric model distortion $N$ may thus be defined as

$$N = \frac{n_x}{n_z} = \frac{n_y}{n_z} \tag{2}$$

A beach slope $\dot{m}$ may be defined by the ratio between a characteristic depth at the surf zone and a horizontal distance perpendicular to the beach. If, for instance, the depth of water at the breaking point $d_b$ is taken as a characteristic depth, then the beach slope becomes

$$m = \frac{d_b}{\lambda_b} \tag{3}$$

where $\lambda_b$ is the breaker distance, measured horizontally between the breaking point and the shoreline.

The model distortion may be written according to its definition as

$$N = \frac{m_m}{m_p} \tag{4}$$

where $m_m$ is the model equilibrium beach slope and $m_p$ is the prototype beach profile. The scale of the beach profile is then given as

$$n_m = \frac{m_p}{m_m} = N^{-1} \tag{5}$$

or

$$\left(\frac{d_b}{\lambda_b}\right)_m = N \left(\frac{d_b}{\lambda_b}\right)_p \tag{6}$$

As $N > 1$ by definition, then it is clear that the beach profile in the model is steeper than the prototype one, i.e. exaggerated by a factor equal to the model distortion.

## REVIEW OF PREVIOUS SCALING LAWS

Scaling laws for modelling equilibrium beach profiles have been previously proposed in literature. A review of some published similarity criteria is presented herewith.

### Valembois (1961)

Both Valembois (1961) and Goddet and Jaffry (1960) have put forward similarity criteria that include in their expression a distortion of the wave height, which is not reasonable for modelling beach processes. Their work have been discussed in detail in Paul (1972) and Kamphuis (1975).

Valembois (1961) also presented a theoretical derivation of the geometric distortion for beach models based on the mechanism of bed material suspension by wave breaking. For the correct reproduction of equilibrium beach profiles, he stated that similarity of the following ratio is required:

$$\frac{\text{fall velocity/depth}}{\text{mass transport velocity/horizontal distance}} \tag{7}$$

This concept for similarity of Eq.(7) was further developed by Guimarães (1983) and will be discussed in a later chapter.

### Noda (1971)

With the work of Fan and Le Méhauté (1969) as a basis, Noda (1971) has conducted wave flume beach experiments, to find scaling laws on beach profile formation by breaking waves. Sand and several lightweight material were used as model material. Based on the experimental results, Noda (1971) proposed the following similarity criteria

$$n_D \cdot n_{(\gamma_s/\gamma)}^{1.46} = n_z^{0.55} \tag{8}$$

and

$$n_x = n_z^{1.32} \, n_{(\gamma_s/\gamma)}^{-0.35} \tag{9}$$

where $n_D$ is the scale of the sediment grain size, $n_{(\gamma_s/\gamma)}$ is the scale of the relative submerged unit weight of sediment, where

$$\gamma_s = (\rho_s - \rho)\ g \tag{10}$$

is the underwater unit weight of sediment.

Yalin (1971)

Based on dimensional analysis for sediment transport under the combined influence of short waves, long waves and unidirectional flow, Yalin (1971) developed expressions for nine dimensionless variables controlling bed phenomena for such flow regimes. These may be regarded as criteria for similarity. He also ilustrates that similarity of only two out of the nine dimensionless variables proposed can be maintained for practical model design.

For the same fluid in model and prototype and considering unidirectional flow experimental data, Yalin (1971) proposed the following relationship

$$n_x = n_z^{3/2} \tag{11}$$

which may be written as

$$N = n_z^{1/2} \tag{12}$$

and agrees with the regime theory recommendation. Eq.(12) may also be found in Le Méhauté (1970), which derived it in a different manner, when studying similarity of sediment motion for littoral transport models, for sand models with prototype material only. Yalin (1971) suggests that Eq.(12) holds for tidal flow models rather than beach models.

Vellinga (1978, 1982)

Beach and dune erosion during storm surges have been studied by Vellinga (1978; 1982) based on two-dimensional model experiments. Only fine sand of different grain sizes was used as model material. He considered the importance of the dimensionless fall time parameter P (Dean, 1973) for the reproduction of beach profiles, where

$$P = \frac{H}{wT} \tag{13}$$

and $w$ is the fall velocity of the sand grains; $H$ is the wave height and $T$ is the wave period.

Vellinga (1978; 1982) proposed the following scaling relationships

$$\frac{n_x}{n_z} = \left(\frac{n_z}{n_w^{\,2}}\right)^{\alpha} \tag{14}$$

and

$$n_t = n_z^{1/2} \tag{15}$$

where the wave conditions were scaled down according to the Froude condition

$$n_H = n_T^2 = n_z \tag{16}$$

$n_w$ is the scale of the fall velocity of the sand grains $n_H$ is the wave height scale, $n_T$ is the wave period scale and $n_t$ is the time scale for hydrodynamical and morphological process.

