Swash on a Natural Beach

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Abstract

A field measurement was conducted in order to clarify the swash motion on a natural beach. It is found on this particular beach, which had a rather steep foreshore slope, that cross spectra calculated between the surface elevations, onshore velocities and the swash agree very well with those given by linear long two-dimensional standing wave theory in the lower frequency region than a certain value. This finding together with the observed $f^{-4}$ ($f$; frequency) high frequency saturation in swash spectra encourages a partial reflection model to describe the fluid motion in the inner surf zone, including the swash. The model developed shows a good agreement with the observed results, in which being employed an analogy to regular waves for the criticality of wave reflection. This model makes it possible to predict the swash spectrum for a given incident wave spectra and a given beach profile, as far as wave breaking takes place on a foreshore slope.

1 INTRODUCTION

The boundary zone between the land and the sea provides interesting wave phenomena known as wave run-up. It is this zone where the incident waves show the swash motion which includes both the up-rush and the down-rush. Wave set-up due to wave breaking outside the swash zone contributes only to the mean water level change and the position of the mean swash motion, namely the shoreline.

The understanding of the wave dynamics in the swash zone on a natural beach is important in order to give a reasonable boundary condition at the shoreline when one tries to simulate the two- or three-dimensional beach transformation due to waves. Conventional assumption that the wave height is linearly proportional to the water depth gives zero wave height at the shoreline, resulting neither wave motion nor sand movement. This is clearly not what we observe in the field or in the laboratory, although one can imagine an extreme case where the wave energy would be completely dissipated before it arrives the swash zone on a very gentle beach.

On the other hand recent field observations reveal that the long period fluctuation may prevail among the fluid motion in the surf zone, especially near the shoreline (Guza & Thornton, 1982; Mizuguchi, 1982b). The long period motion is supposed to be a standing wave in the on-offshore direction, no matter if it is a two-dimensional one or a three-dimensional one like an edge wave. The long period standing wave form an antinode at the shoreline, exhibiting swash oscillations. There has been some discussions whether the observed long period fluctuations are...
two- or three-dimensional. It is critically higher mode or cut-off mode edge waves that were employed to fit the data by Huntley (1976) and Sasaki & Horikawa (1978). It is shown, e.g. by Hotta et al. (1980), that the higher mode edge waves do not quantitatively differ from the two-dimensional standing waves as far as the fluid motion in the near-shore zone is dealt with. Therefore in this paper only two-dimensional motion is assumed as is done by Suhayda (1974).

This study comes from the first motivation mixed with the second findings. That is to test a hypothesis based on the linear long standing wave theory against a field experiment.

2. SWASH OF REGULAR WAVES

The wave run-up on a somewhat artificial beach has been studied for a long time and of their main concern has been the highest point of the up-rush. For example see Hunt (1959). The long history of the studies on its dynamics can be looked at from the following two viewpoints. There are a group of papers in which they looked at the swash motion as the highly nonlinear process and treated it as a bore on a dry bed. For example Shen & Meyer (1963). This approach indicates that the wave front shows a parabolic motion on a frictionless plane beach and that the total excursion width $Y$ is given by the following relation for regular waves (Van Dorn, 1976).

$$Y = \frac{l}{4gT^2\tan^2\beta}$$

where $g$ is the gravitational acceleration, $T$ is the wave period and $\tan \beta$ is the bottom slope.

Another long history can be traced back to Lamb (1932). The swash motion is considered to be a standing wave on a sloping beach. The linear long wave theory gives the following well-known results for the water surface elevation $\eta$ and the offshoreward velocity $u$ for the perfectly reflected waves on a plane beach.

$$\eta = 2a_r v_0(z) \cos \sigma t$$
$$u = 2a_r \frac{\varepsilon}{\beta} \frac{\eta_0(z)}{h_1(z)} \sin \sigma t$$

where $z = \sqrt{\alpha x}$, $\alpha = 4 \sigma^2 / gtan \beta$ and $2a_r$ is the amplitude of a standing wave at the shoreline. The $x$ axis is taken offshoreward along the still water surface from the shoreline. The finite amplitude theory by Carrier & Greenspan (1956) shows that the amplitude of the swash motion does not differ from that given by the linear theory Eq.(2).

