CHAPTER FIFTY ONE

WAVE MEASUREMENT WITH DIFFERENTIAL PRESSURE GAUGES

by

Kevin R. Bodge[1] and Robert G. Dean[2], M. ASCE

ABSTRACT

The potential error of estimating the small pressure gradient under a directional wave field through the subtraction or comparison of relatively large total-head signals from adjacent pressure transducers in an array is avoided through the use of differential pressure transducers which measure directly the pressure gradients. A device which utilizes four differential pressure tranducers placed orthogonally about one absolute pressure transducer, (the "DPG"), was developed and field-tested at the U.S. Army Corps of Engineers Coastal Engineering Research Center Field Research Facility, Duck, North Carolina. The first five directional Fourier coefficients of the directional ocean spectra were developed from the DPG data, and although no other in situ directional wave monitors were available for comparison, the directional peak determined from the DPG agreed well with simultaneous High Frequency (HF) radar data. The DPG instrument is about one-half the size and less than one-sixth the weight of conventional pressure sensor arrays. The field establishment of the orientation of directional-measuring instruments is also discussed.

1. INTRODUCTION

The "pressure-slope array" is a commonly used instrumentation technique to make in-situ point estimates of wave directionality. Such systems traditionally use a subsurface array of transducers which simultaneously measure the absolute pressure at a number of fixed locations under the wave field. Whereas the use of one pressure sensor to obtain wave height requires the measurement of the dynamic pressure sensed by a single gauge, the use of an array of pressure sensors to obtain wave direction requires intercomparison of the dynamic pressures sensed at adjacent gauges. The direction can be estimated by a number of methods. One is to cross-correlate each of the sensors' signals directly (2). Another is to calculate the

[1]Ph.D. Candidate, Coastal and Oceanographic Engineering Department, University of Florida, Gainesville, FL 32611; Formerly, Research Assistant, Department of Civil Engineering, University of Delaware, Newark, DE 19711.

[2]Graduate Research Professor, Coastal and Oceanographic Engineering Department, University of Florida, Gainesville, FL 32611; Formerly, Professor, Department of Civil Engineering and College of Marine Studies, University of Delaware, Newark, DE 19711.

pressure gradients between the sensors and cross-correlate this estimate of slope (4). In any case, the analysis methods are concerned with the difference in pressures sensed between points under the wave field. For a typical array, where pressure sensors might be spaced about 20 feet (6 m) apart, the instantaneous differences in the sensors' records due to wave action can be very small relative to the large total-head values that the sensors are required to record. Hence the pressure gradient across the array is calculated by comparing two large numbers in order to generate a very small number. This technique can lead to inherent inaccuracies, for example due to minor non-linear response or different noise levels in the sensors. The difference between adjacent sensor signals can be increased by enlarging the distance between transducers -- at the expense of a more physically unmanageable array, greater error in the assumption of linear surface slope, and possibly introduction of directional ambiguities for the higher frequencies present.

It is desirable, then, to measure directly the difference in pressure between two points. A differential pressure transducer (DPT) is ideally suited for this task. The DPT generates an electrical voltage proportional to the fluid pressure difference on opposite sides of a mechanical diaphragm, and can thus be considered an inherently more effective instrument for determining pressure gradient (and thereby wave direction) than conventional pressure sensors. Further, the variation with depth of the differential pressure is a maximum over a range of frequencies that is more representative of typical ocean gravity waves. For a pressure transducer located at height s above bottom, the dynamic pressure, P_{dyn}, can be expressed as a function of position (x,y) from linear theory, as:

$$P_{dyn}(x,y,s,t) = \gamma \frac{H}{2} K_p(s)\cos(k_x x+k_y y-\sigma t+\varepsilon) \qquad (1)$$

where $K_p(s) = \frac{\cosh ks}{\cosh kh}$, the pressure response factor

and H = wave height
x,y = coordinate axes in the horizontal plane
k_x = $k \cos\theta$ = wavenumber in the x-direction
k_y = $k \sin\theta$ = wavenumber in the y-direction
θ = wave direction, measured counter-clockwise from the x-axis
σ = wave frequency
h = water depth
γ = specific weight of seawater
ε = phase angle