The empirical coeficient $\alpha$ was found earlier (Vellinga, 1978) to vary with sand grain size, ranging from 0.5 to 0.3 with $\alpha = 0.5$ recommended for very fine sand of $D_{50}$ of the order of 0.1 mm. In a re-analysis of the data, Vellinga (1982) concluded that it should be taken as constant and equal to $\alpha = 0.28$. Hence, Eq.(14) becomes

$$\frac{n_x}{n_z} = \left(\frac{n_z}{n_w^{\,2}}\right)^{0.28} \tag{17}$$

Vellinga (1982) has compared his scaling law with data from three-dimensional model tests, from large-scale two-dimensional tests with irregular waves up to significant wave heights of 2 m, and from field experiments, and Eq.(17) was favorably verified. Also, a computer model for the prediction of dune erosion was developed, see Vellinga (1983).

## DERIVATION OF A SIMILARITY CRITERION

The derivation of a scalling law for modelling equilibrium beach profiles is based on Valembois' (1961) work which was further developed by Guimarães (1983).

Valembois' (1961) expression, see Eq.(7), must be made iqual in prototype and model. This requirement becomes

$$\frac{w_p/d_p}{U_p/\lambda_p} = \frac{w_m/d_m}{U_m/\lambda_m} \tag{18}$$

which leads to

$$\frac{d_p}{d_m} \cdot \frac{U_p}{U_m} = \frac{w_p}{w_m} \cdot \frac{\lambda_p}{\lambda_m} \tag{19}$$

yielding in terms of model scales (prototype value/model value) the following relation

$$n_z = \frac{n_w}{n_U} \cdot n_x \tag{20}$$

where $n_z = n_d$ is the vertical scale and $n_x = n_\lambda$ is the horizontal scale of the model. Also, $n_w$ is the scale of the fall velocity of the grains and the scale of the mass transport velocity is $n_U = U_p/U_m$.

On the other hand, similarity of the Froude number in model and prototype gives

$$\frac{V_p}{V_m} = \left(\frac{d_p}{d_m}\right)^{1/2} \tag{21}$$

or

$$n_V = n_z^{1/2} \tag{22}$$

in which the scale of the horizontal velocity is $N_V = V_p/V_m$.

Considering $n_U = n_V$, substitution of Eq.(22) into Eq. (20) gives the following scaling law

$$n_z = (n_w \cdot n_x)^{2/3} \tag{23}$$

Valembois' similarity concept has been transformed into a similarity criterion, i.e. a scaling relationship, see Eq. (23). For a given prototype situation where the bed material is known and considering given the model material and the horizontal scale, which may be determined according to the prototype geometry and laboratory dimensions, the scaling law of Eq.(23) would permit to determine the correct vertical scale of the model and thus the model distortion. For the particular situation where prototype sand is used in the model, Eq.(23) reduces to Eq.(11).

Le Méhauté (1970) proposed a kinematic similarity condition for both fluvial and coastal modelling

$$\frac{n_x}{n_z} = \frac{n_u}{n_w} \tag{24}$$

where $n_u$ is the scale of the horizontal orbital velocity. Vellinga (1978) showed that Eq.(24) may be written as

$$\frac{n_x}{n_z} = \left(\frac{n_z}{n_w^{\,2}}\right)^{0.5} \tag{25}$$

and this expression for the model distortion, which is the same as Eq.(14) with $\alpha = 0.5$, gave good results for his experimental data with finer sands.

It may be seen that Eq.(23) and Eq.(25) are the same but where derived differently. Vellinga (1978) calls $\alpha = 0.5$ the 'theoretical' value and in later work he proposes the use of $\alpha = 0.28$ and Eq.(17) for a distortion relationship.

Another different derivation of the same Eq.(23) may be found in Hughes (1983) where the modelling law was proposed based on distortion of the morphological time scale, deviating from the Froude similarity criteria by a factor of N.

The physical significance of Eq.(23) may be reasoned as follows. Let $t_s$ be the fall time of the sediment grain, in still water, from the water surface to the bed. Thus $t_s = d/w$ and the scale of the fall time of the grains is

$$n_{t_s} = \frac{n_d}{n_w} = \frac{n_z}{n_w} \tag{26}$$

Let $t$ be the time for wave propagation. Hence, in shallow water $t = \lambda/C_{gr} = \lambda/C$ where $C_{gr}$ is the celerity of the wave group and C is the wave celerity. The Froudian time scale for wave propagation is thus

$$n_t = \frac{n_\lambda}{n_C} = \frac{n_x}{n_z^{1/2}} \tag{27}$$

If Valembois' (1961) similarity concept is written as follows

$$\frac{n_w/n_d}{n_U/n_\lambda} = 1 \tag{28}$$

then, considering $n_U = n_V$, substitution of Eqs.(22), (26) and (27) into Eq.(28) gives

$$n_t = n_{t_s} \tag{29}$$

which means that the scale of the fall time of the sediment grain in still water must be equal to the Froudian time scale for wave propagation, or in other words, the fall time of the sediment grains in the model must be reduced in the same proportion as the breaking wave propagation time.