The standing wave solution, however, is no longer valid when the so-called swash parameter $\epsilon_r$ which is defined as

$$\epsilon_r = 2a_r \sigma^2 / gtan^2 \beta$$

exceeds a certain critical value $\epsilon_{rc}$. Miche (1951) argued that the surface slope at the shoreline should not be smaller than the beach slope and gave $\epsilon_{rc} = 2$. The finite amplitude wave theory gives $\epsilon_{rc} = 1$ as
a necessary condition for the theory to yield a solution (Meyer & Taylor, 1972). The critical value \( r_c = 1 \) also corresponds to a condition that maximum downward acceleration of the swash is equal to the gravitational acceleration along the beach slope. Miche also proposed a hypothesis that the reflection coefficient for the incident waves on a plane beach may be given as follows:

\[
r_0 = 2/ \epsilon_0 \left( \frac{\varepsilon_i}{\varepsilon_{i0}} \right)^{1/5} \left( \frac{\psi}{r_{i0}} \right)^{1/5}
\]

(5)

where

\[
\epsilon_0 = \sqrt{\frac{2}{\pi}} a_0 \sigma \left( \frac{g}{\tan \beta} \right)^{5/2}
\]

(6)

and \( r_0 \) is the reflection coefficient in deep water. The parameter \( \epsilon_0 \) is
equivalent to the swash parameter \( \epsilon_s \) and is calculated under the condition of perfect reflection, using the relation,

\[
a_r = a_0 \sqrt{\frac{\pi}{2 \tan \beta}} \left( \frac{h}{\psi} \right) \left( \frac{a_{i0}}{a_{i0}} \right)^{1/5} \]

(7)

Equation (5) simply states that the possible swash oscillation is always limited by his critical condition.

Guza & Bowen (1976) carried out detailed laboratory experiments in order to investigate the behaviour of the standing waves. They found (1) that the incident waves are perfectly reflected and the observed spatial distribution of the wave height agrees well with Eq. (3) when \( \epsilon_0 \leq 1 \), and (2) that Miche's hypothesis on the reflection coefficient given by Eq. (5) shows a reasonably good agreement with their experimental results when \( \epsilon_0 \geq 1.6 \), being taken into account the effect of bottom roughness. Their study confirmed that the swash oscillation can be considered at least on a rather steep beach as the antinode motion of two-dimensional standing waves, which survived wave breaking.

Comparing these two different approach to the regular wave swash, one may notice that Eq. (1) is quantitatively almost identical to Eq. (5). Substituting \( Y = 2a_r \) into Eq. (1) gives \( \epsilon_s = 2.5 \). Figure 1 compares the experimental data by VanDorn (1976) with those two semi-theoretical results. It is clearly seen that the data agrees with Eq. (5) when the parameter \( L_0 \tan^2 \beta \) is large, in other words, high reflection coeffi-

![Fig. 1 Total excursion of swash (VanDorn, 1976).](image-url)
cients can be expected. As the parameter decreases, Eq.(1) tends to show a better agreement, although the difference between the two lines is small as already pointed out. However it should be noted that, in a strict sense, the wave amplitude should affect the transition.

It is tempting from the above discussion to make a statement that the swash oscillation is, in any case, the antinodal motion at the shoreline of a standing wave, which survives its breaking on the slope. Practically, as pointed by Guza & Bowen (1976), Miche's criterion Eq.(5) provides the "surviving" ratio, as far as wave breaking occurs only on the foreshore slope. The following viewpoint can be added to support this tempting conclusion. It is not an unreasonable conjecture that the swash oscillation, which might be given through a complicated nonlinear process on a gently sloping beach, plays a role to generate an outgoing wave by its periodic or unsteady forcing. The outgoing wave forms a standing wave, coupling with the incident wave. Hence the swash motion may well be treated as that of a standing wave at least as a first approximation. Therefore the following expression would be applicable to describe the wave motion near the shoreline not only for a steep (reflective) beach as confirmed by Guza & Bowen (1976) but also for a gentle (dissipative) beach, again as a first approximation.

\[
\eta = a_0 \{ J_0(z)\cos \sigma t - Y_0(z)\sin \sigma t \} + 2a_r J_0(z)\cos \sigma t \tag{8}
\]
\[
u = \sqrt{\eta} a_1 J_1(z)\cos \sigma t + Y_1(z)\sin \sigma t \tag{9}
\]

where \( a_i \) is the amplitude of the progressive wave, which should change to be zero at the shoreline. The local reflection coefficient \( r = a_i/(a_i + a_r) \) varies with the on-offshore location as \( a_i \) does. The bottom profile in the field scarcely shows a plane beach. Then Eqs.(8) and (9) can be easily extended to the case of a complex beach profile by applying a multi-linear profile approximation.