The x- and y-components of pressure gradient are then:

$$dP(x,y,s,t) = -\gamma \frac{H}{2} k\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} K_p \sin(k_x x+k_y y-\sigma t+\varepsilon)\begin{bmatrix}dx \\ dy\end{bmatrix} \qquad (2)$$

Figure 1 depicts the dynamic response of pressure for a DPT located in twenty feet (6.1 m) of water, three feet (0.91 m) off the bottom, with

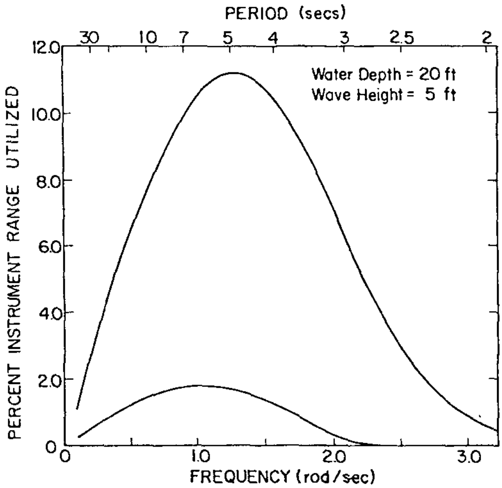

Figure 1. Upper curve: signal from differential pressure transducer sampling 3 feet (0.9 m) along wave ray. Lower curve: difference of signals from two absolute pressure transducers separated 20 feet (6.1 m) along wave ray. (Both curves normalized by typical rated capacity of the measuring instrument.)

a wave height of 5.0 feet (1.5 m). The DPT is assumed to sample two points separated by three feet (0.91 m) coincident with the wave ray. The lower curve represents the difference in pressure monitored by two absolute pressure transducers separated by twenty feet (6.1 m) along the wave ray. The response of each system is normalized by typical values of the rated capacity of the instruments: 35 psia (2460 cm H_2O) for the absolute pressure transducer and ±0.5 psid (70.3 cm H_2O total differential) for the DPT. The differential pressure signal utilizes a greater portion of the instrument dynamic range than the difference between adjacent absolute pressure gauge signals and reaches its greatest response for waves between four and six second periods for the deployment described. The maximum response of the DPT shifts to longer period waves with increasing water depth or instrument deployment depth. The response of the DPT was developed over a pressure sampling space, or gage length, of only three feet (0.91 m) -- less than one-sixth the gage length of conventional pressure sensor array systems. The differential pressure gauge concept, then suggests that more efficiently measured directional information can be obtained with a considerably smaller instrument than is presently used.

2. THE DIFFERENTIAL PRESSURE GAUGE DIRECTIONAL WAVE MONITOR

To test the effectiveness of an array using differential pressure transducers, the differential pressure gauge directional wave monitor

(DPG) was developed and tested under field conditions. The DPG samples the pressure about its center using an absolute pressure transducer and simultaneously samples the pressure gradient along four arms oriented orthogonally about the instrument center using four DPT's, (Figure 2). There are two redundant slope measurements made along each axis in case one of the arm sensors should fail or so that directional spectra calculated from different combinations of gauges might be compared. It was also hoped that the pressure curvature could be developed through the subtraction of collinear slope terms.

3. SELECTION OF TRANSDUCERS AND DPT ARM LENGTH

The selection of the arm length which establishes the pressure difference to be measured by the DPT is dependent upon three criteria: (i) the characteristics of the transducer, (ii) the error in approximating the water surface slope as a linear function between the two sampled points, and (iii) reasonable size limitations of the instrument. The third criterion limits the gage length per transducer to a maximum of about 5 feet (1.5 m) if one imposes a design constraint of easy instrument manageability. At such small spacings, the maximum error in linearly approximating the water surface slope between sampled points is less that one and one-half percent for the shortest waves of interest, (say 3.2 seconds). The necessary rated capacity of the transducer R, is a function of the maximum pressure gradients expected and the ability of the transducer to detect and report a pressure difference of a minimum wave condition beyond the ambient and electrical noise level. From Eq. (2), the gage length between sensors can be found from

