# CHAPTER EIGHTY TWO

### A FINITE ELEMENT METHOD FOR STORM SURGE AND TIDAL COMPUTATION

by

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#### Abstract

The paper reports the current progress in developping a finite element method for the shallow water equations. Some recent developments as the implementation of a semi implicit scheme or the use of an incident wave condition are described. Different realistic applications are presented concerning tidal and storm surge simulations.

#### Introduction

Long waves in coastal areas, when water depth is small compared to the wave length, can be numerically simulated by the resolution of the shallow water equations. The use of a finite element method is then specially well fitted when the shape of the domain to deal with is rather complex, as it often occurs in the field of maritime hydraulics, or when some areas of particular interest require a finer description (entrance of a harbour, for example).

Such a model has been developped, using a special procedure to reduce the storage requirement by deriving from the basic equations separate problems on each of the scalar variables. The implementation of a semi-implicit scheme makes it possible to avoid wave damping when simulating wave propagation on broad areas with a limited number of nodes in a wave length. Finally the possible use of an incident wave condition, the consideration of the atmospheric forcing terms, as well as the use of spherical coordinates, qualify the model to examine the generation and propagation of storm surges.

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Equations and general description of the method

The shallow water equations are the following (\*):  

$$\frac{\partial z}{\partial t} + \operatorname{div} \overline{Q} = 0 \qquad (1)$$

$$\frac{\partial Q_i}{\partial t} + \operatorname{div} (Q_i \overline{u}) + \operatorname{gh} \frac{\partial z}{\partial x_i} - K \land Q_i = F_i \quad (i = 1, 2) \qquad (2)$$
where
$$\begin{cases}
g = \operatorname{acceleration} \operatorname{due} \operatorname{to} \operatorname{gravity} \\
h = \operatorname{depth} \operatorname{of} \operatorname{water} \\
K = \operatorname{dispersion} \operatorname{coefficient} \\
\overline{Q} \quad \left\{ \begin{array}{l} Q_1 = \operatorname{flow} \operatorname{rate} \operatorname{per} \operatorname{unit} \operatorname{length} \\
Q_2 \\
\overline{u} \quad \left\{ \begin{array}{l} u_1 = \operatorname{velocity} \operatorname{field} \\
u_2 \\
z & = \operatorname{free} \operatorname{surface} \operatorname{elevation} \\
\overline{F} \quad F_1 = \operatorname{forces} (\operatorname{Coriolis}, \operatorname{bottom} \\
F_2 \quad \operatorname{friction}, \operatorname{wind}, \operatorname{atmospheric} \\
\operatorname{pressure} \operatorname{gradient} \right\}.
\end{cases}$$

Fractional step algorithm for the convection terms

Taking a simple first order time discretisation of equations (1) and (2) between two time steps  $t^n$  and  $t^{n+1}$ , the discretised unknown fields  $\overline{Q}^n$ ,  $z^n$ ,  $\overline{u}^n$  and  $\overline{Q}^{n+1}$ ,  $z^{n+1}$ ,  $\overline{u}^{n+1}$  are introduced.

Then, using an auxiliary unknown field  $\vec{Q^*}$ , equations (1) and (2) can be decomposed into a pure convection system and the remainder.

\* convection step : 
$$\frac{\partial Q_i}{\partial t}$$
 + div  $(Q_i u^n) = 0$  (i=1,2) (3)  
this step gives  $\bar{Q}^*$ 

\* diffusion and propagation step :

$$\begin{cases} \frac{\partial z}{\partial t} + \operatorname{div} \vec{Q} = 0\\ \frac{\partial Q}{\partial t} & \\ \frac{\partial Q_{i}}{\partial t} - K \Delta Q_{i} + \operatorname{gh} \frac{\partial z}{\partial x_{i}} = F_{i} \quad (i=1,2) \end{cases}$$
(4)

with initial conditions  $z^n$ ,  $\overline{Q}^*$  at  $t^n$ .