3. A MODEL FOR IRREGULAR WAVE SWASH

For irregular waves, one, who put the stress on the high nonlinearity of the process, may introduce a model based on the joint probability distribution of the wave heights and periods. For examples Battjes (1971) and Sawaragi & Iwata (1984). It is generally accepted that the swash motion on a gently sloping beach is described in such a model. However the long period fluctuation in the swash is significant as reported in many field experiments. The individual wave analysis clearly fails to be meaningful when the long period fluctuation exists, unless it is removed before the analysis is applied (Mizuguchi, 1982a). Here a tentative model is proposed based on the previous discussion and illustrated schematically in Fig. 2. The model implies that the Fourier component of the irregular wave behaves independently even through the very nonlinear process like wave breaking, as far as the long period motion is concerned. It is trivial that this model has the shortcomings, as not only the real swash motion but also the waves near the shoreline show some nonlinearity. However it is worth trying to see how well the simple model based on a linear theory can describe a result obtained in a field experiment.
In relation to the critical condition for irregular wave reflection, there have been some field observations on the high frequency saturation of the swash spectra. Huntley et al. (1977) reported the form $f^{-4}$ for the saturated high frequency range. Guza & Thornton (1982) found $f^{-3}$ instead. When the concept of the critical condition could be applied for different frequency component independently, one can expect a kind of universal form for the saturated region, although it is not theoretically clear how the result for monochromatic waves is related to that on the irregular waves. The spectral shape of the high frequency region is also considerably affected by the profile of the main (peak frequency) component. The smoother the profile is, the smaller the power leaking to the high frequency region is. In addition, different techniques employed to measure the swash may produce different results (Guza & Thornton, 1984). Here we will not go further into this discussion.

4. FIELD OBSERVATION

A field observation was carried out around noon on 9th Jan. 1982 at Yonezu-hama beach in Shizuoka Prefecture, Japan. The beach faces to the Pacific Ocean in south and has a straight extension of about 20 km. The beach cusp formation was observed on the previous day. However there remained only their ruins around the high water level on that day. The observation site was almost at the middle of the beach extension and the measuring section chosen on the line of the apex of a beach cusp ruin, so that the node of the edge waves had been expected if it had existed.

In Fig. 3 was shown the experimental setup with the bottom topography along the measuring section. Two artificial channels were placed, being extended into the water only to the position of the lowest run-down point, just covering the swash zone. The natural swash motion was measured between the two channels, by using the photographic technique. Small poles were placed standing on a line, every 50 cm, to give a scale. Several poles and two EMCM (Electro-Magnetic Current Meter) were also installed on the measuring section. The poles were photographed with 16mm camera, to obtain surface elevation records. Horizontal
velocities were measured with EMCM. The EMCM(on) was situated just on the step. The bottom profile was a typical bar-trough one with a step at the foot of the swash zone. The slope of the swash zone was \(1/9\). The beach material of that zone was well-sorted sand with \(\sqrt{d_{75}/d_{25}}=1.7\) and \(d_{50}=0.4\) mm. Visual observation showed (1) that the incident waves were long-crested swells with some white caps due to the strong westerly wind, (2) that the average breaking point was on the offshoreward slope of the bar between the poles (25) and (26), and (3) that the broken wave reformed almost completely in the trough region and made surging type breaker on the foreshore slope. The measurement was done for the duration of 10min 22s with the sampling interval of 0.2s, yielding 3110 total data points.