$$\Delta x = \frac{R}{\gamma \; H \; k \; K_p} \tag{3}$$

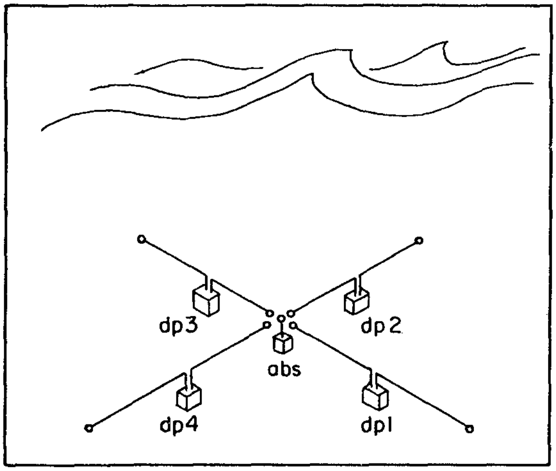

Figure 2. Schematic representation of the DPG directional wave monitor.

evaluated for the maximum wave height of interest (at the most sensitive wave frequency of the DPT at the selected deployment depth).

The site selected for field evaluation of the DPG was the United States Army Corps of Engineers Field Research Facility (FRF) at Duck, North Carolina, which is operated by the Coastal Engineering Research Center (CERC). The instrument was to be placed near and hard-wired to the Facility research pier. After examining the bathymetry near the pier, a nominal deployment depth of 20 feet (6.1 m) was selected. This indicated that the instrument would be most sensitive to waves of about 5 second period, (Figure 1). A design wave height of 16 feet (4.9 m) was chosen and considered with a gage length of 5 feet (1.5 m). This suggested the use of a ±0.6 psid (±42 cm H_2O) transducer which was unavailable. A ±0.5 psid (±35 cm H_2O) transducer was selected instead resulting in a gage length of 4.15 feet (1.27 m) from Eq. (3). The overall dimensions of the final instrument, then, became 9.75 feet (3 m) along each axis and 40 inches (1 m) in height to accommodate the electronics package. The sensors are approximately 3.9 feet (1.2 m) from the seafloor when the instrument is mounted in a supporting cradle.

4. DPG HARDWARE

The _in situ_ instrumentation is contained in a poly-vinyl chloride (PVC) structure that is secured to a steel cradle fixed to the seafloor. The PVC structure, or "instrument" as it is called hereafter, consists of a central tube, or "fuselage," and four arms that extend from near the top of the fuselage, (Figure 3). The fuselage section contains five pressure-sensing isolation diaphragms and a water-tight instrumentation cylinder that contains the pressure transducers. Each of the four arms contain a pressure-sensing isolation diaphragm near its end. The arms bolt into a PVC 4-pipe female junction within the fuselage so that the arms can be disconnected for greater ease in transportation, or potentially, a change in arm orientation. The ends of each arm and the top of the fuselage are punctured with 5/8 inch (1.6 cm) holes. Removable end-caps are fastened to the extreme ends of each arm and the fuselage in order to protect the isolation diaphragm sensors.

A typical DPT (Setra Systems Model 228) measures the pressure difference between the isolation sensor at the end of one arm and one of the five isolation sensors mounted within the fuselage near the center of the instrument. The fifth isolation sensor in the fuselage provides a signal measured by a Setra Systems Model 205-2 50 psia (3515 cm H_2O) absolute pressure transducer. Each isolation sensor consists of a flexible 13 mil DuPont Fairprene® elastomer sealed to an acrylic housing by a 90-10 copper-nickel alloy ring and six Monel® bolts, (Figure 4). The exposed diameter of the elastomer is 1.625 inches (4.1 cm). The isolation sensors are connected to the transducers by flexible tubing -- 1/8 inch (3.2 mm) I.D. stainless steel armored teflon outside the water-tight cylinder and 1/16 inch (1.6 mm) I.D. nylon tubing inside. The tubing is back-filled with an ethanol-water mix (gin) such that the response time of a transducer to a static load placed on its isolation sensor was measured as 0.12 seconds for all sensors.