The convection step is solved using characteristics method.

(\*) These equations are written in cartesian coordinates, but spherical coordinates can also be used in the model.



The main advantages of this method are :

- unconditional stability
- no non-symmetric system to assemble and solve at each time step
- good accuracy (reduced numerical diffusion) when using quadratic shape functions (at least).

The main drawback of classical characteristic method is probably the fact that conservativity is not ensured by the scheme. But a new approach using a weak formulation of equations (3) can be used in order to achieve conservativity [2]. However the results presented in this paper are rather good as for conservativity.

Diffusion-Propagation step

A semi-implicit discretisation has been chosen for system (4) because it has been shown that pure implicit schemes have poor propagation characteristics when used on a coarse mesh (figure 1 and [4]).

The non linear term g h grad z is first splitted up into two terms :

$$g h_m \operatorname{grad}(z) + g(h - h_m) \operatorname{grad}(z)$$

 $\mathbf{h}_{m}$  being a reference depth of water dependant on the location of the point but independent of the time.

Then introducing  

$$\begin{cases}
C = \sqrt{gh_m} : \text{ celerity of waves} \\
\alpha = \frac{1}{\Delta t} \\
\theta, \& [0.5, 1] \text{ two coefficients}
\end{cases}$$

the following discretisation :

$$\frac{\partial z}{\partial x_{i}} = \beta \frac{\partial z^{n+1}}{\partial x_{i}} + (1 - \beta) \frac{\partial z^{n}}{\partial x_{i}}$$
(5)  
$$\frac{\partial Q_{i}}{\partial x_{i}} = \theta \frac{\partial Q_{i}^{n+1}}{\partial Q_{i}^{n+1}} + (1 - \theta) \frac{\partial Q_{i}^{n}}{\partial Q_{i}^{n}}$$
(6)

$$\frac{\partial \mathbf{x}_{i}}{\partial \mathbf{x}_{i}} = \frac{\partial \mathbf{x}_{i}}{\partial \mathbf{x}_{i}} + (\mathbf{I} - \theta) \frac{\partial \mathbf{x}_{i}}{\partial \mathbf{x}_{i}}$$
(6)

leads to the system to be solved :

$$\begin{cases} \alpha^2 \operatorname{zn+1} - K \theta \alpha \Delta \operatorname{zn+1} - \theta \operatorname{div}(C^2 \operatorname{Bgrad} \operatorname{zn+1}) = W \qquad (7) \\ \alpha \overline{\operatorname{qn+1}} - K \theta \Delta \overline{\operatorname{qn+1}} + C^2 \operatorname{Bgrad} \operatorname{zn+1} = \overline{T} \qquad (8) \end{cases}$$

where equation (7) comes from  $\alpha$  (1) +  $\theta$  div (4) and (W,T) are known at each time step.

It must be noticed that equation (7) can be solved easily if  $z^{n+1}$  is given on the boundary. Then,  $\overline{Q}$  can be computed from equation (8) where the term  $C^2 \ \beta \ \text{grad} \ z^{n+1}$  is now a known function and can be rejected into the right hand side.

The problem then amounts to the determination of the trace of z on the boundary. The procedure used for this purpose is an extension of the technique proposed by Glowinski-Pironneau [3] for the Stokes problem. It is described in detail in [1] and [8].

Finite Element Discretisation

All the numerical results presented in this paper have been carried out using a quadratic triangular mesh for the flow rate  $\overline{Q}$  and for the surface elevation z. The elliptic problems of equations (7) and (8) lead to the resolution of 3 symmetric definite matrix problems (one for z and one for each component of the flow rate). At each time step the boundary conditions for z is computed through the resolution of a problems (similar to the 3 previous one) and the resolution of a boundary problem (the size of its definite matrix beeing the number of boundary points).

