CHAPTER THIRTY SIX

STATISTICAL PROPERTIES OF SHORT-TERM OVERTOPPING

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1. INTRODUCTION

It has been recognized recently that large waves tend to form a group in random sea waves. Overtopping tends to occur particularly when a group of high waves attacks a sea wall. If the capacity of a storage reservoir inside the sea wall is not sufficiently large enough to store a total amount of overtopping brought about by a single group of consecutive high waves, and if a drainage facility is not large enough to pump out sufficient water from the storage reservoir before the next overtopping starts, there is a danger of flooding inside the sea wall. Hence, storage and drainage facilities should be planned to be able to cope with the total amount of overtopping produced by a single group of high waves which overtop the sea wall consecutively. The term "short-term overtopping" referred in this study is that caused by a single group of high waves (see Fig.1). This study aims to clarify the following points:

(1) the statistical properties of the amount of short-term overtopping,
(2) the method to evaluate a security factor inside a sea wall against flooding by overtopping and an extension of the theory to the short-term overtopping from a comparatively long sea wall.

2. PROBABILITY DISTRIBUTION OF SHORT-TERM OVERTOPPING

Short-term overtopping from a vertical (steep) sea wall located off a breaking zone is investigated in this paper. Following three assumptions are made to introduce statistical properties of short-term overtopping amount from a sea wall:

(1) characteristics of an overtopping of zero-up-crossing wave from a sea wall can be approximated by a existing theory for periodic wave,

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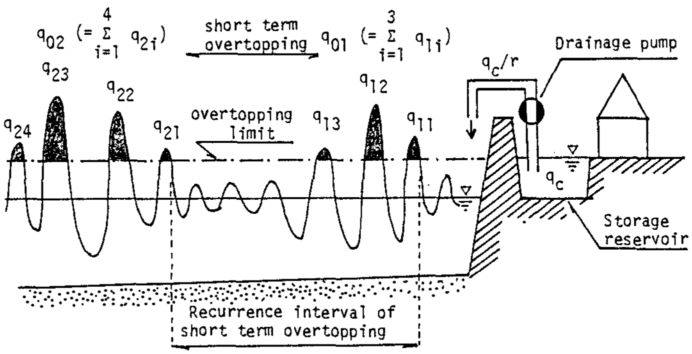

Fig.1 Explanation of the short-term overtopping and schematic illustration of the sea wall, storage reservoir and drainage pump

(2) characteristic of an overtopping is not affected by neighboring waves but can be evaluated only by properties of an individual wave,
(3) statistical distribution of wave height can be approximated as the Rayleigh distribution.

Beside above assumptions, wave period of overtopping waves are assumed to be constant, since high waves in random sea tend to have a constant wave period as recent studies on the joint probability distribution of wave height and period have pointed out[4),7)].
The overtopping equation by Kikkawa et al.[5)] was applied in this study. They gave a simple overtopping equation in terms of a wave height, period and properties of a sea wall as follows.

$$q'/TH\sqrt{2gH} = 2/15 \cdot m_0 k^{3/2}(1-Z/kH)^{5/2} \tag{1}$$

in which q': overtopping amount of a wave from a unit length of a sea wall, H: incident wave height just outside a sea wall, T: wave period, Z: sea wall height, m_0,k: constants which characterize a shape and location of a sea wall (for a vertical sea wall located off a breaking zone, the value of m_0=0.5, k=0.6 are recommended by Kikkawa et al.[5)], g: gravitational acceleration. From eq.(1) an overtopping amount of the mean wave ($\bar{H}$,$\bar{T}$) in case of Z=0 is given by

$$\bar{q} = \sqrt{8}/15 \cdot m_0 k^{3/2} g^{1/2} \bar{T}\bar{H}^{3/2} \tag{2}$$

q' in eq.(1) is normalized with $\bar{q}$ to yield

$$q = q'/\bar{q} = th^{3/2}(1-\frac{z}{h})^{5/2} \tag{3}$$

in which $t=T/\bar{T}$, $h=H/\bar{H}$ and $z=Z/k\bar{H}$.
Analyzing the results presented by Goda[4], it appears that wave periods of high waves tend to distribute around 1.1 times of the mean wave period. t is put equal to 1.1 in this study. Thus a normalized overtopping amount is given only in terms of h when the properties of the sea wall are given. An amount of a short-term overtopping brought about by n high waves $(h_1, h_2, \ldots, h_n)$ which overtop the sea wall consecutively is determined as

$$q_0 = \sum_{i=1}^{n} q(h_i) \tag{4}$$

in which $q(h_i)$ $(i=1,2,\ldots,n)$ is given by eq.(3).
If the above-mentioned n wave heights are classified into following ranks