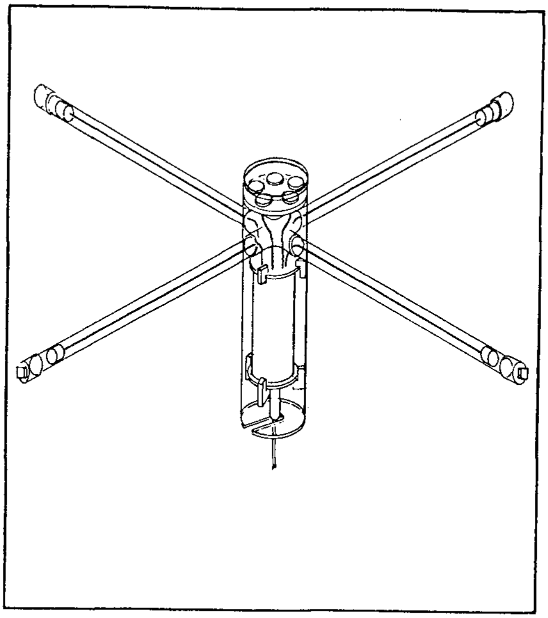

Figure 3. Rendering of the DPG instrument.

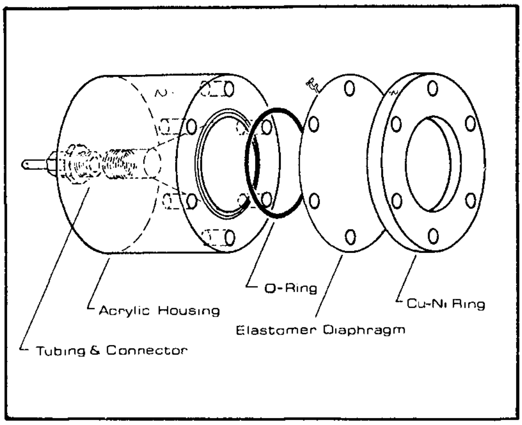

Figure 4. Assembly drawing of an isolation sensor chamber.

The instrument is anchored to the seafloor using a steel cradle which consists of two mutually perpendicular pieces of channel iron in plan view with a hole in the center, and legs to raise the channel off the seabed. The arms of the instrument fit into the channel and are made fast with heavy electrical cable ties. The cradle is anchored to the seafloor by chain that runs taut between each arm of the cradle and screw-anchors in the seabed. Alone, the cradle weighs about 200 pounds (91 kg) in air and about 280 pounds (127 kg) with the instrument attached (cable excluded).

The instrument is cabled to shore for power requirements and analogue data delivery. Eighty feet (24 m) of the seaward end of the cable is stripped of its armoring and stored in a plexiglass box immediately shoreward of the instrument. The top plate of the box is removable in order to access the extra cable if the instrument is to be moved or taken to the surface.

5. CALIBRATION AND WAVE TANK EVALUATION

A relatively simple bench calibration system was designed consisting of two pressure chambers which secure around each isolation sensor of a DPT using a short length of motorcycle tire inner tube, (Figure 5). Each chamber is equipped with a pressure transducer and a bicycle tire valve. The two chambers are connected by tubing with a valve in the center to isolate, bleed, or allow the chambers to communicate with one another. The chambers are pressured with a bicycle tire pump and the pressures which are sensed simultaneously in each chamber and by the corresponding DPG transducer are recorded on a multi-channel strip chart. The response time for the DPG transducers was determined by measuring the passage of time between loading events as reported by the chamber transducers and by the DPG transducer. If an anamolous pressure difference or response lag was observed during a

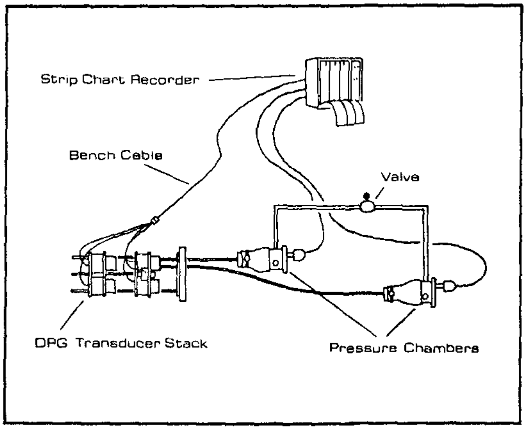

Figure 5. The DPG bench calibration system. One pair of sensors shown for clarity.

calibration test, it was assumed that air was present in the lines and the isolation sensors and connecting tubing were re-back-filled and re-calibrated. After the final back-filling and calibration, each of the transducers on board the DPG was found to respond linearly (1:1) to the pressure loads imposed on its isolation sensors.