## EXPERIMENTAL PROCEDURE AND RESULTS

The proposed scaling law of Eq.(23) may be experimentaly verified if model tests designed according to it give a good reproduction (in the model) of the prototype beach profiles. By good reproduction of the prototype profiles it is meant that the scale of the beach profiles should be given by Eq. (4) or Eq.(5), i.e, the model distortion is obtained when dividing model beach profile by prototype beach profile.

It is considered in this paper that the coastal mobile bed model is distorted, even though some researchers have proposed to develop model studies in geometrically undistorted coastal models based on the concept of equality of the dimensionless fall time parameter in model and prototype (Dalrymple and Thompson, 1976; Vellinga, 1982; Kamphuis, 1983) or of a similar parameter (Hallermeier, 1984).

Two-dimensional model tests were performed in a wave flume at INPH/PORTOBRÁS, Instituto de Pesquisas Hidroviárias, Empresa de Portos do Brasil S.A., Rio de Janeiro. Detailed information on the experimental equipment and procedure may be found in Guimarães (1983).

### Sediment Characteristics

Two model bed materials were readily available for testing, fine sand, mined from Cabo Frio dunes, Rio de Janeiro, and cellulose acetate, a lightweight material commonly used for modelling in INPH/PORTOBRÁS.

Model beaches were formed with the two different bed materials (fine sand or cellulose acetate) and were submitted to different monochromatic wave climate. Model tests were run maintaining constant the horizontal scale, and for several vertical scales. The simulated prototype beach profile is composed of fine sand grains and is located in Sergipe State, northeast of Brazil, see Figure 1.

Two sand samples were colected in the field and analysed in the laboratory with the purpose of obtaining the necessary information on the prototype bed material. Table 1 gives the sediment characteristics for both model and prototype materials and Figure 2 gives the grain size distribution curves.

The prototype average beach slope was defined based on 12 shore-normal profiles spaced at 500m intervals, covering both the beaches north and south of the Sergipe river inlet, see Figure 1. For an appraisal of the beach profiles evolution, nautical charts of the region were used with surveys from 1970, 1976 and 1979. The average beach slope obtained was approximately 1/400. This value was considered constant and was used as the prototype equilibrium beach slope, i. e. $m_p \approx 1/400 = 0.25\%$.

Horizontal Scale Definition

The definition of the horizontal scale was based on the design of a three-dimensional mobile bed model of Sergipe river inlet, presently (1984) underway at INPH/PORTOBRÁS. This model study is related to future port expansions inside the Sergipe river, the inlet behaviour and the sedimentation of the access channel to be dredged through the offshore bar. It was considered that a 200m wide channel in the prototype should be reproduced with a model width of 0.40m, to permit measurements of the small scale parameters with the available equipment at INPH. These dimensions lead to a horizontal scale of $n_x = n_y = 500$.

Fall Velocity Scales Selection

The fall velocity of the sediment grains was obtained through laboratory measurements. A 2m long glass tube with 3cm diameter was used to measure the fall time of several small sized sediment samples, considering the grain size distribution curves. For each diameter the fall time was recorded 6 times and an average value was found, see Table 2. The distribution curves for the fall velocity were then found (percent finer versus fall velocity curves) for the different materials, see Figure 3, based on the measured data as a function of grain diameter and also on the grain size distribution curves.

Using Figure 3 it was possible to find the scales for the fall velocity of the sediment grains, calculating an average ratio for each given constant value of percent finer as follows

$$\frac{w_{1_p}}{w_{1_m}} = \frac{w_{2_p}}{w_{2_m}} = \ldots = \frac{w_{i_p}}{w_{i_m}} = \text{constant} = n_w \tag{30}$$

For the model with sand material, the fall velocity scale was found to be $n_w = 0.671$. For lightweight material in the model, $n_w = 1.449$. These values are also shown in Table 3. It is interesting to note that these values include the effect of the grain size distribution curves and different scale factors would be obtained if median diameter values for the fall velocity would be used.

Vertical Scales Selection

In order to define the vertical scales to run the model tests, a preliminar calculation was made for the similarity criteria of Noda (1971), Eq.(9), Yalin (1971), Eq.(11), Vellinga (1982), Eq.(17) and modified Valembois, Eq.(23). The results are presented in Table 3.