Examples of the obtained raw data are shown in Fig. 4. This figure shows the following features; (1) Short period waves disappear almost suddenly in the swash record, although they are still seen in the surface elevation at the pole (10), which was in the swash zone. (2) Both at the two locations where the horizontal velocity was measured as well as the surface fluctuation, they are not correlated, in contrast to the in-phase relation for the progressive wave theory. (3) The downward motion of the swash, for instance that of about 60s from the beginning of the data, shows a very good correlation with the strong offshoreward velocity, and the upward motion does with onshore one. The latter two features indicate a fact that the long period fluctuations were significant, forming the swash oscillations as the antinode of standing waves.
Fig. 4 Examples of raw data obtained. The dotted line on the top denotes the swash on a artificial channel. The broken lines denote onshore velocity.

5. ANALYSIS OF THE DATA

First, the individual wave analysis was applied to the surface elevation data, partly for the purpose to show how inappropriate to apply the method automatically. The results obtained in Table 1 show peculiar behaviors of the wave transformation in the surf zone. As reported by Mizuguchi (1982a) the individual waves propagate independently in its literal sense in a normal circumstance, showing almost no change of the wave period distribution. However this results shows a considerable change of the wave period in the surf zone, being affected by the standing waves of significant magnitude. The standing waves have a node and antinode structure, which results in the dominance of shorter period waves at the node and longer period waves at the antinode. Table 1 shows that the significant node lay at the pole (16) where minima of both wave period and height were observed. Figure 4 also shows only
Table 1 Wave statistics of the surface elevations

<table>
<thead>
<tr>
<th>Location</th>
<th>Mean water level*</th>
<th>Water depth (cm)</th>
<th>( \eta_{rms} ) (cm)</th>
<th>( H_{1/3} )**</th>
<th>( T_{1/3} ) (s)</th>
<th>( H_{rms} ) (cm)</th>
<th>( \bar{T} ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swash (Natural)</td>
<td>190.9</td>
<td>-</td>
<td>16.9</td>
<td>57.1</td>
<td>10.6</td>
<td>45.1</td>
<td>9.5</td>
</tr>
<tr>
<td>Swash (Channel)</td>
<td>194.7</td>
<td>-</td>
<td>22.7</td>
<td>77.0</td>
<td>10.9</td>
<td>60.9</td>
<td>9.5</td>
</tr>
<tr>
<td>No. 10</td>
<td>181.9</td>
<td>10.4</td>
<td>5.9</td>
<td>24.2</td>
<td>10.3</td>
<td>17.0</td>
<td>7.4</td>
</tr>
<tr>
<td>&quot; 12</td>
<td>174.2</td>
<td>23.7</td>
<td>8.5</td>
<td>32.8</td>
<td>6.0</td>
<td>23.4</td>
<td>4.0</td>
</tr>
<tr>
<td>&quot; 13</td>
<td>174.8</td>
<td>47.9</td>
<td>5.2</td>
<td>32.4</td>
<td>4.9</td>
<td>22.3</td>
<td>3.5</td>
</tr>
<tr>
<td>&quot; 16</td>
<td>173.9</td>
<td>60.6</td>
<td>7.3</td>
<td>28.5</td>
<td>4.4</td>
<td>19.9</td>
<td>3.0</td>
</tr>
<tr>
<td>&quot; 20</td>
<td>176.3</td>
<td>82.0</td>
<td>7.6</td>
<td>30.5</td>
<td>5.5</td>
<td>21.0</td>
<td>3.5</td>
</tr>
<tr>
<td>&quot; 21</td>
<td>-</td>
<td>(83.5)</td>
<td>7.8</td>
<td>31.8</td>
<td>6.0</td>
<td>21.9</td>
<td>3.6</td>
</tr>
<tr>
<td>&quot; 25</td>
<td>176.3</td>
<td>85.5</td>
<td>13.4</td>
<td>59.5</td>
<td>6.9</td>
<td>40.0</td>
<td>4.3</td>
</tr>
<tr>
<td>&quot; 26</td>
<td>176.3</td>
<td>123.0</td>
<td>13.2</td>
<td>50.7</td>
<td>6.4</td>
<td>35.5</td>
<td>3.9</td>
</tr>
</tbody>
</table>

* Reference is made to an arbitrary level as is in Fig. 3.
** Zero-down crossing method is applied with a band width of 1 cm for the mean water level (Mizuguchi, 1982a).

short period fluctuation at this position. This result shows that it is essential to separate the long period fluctuations which are of standing waves, in order to discuss the transformation of the progressive waves in the surf zone. Table 1 also shows that the significant breaking wave height and period were about 60cm and 7s respectively.