The tubing length of the arm sensors is four times greater than that of the center sensors so that an ambient temperature change induces a slight differential pressure across each DPT. (Volumetric expansion of a fluid is linearly proportional to the change in temperature and initial volume of fluid, so that the expanded volume in the arm sensors due to a temperature increase will be four times that of the center sensors.) Temperature changes under twenty feet (6.1 m) of ocean water are typically of low frequency, so it is assumed that any temperature-induced drift during a sampling interval will be interpreted as a very long period wave or as a mean which changes from record to record. If the mean of a DPT record was known at some measured ambient temperature, then for any other sampling interval, the record mean could be used to approximate the water temperature during the time of that record, (neglecting the small lag of the transducer readings due to the thermal mass of the instruments).

An early version of the DPG was tested in the Coastal Engineering Research Center's large outdoor wave tank at Fort Belvoir, Virginia. This initial version of the instrument used semi-rigid 1/8 inch (3.2 mm) I.D. sensor tubing, and it is believed that three of the four arm tubings broke during the instrument assembly; (the tubing was subsequently changed to a flexible type for the field evaluation). The absolute pressure gauge and differential pressure channel dP3 appeared to be in satisfactory order, however, and their signals were recorded on a strip chart while the instrument operated under large amplitude, regular waves generated for another test in progress. The wave profile recorded by the tank's resistance wave staffs indicated non-linear waves of 5 second period and crest-to-trough height between 3.0 feet (0.925 m) and 3.36 feet (1.025 m). Because of the highly non-linear nature of the waves, stream function theory (3) was used to calculate the water surface displacement inferred by the signals of the DPG's absolute pressure gauge — approximately 3.2 feet (0.98 m) from crest to trough with a period of 5 seconds.

The DPG orientation relative to the unidirectional wave train was approximated from the signal of dP3 for each of the two DPG orientations tested. The average total magnitude of the dP3 signal was estimated from the strip chart and the maximum differential pressure expected along the direction of wave travel s, was calculated from stream function theory. The angle θ between the wave ray and the dP3 arm is then:

$$\theta = \cos^{-1} \left[\frac{|dP/dx|_{\text{measured by dP3}}}{|dP/ds|_{\text{max. calculated}}} \right] \qquad (4)$$

where $(dP/ds)_{max}$ is the value applicable for $\theta=0°$ from stream function theory. The orientation of arm 3 of the DPG was measured approximately by divers as 55° to the tank centerline, or wave ray, for the first tests and 35° for the second tests. The calculated angle from (4) using the instrument data was 51.3° and 35.4° for each of the two sets of tests, respectively. The directional agreement -- as well as the ease with which the instrument was installed and removed -- was encouraging. Details of the wave tank evaluation are given in Bodge (1).

6. FIELD INSTALLATION AND ORIENTATION MEASUREMENT

The DPG field prototype was deployed at the CERC Field Research Facility on May 14, 1982, using the Facility's Coastal Research Amphibious Buggy (CRAB). The CRAB is a large tripod that is capable of driving across the beach and into the surf zone under its own power. The DPG was hoisted beneath the operator's platform, and the CRAB driven toward the end of the pier where the cable was mated to the instrument and then driven to the installation site where the instrument and cradle were lowered and secured to the seafloor by divers. The relatively small and lightweight DPG hardware, combined with the flexibility of the CRAB, afforded a very smooth installation -- despite less than ideal wave conditions.

The DPG is installed on the relatively stable 20 foot (6.1 m) depth contour, approximately 735 feet (224 m) south of the pier and 1800 feet (549 m) offshore. The orientation of the instrument was determined using a submersible digital compass mounted at the end of a five-foot (1.5 m) length of aluminum angle which was placed atop the end of each PVC instrument arm. In this way, the digital compass was at least six feet (1.8 m) from the steel cradle. Compass headings were recorded for each arm twice -- once by each of two divers -- and then averaged. The orientation of each arm was also measured by securing a diver's compass around the end of each arm using the compass wrist strap. Measurements were taken three times for each arm using two different wrist compasses and then averaged. All of the values recorded for each technique along each arm agreed reasonably well within a technique.