Based on the variation of vertical scales results shown in Table 3, the tests programme was defined with $n_z$ = 40, 50, 75 and 100 for sand models and with $n_z$ = 50, 75 and 100 for lightweight material. These would provide the experimental verification of the similarity criteria if good reproduction of beach profiles are obtained in the model.

Wave Flume Tests and Results

A total of 69 tests were performed in the two-dimensional wave flume of INPH/PORTOBRÁS, of which 42 with sand in the model and 27 with celulose acetate in the model.

The wave flume is 45.5m long and 0.80m wide at the test section. Initial water depth in the model was 1.05m and the mean sea level was adjusted in the flume according to each vertical scale. A glass window 6.0m long permitted observation of the beach profile development. Regular waves were generated by a flap-type wave paddle, 1.5m wide and 2.0m high. Waves were recorded by a resistance type wave probe located in the flume at a depth corresponding to - 20m in the prototype. A one-channel Watanabe recorder gave a permanent recording of the measured wave heights on chart paper. Wave periods were recorded with a digital stop-watch.

The prototype wave climate was reproduced in the model based on measurements presented and analysed in Motta (1965) and Bandeira (1972) Tables 4 and 5 show wave and sea level parameters for both model and prototype, according to the several vertical scales used.

Prior to the start of each test the beach profile was planed and compacted to a uniform slope, calculated with Eq.(4), which would reproduce in scale the prototype beach profile (see Table 5). At the beginning of each test an exaggerated model wave height was produced in the flume for some minutes, in order to form ripples everywhere on the sloping model beach ensuring that a turbulent boundary layer was formed in the model, as suggested by Motta (1967).

During each test the beach profile would develop itself with the continuous action of the regular waves. Equilibrium was reached after 30 hours of testing for sand as the model material. The lightweight model beaches would reach an equilibrium profile after 5 hours of continuous testing. These model times for profile development are in accordance with the ones reported by Paul (1972), which were 24 hours for sand and 6 hours for bakelite beach profiles.

The monitoring of the equilibrium beach profiles in the flume was

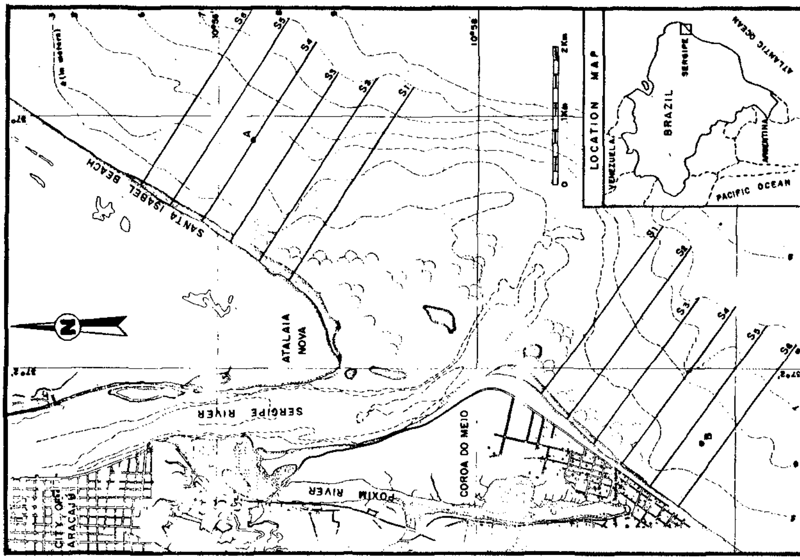


FIGURE 1 : LOCATION OF PROTOTYPE BEACH IN SERGIPE STATE, BRAZIL

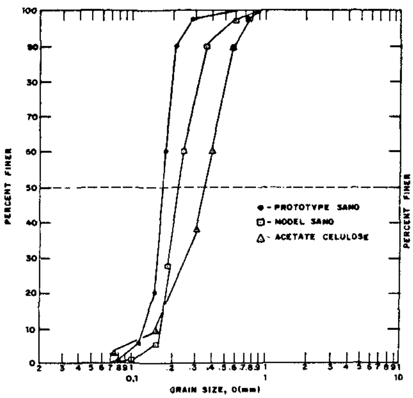


FIGURE 2 – GRAIN SIZE DISTRIBUTION CURVES

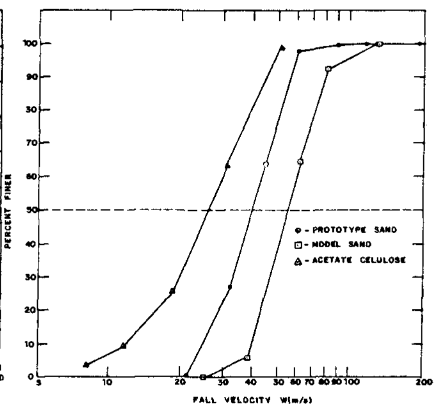


FIGURE 3 – FALL VELOCITY DISTRIBUTION CURVES

done visually after 20 hours of model testing for sand and 3 hours for cellulose acetate. Every beach profile was traced on the window of the flume and whenever no considerable changes would be detected with the continuation of the tests, the profile was then considered to be in equilibrium and the test stopped. An example of the model profile development is shown in Figure 4.