This technique is advantageous on the point of view of computational efficiency. Two "domain" matrices have to be stored, one for the variable z, another for each component of  $\overline{Q}$  (it is the same matrix for the two components). They are symmetric and sparse and small enough to be processed by an in-core solver such as an incomplete Choleski preconditioned conjugate gradient. In addition, the matrix of the boundary operator has to be factorized and stored.

#### Boundary conditions

On the coast a simple prescribed flux condition can be prescribed :  $\overline{Q} = \overline{0}$ .

On the open boundaries, a specified flux condition (both components) could be too reflexive. In order to specify a wave rather than a flow rate and above all to let waves generated inside leave the domain easily, an incident wave boundary condition may be used. Such a condition, described in details in LABADIE and LATTEUX [8], is written in the following way:

$$\begin{cases} -C^{2} z + K \frac{\partial Q}{\partial n} \cdot \vec{n} + c \vec{Q} \cdot \vec{n} = (\vec{n} \cdot \vec{u} - 1) c^{2} \psi \qquad (9) \\ K \frac{\partial \vec{Q}}{\partial n} \cdot \vec{\tau} + c \vec{Q} \cdot \vec{\tau} = (\vec{\tau} \cdot \vec{u}) c^{2} \psi \qquad (10) \end{cases}$$

Where n and  $\overline{\tau}$  denote boundary exterior normal and tangent vectors,  $\overline{u}$  is a unit vector indicating the direction of the wave to specify and  $\psi$  is a scalar function of space and time describing the incident wave entering the domain through the open boundary (z and  $\overline{Q}$ must be discretised by equations (5) and (6)). The conditions (9) and (10) allow any wave normal to the boundary to leave the domain





without any reflexion. In the simple study of storm surges described further,  $\psi$  can be chosen as follows :

$$\Psi = \Psi_0 = \frac{P - P_0}{\rho_g}$$

with P : atmospheric pressure  $P_0$  : reference atmospheric pressure : 1013 mb

This expression denotes simple static equilibrium at the edge of the continental shelf. But, as we shall see further, taking = o seems to give better results. Other terms may be added in order to take into account tidal forcing.

# Numerical results

A previous paper [6] reported a first realistic computation concerning the tidal flow pattern in the vicinity of the new outer harbour of Dunkirk.

The present one gives the results of simulation of storm surges on the Nort-West European Continental Shelf and of tide in the English Channel.

#### The November 1973 storm surge

A series of storm surge computation models of the North-West European Continental Shelf has been developped since 1969 at the I.O.S. (Institute of Oceanographic Sciences, BIDSTON, UK) using finite difference discretisations and a review of this work is given by HEAPS [7]. As a test for an extended prediction covering a period of 44 days, FLATHER [5] computed the surges from 4 november to 18 december 1973 by means of an improved model including tidal influence. Using our model on the spherical coordinate mesh of figure 2 and taking meteorological data from a tape furnished by JONSMOD (JOINT NORTH SEA MODELLING GROUP) we computed the surge of 19 and 20 november.

Figure 3 presents an exemple of surface elevation and currents just before the very high surge of 19 november 13 h TU. It is an instant when the currents reach their maxima. Although the meteorological data have different origins, the results of this computation and previous results from FLATHER [5] are very similar. The comparison between measurements, FLATHER's results and this finite element computation is done by means of the plots of figure 4a and 4b. In order to simulate the surge of 19-20 november, the computation was started 3 days before (16/11/73 at 3 h).

It can be shown that the behaviour of the finite element model is rather good in general and that the two peaks of 16 and 19 November are very well reproduced, although interaction with the tide was not considered in this simulation.



Dot Line	:	Observed surges
Fine Line	:	Results from FLATHER (1.0.S.)
Thick Line	:	Finite element computation



Figure 4 a: STORM SURGES AT CONTINENTAL PORTS

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Figure 4b : STORM SURGES AT BRITISH PORTS





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The December 1982 storm surge

The November 1973 storm surge was mostly what we should call a North Sea storm surge. Indeed, the depression moved approximately East-South-East from somewhere between Iceland and the Faroes, subsequently crossing southern Scandinavia, and was therefore located far away from the English Channel.