As an experiment, the divers' compasses were placed on an arm above the end of the steel channel and then slid towards the end of the arm. The heading changed as the compass moved across the end of the steel and then remained relatively stable. It was therefore thought at the time that the readings taken at the ends of the PVC arms were sufficiently far from the steel cradle to avoid magnetic bias. However, this was found not to be the case. Since each arm is perpendicular to its neighboring arm, adjacent arm bearings are constrained to be 90° apart. Although the digital compass measurements approximate 90° separation, those of the wrist compasses clearly do not, (Table 1).

If the measured orientation β_n' is expressed in terms of the actual orientation, β_n, of the arm and the error, ε_n, associated with that arm that is introduced by the metal mass of the cradle, one can write:

Table 1: ORIENTATION HEADINGS* OF THE DPG ARMS					
ARM	WRIST COMPASS		DIGITAL COMPASS		RESOLVED
	apparent	actual	apparent	actual	averaged
1	228.33	241.75	238.80	241.89	241.82
2	155.33	151.75	153.90	151.89	151.82
3	78.00	61.75	61.20	61.89	61.82
4	325.33	331.75	333.65	331.89	331.82

*Headings are with respect to magnetic north, 6/15/82.

$$\beta_n' = \beta_n + \varepsilon_n \tag{5}$$

Since the compasses were placed upon each arm at about the same distance from the center of the instrument, and if one assumes that the effect of the metal mass upon the compass readings is the same for each arm (i.e., radially inward), then the errors associated with each of two collinear arms should be equal and opposite in sense. Accordingly,

$$\sum_{n=1}^{4} \varepsilon_n = 0 \tag{6}$$

If one expresses the actual orientation of each arm in terms of the arm with the smallest value of apparent direction, (arm 3, in this case), Eq. (5) may be written:

$$\beta_3' = \beta_3 + \varepsilon_3 \tag{7a}$$

$$\beta_4' = \beta_3 + 90^0 + \varepsilon_4 \tag{7b}$$

$$\beta_1' = \beta_3 + 180^0 + \varepsilon_1 \tag{7c}$$

$$\beta_2' = \beta_3 + 270^0 + \varepsilon_2 \tag{7d}$$

From the sum of Eqs. (7) and substituting (6);

$$\beta_3 = \frac{1}{4}\left[\left(\sum_{n=1}^{4} \beta_n'\right) - 540^0\right] \tag{8}$$

The corresponding orientation for each of the other three arms may be found by successively adding 90° to the orientation of the adjacent arm. The agreement between the corrected averages of the arm headings as found using the expensive digital compass and the two simple wrist compasses is surprisingly good, within 0.3° (Table 1). This investigation indicates the importance of establishing the inherent inaccuracies in a compass and/or of careful redundant checks of instrument orientation when working near a steel structure.

7. DATA ANALYSIS AND RESULTS

If only the absolute pressure signal P, and one slope signal from each of the two axes, dP_x and dP_y, are processed, then the DPG yields the same information as a heave-pitch-roll buoy. The first five coefficients in a Fourier series representation of the directional wave spectrum ("directional Fourier coefficients," or DFC's, for short), can be found using the analysis technique described by Longuet-Higgins, Cartwright, and Smith (4).

If the absolute pressure and the four slope signals are measured and the curvature across each axis established, the first seven DFC's could be developed by the aforementioned technique (4). However, attempts to calculate curvature through the subtraction of collinear slope measurements were generally unsuccessful. This is not surprising since it requires the calculation of very small curvature terms by subtracting two already small (measured) slope terms. In principal, differential pressure transducers can be used to measure directly pressure curvature, however, and this technique will constitute the subject of a future investigation.