At the end of each test the flume was drained, as slowly as possible, and the beach profile coordinates were recorded through the glass window. Up to 58 depth measurements were taken every 10cm along the profile seawards from the berm crest of the beach. The model beach

TABLE 1. Sediment Characteristics for Model and Prototype

| Sediment Characteristic | Prototype Sand | Model Sand | Celulose Acetate |
|---|---|---|---|
| $D_{50}$ | 0.17 mm | 0.22 mm | 0.37 mm |
| $\rho_s$ | 2630 kg/m$^3$ | 2650 kg/m$^3$ | 1340 kg/m$^3$ |
| $\gamma_s/\gamma$ | 1.558 | 1.65 | 0.34 |

Note: sea water density was taken as 1028 kg/m$^3$.

TABLE 2. Fall Velocity Measured Data

| Sediment Grain Size (mm) | Fall Velocity w (mm/s) | | | Calculated values of w (mm/s) SPM curves for 20°C (1) |
|---|---|---|---|---|
| | Prototype Sand | Model Sand | Celulose Acetate | |
| 0.074 | 21 | 25 | 8 | 5 |
| 0.149 | 32 | 39 | 12 | 16 |
| 0.250 | 61 | 62 | 19 | 35 |
| 0.420 | 90 | 83 | 31 | 72 |
| 0.840 | 120 | 133 | 51 | 135 |
| 2.0 | 190 | - | - | 290 |
| $D_{50}$ (2) | 40 | 59 | 27 | - |

(1) See U.S.Army Corps of Engineers (1977).

(2) Prototype sand $D_{50}$ = 0.17 mm; model sand $D_{50}$ = 0.22 mm; celulose acetate $D_{50}$ = 0.37 mm; see Figures 2 and 3.

TABLE 5. Testing Data

| $n_z$ | N | $m_p$ | $m_{m(1)}$ | (MSL) (m) $P_{(2)}$ | (MSL) (cm) $m_{(3)}$ |
|---|---|---|---|---|---|
| 40 | 12.5 | 1/400 | 1/32 | + 1.5 | + 3.75 |
| 50 | 10 | 1/400 | 1/40 | + 1.5 | + 3.0 |
| 75 | 6.67 | 1/400 | 1/60 | + 1.5 | + 2.0 |
| 100 | 5 | 1/400 | 1/80 | + 1.5 | + 1.5 |

(1) Initial model beach slope, calculated with Eq.(4).

(2) Above the zero reference level of the nautical charts.

(3) Above the initial water depth at the model.

TABLE 3. Calculation of Vertical Scales according to Similarity Criteria

| Similarity Criteria | Equation No. | $n_x$ | $n_{\gamma_s/\gamma}$ | | $n_w$ | | Vertical Scale $n_z$ | |
|---|---|---|---|---|---|---|---|---|
| | | | SM | LWM | SM | LWM | SM (1) | LWM (2) |
| Noda (1971) | Eq.(9) | 500 | 0.944 | 4.583 | - | - | 100 | 165 |
| Yalin (1971) | Eq.(11) | 500 | - | - | 0.671 | 1.449 | 63 | 63 |
| Vellinga(1982) | Eq.(17) | 500 | - | - | 0.671 | 1.449 | 108 | - (3) |
| This study | Eq.(23) | 500 | - | - | 0.671 | 1.449 | 48 | 81 |

(1) SM: Sand model

(2) LWM: Lightweight model, with celulose acetate

(3) Eq.(17) was empirically developed for sand models only

TABLE 4. Wave Parameters

| $n_z$ | $T_p$ | | | $T_m$ | | | $H_p$ | $H_m$ |
|---|---|---|---|---|---|---|---|---|
| | (s) | (s) | (s) | (s) | (s) | (s) | (m) | (cm) |
| 40 | 8 | 10 | 12 | 1.26 | 1.58 | 1.90 | 1.0<br>1.5<br>2.0 | 2.5<br>3.75<br>5.0 |
| 50 | 8 | 10 | 12 | 1.13 | 1.41 | 1.70 | 1.0<br>1.5<br>2.0<br>2.5<br>3.0 | 2.0<br>3.0<br>4.0<br>5.0<br>6.0 |
| 75 | 8 | 10 | 12 | 0.92 | 1.15 | 1.39 | 1.0<br>1.5<br>2.0 | 1.33<br>2.0<br>2.67 |
| 100 | 8 | 10 | 12 | 0.8 | 1.0 | 1.2 | 1.0<br>1.5<br>2.0 | 1.0<br>1.5<br>2.0 |

profile slope was defined as the best fit straight line through the experimental data.