Secondly the obtained surface elevations were studied in the frequency domains. Figure 5 shows the two-sided power spectra of \( S(f) \) at the representative locations as well as that of the swash on the natural beach. There are following two points to be noted.

1) There are seen three distinct regions of the high frequency saturation. One is the \( f^{-2} \) law for the high frequency region of the incident wave spectra around the breaker line. This is consistent with the result reported by Thornton (1977), however the physical argument is second and third ones, which are more interesting here, are the \( f^{-4} \) observed in the intermediate frequency range from 0.2Hz to 0.5Hz in the surf zone, and the \( f^{-4} \) for the entire high frequency region of the swash. The simplest argument to give the \( f^{-2} \) law is to assume that the power spectra is determined by the depth-controlled wave breaking where the wave height is almost proportional to the water depth \( d \) as done by Sawaragi & Iwata (1980). This assumption yields the relation \( S_{\eta}(f) \propto d^{-2} f^{-4} \), based on a dimensional analysis. This relation may hold only for the lower frequency than the limit of the long wave assumption. However in this observation the \( f^{-2} \) law is seen to the higher frequency range than this limit. Third one, high frequency saturation of the swash, has been recently investigated as already mentioned. It is useless to repeat the argument, unless one can add the more convincing model to describe the swash motion on a natural beach. Here it is taken for granted that there is a saturation for the swash spectra and the form obeys the \( f^{-4} \) law as obtained.
2) The peak frequency of the swash is 0.1 Hz as shown by the arrow. However the power corresponding to the peak almost disappears making a deep trough for the spectrum measured at the pole (13), which is just off the step at the foot of the swash zone. This fact suggests that the powers around the peak frequency are of standing waves.

Now it will be shown how well the linear long standing wave theory can describe the observed fluid motion including the swash. In the following three figures the results denoted by the solid lines for the perfect reflection model were calculated by using Eqs.(1) and (2) for the approximated multi-linear profile shown in Fig. 3. The dotted lines are those obtained by a partial reflection model which will be explained in the next section. Figure 6 shows the cross spectra between the surface elevations at the two different locations, (13) and (16), both of which were in the wave reforming zone. The agreement between
Fig. 6 Cross spectra between surface elevations at poles (13) and (16). The solid line denotes results for perfect reflection and the dotted lines those for a partial reflection model. Coherence is given by the ratio of transfer function to the root of power ratio.

Fig. 7 Cross spectra between surface elevation and onshore velocity at pole (21).
the theory and the measurement is remarkably good for the lower frequency region than 0.1Hz, except very near the zero frequency. Figure 7 shows the cross spectra between the surface elevation and the onshore velocity at the point (21). In this figure one can obtain the same conclusion as in Fig. 6. The third figure, Fig. 8 shows the cross spectra between the swash and the surface elevation at the pole (21). The good agreement between the experiment and the perfect reflection model is only found around the peak frequency 0.1Hz, where the significant power of the spectra was observed. The following conclusions can be drawn from these three figures. (1) The long period fluid motion inside the surf zone is well described by the linear long standing wave theory. (2) The measured data in all of the figures shows large deviation from the theory in the higher frequency range beyond 0.1Hz. This corresponds to the inception of wave breaking. (3) In the low frequency range in Fig. 8, the swash motion tends to be about twice larger than that predicted by the theory, although the agreements are reasonably good in other two Figs. 6 and 7. This indicates that there might be taking place some nonlinear processes which work only in the swash zone to produce the longer period motion, transferring the energy from the shorter period one to the longer period one.