If one directly cross-correlates the instrument's signals using the technique outlined by Borgman (2), among others, higher order DFC's can generally be obtained. If one directly cross-correlates the instrument's signals using the technique outlined by Borgman (2), among others, higher order DFC's can be obtained if the gages are separated sufficiently to allow estimation of an accurate separation-related phase difference in the signals of the two gages. If the gages are so closely located that it is impossible to obtain meaningful phase differences due to their separation, it is possible to estimate only the first five DFC's. If it is known that waves approach only over a limited range of directions, it is possible to modify the analysis procedure to improve the directional description substantially. Altering the orientation of the arms as shown in Figure 6 would appear to improve the quality of the directional distribution information for waves approaching over a limited range of directions.

Inspection of the time series signals from the submerged DPG indicated that one of the differential transducers along the instrument's x-axis was suspect. Accordingly, only the absolute pressure signal and one measure of slope along each axis were processed. In particular, the pressure slope along each axis was calculated by dividing each differential pressure signal by the gauge length between DPT sensors, and the absolute pressure and pressure slope time series were then transformed to the frequency domain. The linear pressure response function and a low pass filter were applied to obtain the water surface displacement and slope spectra, and the analysis technique mentioned above, (4), was utilized in order to calculate the first five directional Fourier coefficients.

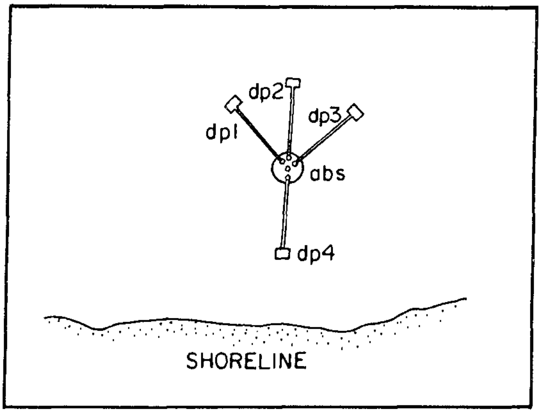

Figure 6. Plan view of DPG proposed to potentially improve resolution of directional wave distribution.

For several data sets, directional spectra were calculated using P, dP_x, and one of the dP_y signals — and then re-calculated using P, dP_x, and the other dP_y signals from the redundant y-axis differential pressure transducer. Agreement between the calculated spectra using the two different y-axis DPT signals was satisfactory (1).

No other *in situ* directional wave monitors were operational during the first few months following the DPG installation. Therefore, the wave height and period information calculated from the DPG's absolute pressure sensor were compared to height and period estimates from nearby surface-piercing Baylor gauges, (Table 2). The direction of peak wave energy reported by the DPG was compared to visual estimates and HF radar images of the wave fields, (Table 2). The agreement between observed and DPG-generated height, peak period, and directional estimates, (all taken from the spectra for the DPG estimates), is encouraging.

It is theoretically possible to develop the water surface displacement energy spectrum from the slope spectra of the differential gauges:

$$S_{\eta\eta}(\sigma) = \frac{S_{\eta_x\eta_x}(\sigma) + S_{\eta_y\eta_y}(\sigma)}{k^2} \tag{9}$$

where $S_{\eta\eta}$, $S_{\eta_x\eta_y}$, and $S_{\eta_y\eta_y}$ represent the auto-spectra of water surface displacement, and x-axis and y-axis slopes respectively. Theoretically, it is also possible to estimate the principal wave direction without the absolute pressure signal if one assumes that there exists only one wave direction θ per frequency σ. Specifically,

Table 2: COMPARISON OF DPG RESULTS WITH FRF MEASUREMENTS AND OBSERVATIONS

8 June - 11 June, 1982

DATE	TIME		DPG	FRF OBSERVATIONS
6/8	0700	dirxn	59°	58°[a] (50°)[b]
		H_{sig}	1.39	1.41[c]
		period	8.23	8.00[c]
	1300	dirxn	59°	- -
		H_{sig}	1.28	1.33[c]
		period	8.79	9.57[c]
6/9	0700	dirxn	67°	68°[a] (50°)[b]
		H_{sig}	1.34	1.33[c]
		period	9.44	9.66[c]
6/10	0700	dirxn	65°	66°[a] (55°)[b]
		H_{sig}	1.23	1.56[c]
		period	10.19	10.56[c]
	1300	dirxn	85°	- -
		H_{sig}	1.71	1.82[c]
		period	11.07	10.34[c]
6/11	0700	dirxn	73°	69°[a] (60°)[b]
		H_{sig}	1.32	1.53[c]
		period	11.07	10.89[c]