The experimental results obtained during the tests are shown in Tables 6 and 7.

DISCUSSION

The model experiments were designed in accordance to the calculated vertical scales, see Tables 3 and 5. Given the values of $n_z$ and knowing the prototype beach slope $m_p$ for the selected horizontal scale $n_x = 500$, it is possible to calculate the beach slope to be expected to form in the model $m'_m$, using Eqs. (2) and (4) as follows

$$m'_m = m_p \cdot N = m_p \cdot \frac{n_x}{n_z} \quad (31)$$

From simple comparison of the values for $m_m$ and $m'_m$ (see Tables 6 and 7) it can be verified if the prototype beach profile was well reproduced in the model.

Results with Lightweight Material

As it can be seen from Table 6, the use of cellulose acetate as model material did not give good results, and none of the similarity criteria used was verified.

For $n_z = 50$ the results were very different than the calculated value. For $n_z = 75$ the results did not depart so much from the calculated beach slope when the model wave height was small. This was due to the fact that not much sediment was moved by such small model waves and the measured profile would retain the influence of the initial beach slope. The same tendency was seen for the $n_z = 100$ test results , as shown in Figure 5.

Results for the Sand Model

For fine sand as model material the experimental results for beach slope agreed well with the profile calculated values when the vertical scale tested was $n_z = 50$, see tests numbers 10 to 18 in Table 7. This vertical scale value ($n_z = 50$) is approximately the same than the one calculated with Eq. (23), see Table 3.

Hence these experimental results proved valid the similarity criterion based on the work by Valembois (1961). The use of this scaling law Eq. (23) showed that the beach slopes were correctly reproduced in the model.

For $n_z = 40$ the beach slope results presented in Table 7 (tests no.1 to no. 9) did not depart much from the calculated value.

For $n_z = 50$ and high waves in the model (tests no. 19 to no. 24 ) the measured beach slope values shown a deviation from the calculated one. This may be explained by the fact that the prototype beach profile was kept constant throughout the analysis and its value was taken from nautical charts which are based on calm weather site surveys, during the summer months.


FIGURE 4. Model Profile Development

When using Eqs. (9) and (17), the predicted values for the vertical scale are $n_z$ = 100 and $n_z$ = 108 respectivelly (see Table 3 ). For $n_z$ = 75 and $n_z$ = 100 the measured values for beach slope did not agree with the calculated values, see Table 7, tests no. 25 to no. 42 . This seem to indicate that both Eqs. (9) and (17) do not give valid predictions for beach profiles when they are used as scaling laws.

If Eq. (11) is used, the predicted value for the vertical scale ($n_z$ = 63) falls between the tested values of $n_z$ = 50 and $n_z$ = 75 and it may be noticed that Eq. (23) is an improved version of Eq .(11) because it takes into account the influence of the fall velocity scale.


FIGURE 5. Test with Cellulose Acetate

**Table 6. Experimental Results with Cellulose Acetate in the Model**

| Test No | Vertical Scale $n_z$ | Model Wave Height $H_m$(cm) | Model Wave Period $T_m$(s) | Measured Beach Slope $m_m$(%) | Calculated (Expected) Beach Slope* $m'_m$ (%) |
|---|---|---|---|---|---|
| 1 | | | 1.13 | 3.16 | |
| 2 | 50 | 2.0 | 1.41 | 1.80 | 2.5 |
| 3 | | | 1.70 | 3.71 | |
| 4 | | | 1.13 | 3.18 | |
| 5 | 50 | 3.0 | 1.41 | 3.40 | 2.5 |
| 6 | | | 1.70 | 2.69 | |
| 7 | | | 1.13 | 6.76 | |
| 8 | 50 | 4.0 | 1.41 | 4.65 | 2.5 |
| 9 | | | 1.70 | 3.69 | |
| 10 | | | 0.92 | 1.7 | |
| 11 | 75 | 1.3 | 1.15 | 1.68 | 1.67 |
| 12 | | | 1.39 | 1.10 | |
| 13 | | | 0.92 | 1.77 | |
| 14 | 75 | 2.0 | 1.15 | 1.95 | 1.67 |
| 15 | | | 1.39 | 1.03 | |
| 16 | | | 0.92 | 3.36 | |
| 17 | 75 | 2.7 | 1.15 | 2.76 | 1.67 |
| 18 | | | 1.39 | 1.65 | |
| 19 | | | 0.8 | 1.19 | |
| 20 | 100 | 1.0 | 1.0 | 0.96 | 1.25 |
| 21 | | | 1.2 | 1.14 | |
| 22 | | | 0.8 | 1.16 | |
| 23 | 100 | 1.5 | 1.0 | 0.76 | 1.25 |
| 24 | | | 1.2 | 1.03 | |
| 25 | | | 0.8 | 1.59 | |
| 26 | 100 | 2.0 | 1.0 | 1.46 | 1.25 |
| 27 | | | 1.2 | 1.66 | |