In concluding this section, it should be noted the magnitude of the power spectra of the alongshore velocity component was one-twentieth or less than that of the onshore component and can be maintained that there existed no significant edge waves.
Table 2 A model to predict the swash spectra as well as the reflection coefficient

<table>
<thead>
<tr>
<th>Input data: incident wave spectra + bottom profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear long standing wave theory (on a multi-linear profile)</td>
</tr>
<tr>
<td>The expected spectra of swash</td>
</tr>
<tr>
<td>Saturation model (cut-off frequency + high frequency saturation)</td>
</tr>
<tr>
<td>The predicted spectra of swash S(f)</td>
</tr>
<tr>
<td>Linear long standing wave theory</td>
</tr>
<tr>
<td>Standing wave components</td>
</tr>
<tr>
<td>Reflection coefficient</td>
</tr>
</tbody>
</table>

6. PARTIAL REFLECTION MODEL AND THE SWASH SPECTRA

The observed swash motion as well as the fluid motion in the surf zone encourage the model discussed in the section 3. Table 2 illustrates the partial reflection model in terms of the flow of computation. The expected spectrum is defined as that which would be realized if all the incident wave powers were perfectly reflected. The concept of the saturation is introduced to calculate the predicted swash spectrum from the expected one. The assumption that the swash oscillations consist only of standing waves makes them equivalent evaluating the reflection coefficient at the specified location and determining the swash motion at the shoreline.

In order to obtain the predicted spectra, we exploit the following two results. First one is that there is a high frequency saturation. The $f^{-4}$ law is employed here simply as it is the case observed. Second one is that the irregular wave trains may show a similar behavior as that of monochromatic waves in terms of the "surviving" process through the run-up. The method is not yet well established to relate properties of the irregular wave trains with those of the monochromatic waves. Here rms swash amplitude $2a_*$ and mean frequency $f_*$ are defined by the following relations after Longuet-Higgins (1969).

$$2a_* = \sqrt{2 \eta R} = \int_0^\infty S \eta_* (f) df$$

(10)

$$f_* = \sqrt{\frac{\int_0^\infty S \eta_* (f) df \int_0^\infty S \eta_* (f) df}{\int_0^\infty S \eta_* (f) df}}$$

(11)
These relations were applied to the measured swash spectra both on the natural beach and on the artificial channel. The representative swash parameter obtained by using these $2\alpha$ and $f$ are respectively 1.18 and 1.50. These values are reasonably similar to those for the monochromatic waves so that the analogy between irregular waves and regular waves can be considered to work, as far as this experiment is concerned. However it could be wrong to take the zero frequency as the lower limit of the integral in Eqs. (10) and (11). The lower limit should be chosen such that the integral contains only the power of the spectral components affecting the phenomena considered. In this experiment the low frequency energy shown in Fig. 5 is not large and do not contribute to the integral. It is also true that the fact that the run duration of the experiment is the order of ten minutes automatically excludes the lower frequency energy contribution. The difference between the values on the natural beach and on the artificial channel is mainly due to the last moment up-rush which was only observed on the artificial channels as shown in Fig. 4. The reason should be attributed to the energy loss for the sand movement as well as seepage on a natural beach.

\[
\begin{align*}
&\text{(12)} + (\text{On}) \\
&\text{(16)} + \text{(20)} \\
&\text{(21)} + (\text{Off}) \\
&\text{(25)} + \text{(26)} \\
\end{align*}
\]

Fig. 9 Representative swash parameters versus critical frequencies

Fig. 10 Comparison of predicted swash spectra with observed one
Then the critical frequency $f_c$ for the predicted spectra which is defined as the highest frequency for perfect reflection or as the lowest frequency of the saturation range is calculated so that the representative swash parameter is equal to the observed value 1.18. Figure 9 shows the changes of estimated swash parameters $\eta$ versus different critical frequencies. Here four pairs of the data were used. It is necessary to use a pair of data to estimate the expected swash spectrum, as standing waves give nodes for particular combinations of an offshore location and a frequency. A practical method to have an average of the two quantities is needed, in order to avoid infinite transfer function that the nodes give. In Fig. 9 it is seen that the two results calculated by the pairs of the data in the trough region are almost identical, showing that there is no significant wave transformation in that zone. The critical frequency $f_c$ for these data is determined 0.095Hz. The result based on the data at the pole (12) show smaller values and do not reach the measured swash parameter, indicating that the pole was actually located within the swash zone and some part of the energy was already dissipated before reaching the pole (12). The results from the offshoreeward data, on the contrary, show larger value for the given critical frequency compared with the other results. The application of this model yields that the critical frequency $f_c$ should be 0.075Hz.