The December 1982 storm surge was different because two large depressions successively joined the English Channel while crossing the North Sea. Our first results obtained using the "incident wave condition" and taking  $\Psi = (P-Po)/\rho$  g showed overestimations of water levels in almost every port. Taking  $\Psi = 0$ , i.e. without generating external waves, but only letting internal waves leave the domain, the results obtained were much better (see figure 5). The reason is that taking  $\Psi = (P-Po)/\rho$  g, an important external wave normal to the boundary is generated and that was probably not the case in the reality.

Two other storm surges similar to the previous one were simulated and the incident wave condition with  $\Psi = 0$  always gave better results than with  $\Psi = (P-Po)/\rho g$ . Our conclusion is that the hydrostatic approximation doesn't reproduce correctly an external surge. In particular the effects of the wind are omitted in the expression of  $\Psi$ . Some investigation has to be made on the subject. However the good agreement between observed and computed elevations is very encouraging since we can improve our model using tidal influence, better meteorological data, optimised mesh...

Tidal simulation in the English Channel

The tidal phenomenon in the English Channel can be considered as an interesting case for the application of the model on a large domain : tide and currents are rather well known, and the high tide ranges, small depths and strong currents provide a good test for the treatment of the non-linearities of the equations (propagation and friction terms).

The domain extends westwards till to the edge of the continental shelf, and covers on the north-east a small part of the North Sea.

Computation has just begun, under simple conditions : flow discharge prescribed on the boundaries, corresponding to a mean spring tide, and totally implicit numerical scheme. Besides, the advection step is not considered here.

The mesh is rather coarse for this first trial on the English Channel (fig. 6): there are 220 triangular elements, and 521 nodes of discretization for the quadratic approximation. This rough discretization allows a very fast computation : the whole tide is simulated within less than 10 minutes C.P.U. on a CRAY 1 computer.



Fig.6-Finite element grid of the English Channel



Fig. 7\_Computed tidal currents at H.W. at BREST



b) Measured co-tidal lines



a) Computed co-tidal lines

Fig.8 \_ Comparison between computed and measured tide ranges in the English Channel (Mean Spring Tide) Fig. 7 displays the tidal current pattern at High Water (Brest). Fig. 8 exhibits the comparison between computed and measured tide range over the English Channel. The main features of the complex tidal pattern in this region, as the significant difference between english and french coast, due to the rotation of the Earth, and the wave amplification in the Gulf of Saint-Malo, are qualitatively well reproduced. However a damping of the computed tidal wave is obvious, coming from the use of an implicit scheme : with only 40 nodes in a wave length, the tide range has generally been weakened by 10 to 20 %.

Further computations performed with a semi~implicit scheme, and possibly with a finer mesh, will probably strongly improve these results.

#### Conclusion

A finite element method has been developped to solve the shallow water equations, and has proved to be particularly well fitted for the simulation of hydrodynamics in complex shaped domains, whatever their size, with reasonable in core memory requirements.

It is the feeling of the authors that, thanks to the advantages of a grid of finite element type (large elements far away, small elements near the region of interest) which decreases the number of required unknowns, the cost of such a model becomes comparable to the one of a classical finite difference model.

The use of a semi-implicit scheme enables the model to avoid wave damping even with a coarse discretization of a wave length ; the implementation of an incident wave condition and the consideration of the atmospheric forcing terms make it quite convenient for the study of the generation and propagation of storm surges on broad areas, just as the North-West European Continental Shelf ; first results concerning the simulation of a measured storm surge has turned to be very satisfactory.

Next steps will concern the tuning of the model on other observed storm surges, the examination of tide-storm surge interaction and, thanks to a better model of turbulence, a better modelling of 2 D boundary layers developped near obstacles and of associated boundary conditions.

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