$$* \quad m'_m = m_p \cdot \frac{n_x}{n_z}$$

**Table 7. Experimental Results with Sand in the Model**

| Test No | Vertical Scale $n_z$ | Model Wave Height $H_m$(cm) | Model Wave Period $T_m$(s) | Measured Beach Slope $m_m$(%) | Calculated (Expected) Beach Slope* $m'_m$ (%) |
|---|---|---|---|---|---|
| 1 | | | 1.26 | 3.09 | |
| 2 | 40 | 2.5 | 1.58 | 3.28 | 3.13 |
| 3 | | | 1.90 | 3.64 | |
| 4 | | | 1.26 | 3.06 | |
| 5 | 40 | 3.75 | 1.58 | 3.33 | 3.13 |
| 6 | | | 1.90 | 3.15 | |
| 7 | | | 1.26 | 2.99 | |
| 8 | 40 | 5.0 | 1.58 | 3.09 | 3.13 |
| 9 | | | 1.9 | 3.48 | |
| 10 | | | 1.13 | 2.47 | |
| 11 | 50 | 2.0 | 1.41 | 2.51 | 2.5 |
| 12 | | | 1.70 | 2.51 | |
| 13 | | | 1.13 | 2.47 | |
| 14 | 50 | 3.0 | 1.41 | 2.49 | 2.5 |
| 15 | | | 1.70 | 2.52 | |
| 16 | | | 1.13 | 2.53 | |
| 17 | 50 | 4.0 | 1.41 | 2.54 | 2.5 |
| 18 | | | 1.70 | 2.57 | |
| 19 | | | 1.13 | 2.61 | |
| 20 | 50 | 5.0 | 1.41 | 2.58 | 2.5 |
| 21 | | | 1.70 | 2.78 | |
| 22 | | | 1.13 | 2.69 | |
| 23 | 50 | 6.0 | 1.41 | 3.97 | 2.5 |
| 24 | | | 1.70 | 3.88 | |
| 25 | | | 0.92 | 2.30 | |
| 26 | 75 | 1.3 | 1.15 | 1.63 | 1.67 |
| 27 | | | 1.39 | 2.32 | |
| 28 | | | 0.92 | 2.22 | |
| 29 | 75 | 2.0 | 1.15 | 1.80 | 1.67 |
| 30 | | | 1.39 | 2.29 | |
| 31 | | | 0.92 | 2.43 | |
| 32 | 75 | 2.7 | 1.15 | 1.94 | 1.67 |
| 33 | | | 1.39 | 2.13 | |
| 34 | | | 0.8 | 0.58 | |
| 35 | 100 | 1.0 | 1.0 | 0.88 | 1.25 |
| 36 | | | 1.2 | 1.03 | |
| 37 | | | 0.8 | 1.06 | |
| 38 | 100 | 1.5 | 1.0 | 1.09 | 1.25 |
| 39 | | | 1.2 | 0.92 | |
| 40 | | | 0.8 | 0.76 | |
| 41 | 100 | 2.0 | 1.0 | 1.03 | 1.25 |
| 42 | | | 1.2 | 1.41 | |