NOTES:
"dirxn" listed is the principal direction (peak energy), true north.
"H_{sig}" is the significant wave height in meters.
"period" corresponds to the frequency band of greatest energy in seconds.
[a]CERC Radar (± 2°)
[b]Visual estimate from the end of the pier
[c]CERC Baylor Gauge near the end of the pier

$$\tan 2\theta\,(\sigma) = \frac{2\,S_{\eta_x \eta_y}(\sigma)}{S_{\eta_x \eta_x}(\sigma) - S_{\eta_y \eta_y}(\sigma)} \tag{10}$$

There exist four roots in the arc-tangent of 2θ, and it can be shown that two of these roots are associated with maxima (and separated by 180°) and the other two (also separated by 180°) are associated with minima and separated from the first two roots by 90°. Two of these roots (the minima) can be eliminated by considering the signs of the numerator and denominator. The remaining two might be resolved by considering the physical environment of the instrument deployment site; i.e., ruling out the possibility of dominant waves originating from the beach. Equations (9) and (10) were tested on a frequency-by-frequency basis with DPG data and compared to the total energy from the absolute channel and the peak-energy direction found from the directional spectra, respectively. Agreement was poor (with errors often greater than 40% in energy and 70° in direction) -- presumably due to spectral leakage and because the noise of the differential pressure signals is potentially greater than that of the absolute pressure signals and because the wave energy is spread over many directions in reality.

Similarly, a directional estimate was considered (and found likewise unreliable) which uses only one differential pressure signal and the absolute pressure signal:

$$\theta(\sigma) = \cos^{-1} \frac{1}{k} \sqrt{\frac{S_{\eta_x \eta_x}(\sigma)}{S_{\eta\eta}(\sigma)}} \tag{11}$$

or

$$\theta(\sigma) = \sin^{-1} \frac{1}{k} \sqrt{\frac{S_{\eta_y \eta_y}(\sigma)}{S_{\eta\eta}(\sigma)}} \tag{12}$$

assuming one direction per frequency. Non-linearity in wavenumber, k, may contribute to the observed error of (11) and (12).

8. CONCLUSIONS

From the present work, it appears that differential pressure gauges provide an effective means of measuring the pressure gradient under a wave field -- enabling estimation of at least the first five coefficients in the Fourier series representation of the directional wave spectrum. The use of differential pressure sensors enables one to directly measure pressure gradients over small arrays instead of comparing large-valued point measurements of pressure over relatively large arrays. It appears that the development of higher order terms, such as curvature, are also best made directly.

ACKNOWLEDGEMENTS

The DPG was reduced to practice under U.S. Army Corps of Engineers Coastal Engineering Research Center Contract No. DACW72-81-C-0025 and further supported by the Exxon Research Foundation while the authors were at the University of Delaware. The authors wish to thank James Coverdale, Ed Green, and Roland Essex of the University of Delaware Civil Engineering Shop Facility, Dr. Todd Walton of CERC, Curtis Mason and the staff of the FRF, and Brian Jacobs, "Chip" Fletcher, David Wiley, and Karen Francheschini for their dedicated assistance to the project.

APPENDIX 1. - REFERENCES

1. Bodge, K. R., "The Design, Development, and Evaluation of a Differential Pressure Gauge Directional Wave Monitor," U.S. Army Corps of Engineers Coastal Engineering Research Center, MR82-11, October, 1982.

2. Borgman, L. E., "Directional Spectra Models for Design Use," Proceedings, Offshore Technology Conference, 1969.

3. Dean, R. G., "Evaluation and Development of Water Wave Theories for Engineering Application," Special Report No. 1, U.S. Army Corps of Engineers Coastal Engineering Research Center, Vols. I and II, Nov., 1974.

4. Longuet-Higgins, M. S., Cartwright, D. E., and Smith, N. D., "Observations of the Directional Spectrum of Sea Waves Using the Motions of a Floating Buoy," Ocean Wave Spectra, Proceedings of a Conference, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.