Figure 10 shows the comparison of the three predicted swash spectra thus-obtained with the observed swash spectrum. The predicted results (denoted by the dotted line or the broken line) based on the data obtained in the trough zone agrees very well with the observed one, except in the frequency domain lower than 0.05Hz. The reason of this difference is already mentioned in the discussion for Fig. 8. The poorer agreement of the chain line may mean that the model shown in Table 2 should be applied only for the data obtained in the location between which and the shoreline there is no wave breaking except that on the foreshore slope.

As shown in Table 2, reflection coefficient at any location can be estimated either if the transformation of progressive wave components could be estimated or when wave spectrum is measured. In Fig. 11 are shown the reflection coefficients calculated by using the observed swash spectra together with the observed local spectra. The results based on the data in the trough region (denoted by open circles or open triangles) are supposed to show the reflection coefficients at the slope of swash zone. The higher reflection coefficients than unity in the low frequency region again correspond to the larger swash oscillation compared with the theory as shown in Fig. 8. The most reliable frequency region around 0.1Hz shows perfect reflection for the lower frequency than the critical frequency 0.095Hz and the decrease of the reflection coefficient following $f^{-2}$ law in the higher frequency region. The $f$ law can be understood as follows. As discussed in the previous section the high frequency saturation due to the depth-controlled wave breaking results in the $f^{-1}$ law. Then the expected swash spectra would be constant in that frequency region, as the linear long standing waves show the following behavior as $\alpha x$ is large,

$$\eta = 2\alpha F_0 (\sqrt{\alpha x}) \sim f^{-1/2} \quad (12)$$

Taking this into consideration, the $f^{-4}$ saturation law of the swash
spectra simply indicate the $f^{-2}$ dependence of the reflection coefficient at the slope. The reflection coefficients for the combination of the poles (25) and (26) cannot be free from the effect of wave breaking on the shoreward slope of the bar and are considerably smaller than those for the swash slope only. In Fig. 11 is also plotted the reflection coefficient calculated from the phase lag between the water surface elevation and the onshore velocity at the pole (21). The result shows a quantitative agreement with others. The surveying of the position of EMCM was not so accurate that one should expect some errors for the reflection coefficients especially for high frequency region.

Returning back to Figs. 6-8, one can see the agreement improved between the results of this partial reflection model and the observed results. Especially in Fig. 7, where it is assumed that the progressive components of the incident wave do not change during the travel over the distance, the agreement is excellent. The partial reflection model gives slightly higher values than the observed ones in the high frequency region in Fig. 9. This is not significant and not worth of further consideration at present. However in Fig. 8 the observed velocity is considerably smaller in the higher frequency region than that calculated by the linear long wave theory. This corresponds to the fact reported by Guza & Thornton (1980) and Mizuguchi et al. (1980), that the linear (long) wave theory gives larger values of transfer function for the progressive waves from the water surface elevation to the onshore velocity in the nearshore zone. Nonlinearity can partly explain the difference, as the air entrained may account for the rest.

Finally Fig. 12 shows the result of the comparison of the proposed model to the data around the breaker line. The agreement is still good particularly around the peak frequency region. In this sense, it should be pointed out that the measured swash oscillation makes it possible to rather easily evaluate the standing wave components by applying the two-
Fig. 12 Cross spectra between surface elevations around breaker line.

dimensional liner long standing wave theory for the multi-linearly approximated bottom topography.

7 CONCLUSION

A model to describe the swash motion on a natural beach is developed based on three hypotheses. First one is that the swash oscillations are considered to be antinode motion of the linear long standing waves. Second one is that the swash spectra exhibit the high frequency saturation for which the observed $f^{-1}$ law is assumed. Third one is that a concept of the saturation developed for the regular waves swash is also applicable to the irregular wave trains, as an appropriate method to make them related being employed. A field observation was conducted on a natural beach with a rather steep foreshore slope. The results, in a general sense, confirm the applicability of the model. With regard to the three hypotheses, first one is strongly supported by the observation. The validity of the concept of the saturation for the irregular waves is not so clear and needs further study, although the model works very well for this experiment. The minor discrepancy found in the very low frequency region suggests that there might be the cases where one should take into account nonlinearity to achieve a better agreements. It is plausible that the gentle slope of the swash zone provides such cases. Another limitation of this model is that the effect of wave breaking can not be included, which takes place in other area than the swash zone.

REFERENCES
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