**Table 8. Fall Time Parameter for Sand Model Tests**

| Test No | $n_z$ | N | $P_m$ | $P_p$ | $n_P$ |
|---|---|---|---|---|---|
| 1 | | | 0.336 | 3.125 | 9.29 |
| 2 | 40 | 12.5 | 0.268 | 2.5 | 9.32 |
| 3 | | | 0.223 | 2.083 | 9.34 |
| 4 | | | 0.504 | 4.687 | 9.29 |
| 5 | 40 | 12.5 | 0.402 | 3.75 | 9.32 |
| 6 | | | 0.334 | 3.125 | 9.34 |
| 7 | | | 0.673 | 6.25 | 9.29 |
| 8 | 40 | 12.5 | 0.536 | 5.0 | 9.32 |
| 9 | | | 0.446 | 4.167 | 9.34 |
| 10 | | | 0.30 | 3.125 | 10.42 |
| 11 | 50 | 10 | 0.24 | 2.5 | 10.40 |
| 12 | | | 0.199 | 2.083 | 10.45 |
| 13 | | | 0.45 | 4.687 | 10.42 |
| 14 | 50 | 10 | 0.361 | 3.75 | 10.40 |
| 15 | | | 0.299 | 3.125 | 10.45 |
| 16 | | | 0.60 | 6.25 | 10.42 |
| 17 | 50 | 10 | 0.481 | 5.0 | 10.40 |
| 18 | | | 0.399 | 4.167 | 10.45 |
| 19 | | | 0.75 | 7.812 | 10.42 |
| 20 | 50 | 10 | 0.601 | 6.25 | 10.40 |
| 21 | | | 0.499 | 5.208 | 10.45 |
| 22 | | | 0.90 | 9.375 | 10.42 |
| 23 | 50 | 10 | 0.721 | 7.5 | 10.40 |
| 24 | | | 0.598 | 6.25 | 10.45 |
| 25 | | | 0.239 | 3.125 | 13.05 |
| 26 | 75 | 6.7 | 0.192 | 2.5 | 13.05 |
| 27 | | | 0.158 | 2.083 | 13.14 |
| 28 | | | 0.368 | 4.687 | 12.72 |
| 29 | 75 | 6.7 | 0.295 | 3.75 | 12.72 |
| 30 | | | 0.244 | 3.125 | 12.81 |
| 31 | | | 0.497 | 6.25 | 12.57 |
| 32 | 75 | 6.7 | 0.398 | 5.0 | 12.57 |
| 33 | | | 0.329 | 4.167 | 12.66 |
| 34 | | | 0.212 | 3.125 | 14.75 |
| 35 | 100 | 5 | 0.169 | 2.5 | 14.75 |
| 36 | | | 0.141 | 2.083 | 14.75 |
| 37 | | | 0.318 | 4.687 | 14.75 |
| 38 | 100 | 5 | 0.254 | 3.75 | 14.75 |
| 39 | | | 0.212 | 3.125 | 14.75 |
| 40 | | | 0.424 | 6.25 | 14.75 |
| 41 | 100 | 5 | 0.339 | 5.0 | 14.75 |
| 42 | | | 0.282 | 4.167 | 14.75 |

### Influence of the Fall Time Parameter

Sayão and Kamphuis (1983) have shown that the dimensionless fall time parameter (see Eq. 13) gives an indication of whether or not a shoreline is receding or progressing, being $P \simeq 1$ a critical value as shown in the Shore Protection Manual (US Army Corps of Engineers,1977). For $P > 1$ recession of the shoreline occurs, while for $P < 1$ progression results.

Table 8 shows the values of the dimensionless fall time parameter for the tests where sand was used as model material. The scale of P is by definition given as

$$n_P = \frac{n_H}{n_w\, n_T} = \frac{n_z^{1/2}}{n_w} \tag{32}$$

and if Eq. (23) is used

$$n_P = \frac{n_x}{n_z} = N \tag{33}$$

and from Table 8, it is found that for tests with $n_z = 50$ the values of $n_P$ are approximately the same as the model distortion values.

It may be seen also from Table 8 that all P-values in the model are smaller than the critical value, whereas all prototype P-values are greater than the critical value. This indicates that there is a progressing beach in the model and a receding one in the prototype which will cause similarity problems if coastal sediment transport and coastal erosion are under investigation.

For almost all measurements of beach slope in the model it was noticed that the resulting type was a berm (summer) profile and the shoreline was progressing, as can be seen for example in Figure 4.

As the prototype values for the dimensionless fall time parameter are $P_p > 1$, the site beach should be a receding one. Bandeira (1972) has calculated littoral drift along the Sergipe shoreline to be of the order of 800,000 cubic meters / year, with a southward direction. Hence, if the beach profiles are far enough south from the mouth of Sergipe river they would be expected to show a receding trend. No further conclusions could be made due to lack of measured prototype beach profiles.

## CONCLUSIONS AND RECOMMENDATIONS

The use of lighweight material cellulose acetate did not give good reproduction of beach profiles in the model, hence this material should not be used for modelling beach processes.

For fine sand as model material the experimental results proved valid the similarity criterion based on Valembois (1961), see Eq.(23), among all criteria used. The use of Eq. (23) showed that the beach profiles were correctly reproduced in the model, as far as beach slope is concerned.

The use of Eq.(23) means in fact that Vallinga (1982) coefficient is $\alpha = 0.5$, which also agrees with the theoretical value proposed by Le Méhauté (1970).

To design a model investigation of beach processes, where the fine sand grain size distribution curve to be used in the model is known, the vertical scale may be calculated with the aid of Eq.(23) and the scale of the fall velocity of the sand grains should be measured in the laboratory. It is to be noted the discrepancy between measured and calculated values of w (see Table 2).

To improve the data base of the present experiments further testing with finer sand (say $D_{50} \simeq 0.1$mm) would be required, so that the dimensionless fall time parameter in the model would be greater than the critical value. For further empirical research it is recommended that P-values for both model and prototype could be maintained in the same region, i.e., above or below the critical value. That may be sufficient, rather than preserving full similarity for the dimensionless fall time parameter between model and prototype as suggested by other researchers. Also, it would give more flexibility for model material selection.

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