$$\begin{array}{llr} \text{rank} & 1 : & 0 < h_i \le z \\ \text{"} & 2 : & z < h_i \le z+\Delta h \\ \cdots & \cdots & \cdots \\ \text{"} & j : & z+(j-2)\Delta h < h_i \le z+(j-1)\Delta h \\ \cdots & \cdots & \cdots \end{array} \tag{5}$$

and if these n waves belong to the ranks $j_1, j_2, \ldots, j_n$ respectively and since the time series of zero-up-crossing wave height has close properties to those of the Markov chain[6], the probability that they appear in this order is given as

$$p(j1,j2,\ldots,jn) = p_{j1}p_{j1j2}\cdots p_{j(n-1)jn} \tag{6}$$

in which p_{ji}: probability that wave h_i belongs to the rank ji, $p_{jij(i+1)}$ $(i=1,2,\ldots.)$: probability that consecutive two waves belong to the rank ji and j(i+1) in this order respectively and so on. These probabilities are given as follows in this study[6].

$$p_{ji} = \int_{z+(ji-2)\Delta h}^{z+(ji-1)\Delta h} P(h)dh \tag{7}$$

and

$$p_{jij(i+1)} = \frac{\int_{z+(ji-2)\Delta h}^{z+(ji-1)\Delta h} \int_{z+[j(i+1)-2]\Delta h}^{z+[j(i+1)-1]\Delta h} P(h_1,h_2)dh_1dh_2}{\int_{z+(ji-2)\Delta h}^{z+(ji-1)\Delta h} P(h)dh} \tag{8}$$

from the third assumption, $P(h)$ and $P(h_1,h_2)$ are given as[6)]

$$P(h) = \pi h/2\cdot\exp(-\pi h^2/4) \tag{9}$$

$$P(h_1,h_2) = h_1h_2\cdot I_0[h_1h_2\rho/A]\cdot\exp[-(h_1^2+h_2^2)/\pi A]/A \tag{10}$$

and

$$A = 4/\pi^2 - \rho^2$$

in which I_0: modified Bessel function of order 0, ρ : the correlation parameter which has the following relation with the correlation coefficient of the consecutive two wave heights[1),6)].

$$\gamma_h = \{E(\pi\rho/2) - (1/2)(1-\pi^2\rho^2/4)K(\pi\rho/2) - \pi/4\}/(1-\pi/4) \tag{11}$$

in which γ_h: correlation coefficient of consecutive wave heights, K and E: complete elliptic integrals of the 1st and 2nd kinds. When Δh in eq.(5) is sufficiently small, eqs.(7) and (8) can be replaced by eqs.(7)' and (8)'[6)].

$$p_{ji} = \pi h_i/2\cdot\exp(-\pi h_i^2/4)dh \tag{7'}$$

and

$$p_{jij(i+1)} = 2h_{i+1}/\pi A\cdot I_0(h_ih_{i+1}\rho/A) \cdot\exp[-(h_i^2 + h_{i+1}^2)/\pi A + \pi h_i^2/4]dh \tag{8'}$$

The probability that a short-term overtopping amount becomes q_0, when the above mentioned n' waves overtop consecutively in this order from a unit length of the sea wall, is given as

$$P_1(q_0)dq = \int\int\cdots\int_D p_{j1}p_{j1j2}\cdots p_{j(n-1)jn}dh_1dh_2\ldots dh_n \tag{12}$$

where D is the region which is determined as

$$D \;:\; q_0 < \sum_{i=1}^{n} q(h_i) \leq q_0+dq \tag{13}$$

D is schematically shown in Fig.2 when n=2 for example.

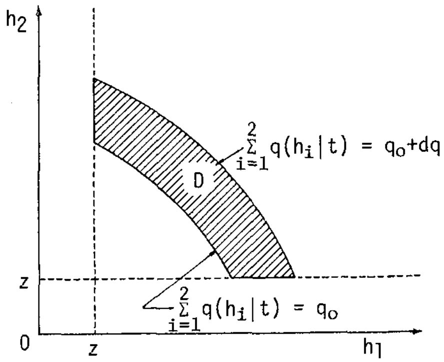

Fig.2 Region D (hatched area)

Region D (hatched area) the boundaries of which are given by the equations

$$\sum_{i=1}^{2} q(h_i|t) = q_0$$

$$\sum_{i=1}^{2} q(h_i|t) = q_0 + dq$$

$$h_1 > z \qquad h_2 > z$$

in which z: normalized sea wall height (eq.3). No overtopping takes place when an incident wave height is not larger than z. Since the region D and the probabilities p_{ji}, and $p_{jij(i+1)}$ in eq.(12) have complex forms, this equation is integrated numerically in this study. In such a case, the more number of waves in a high wave group increases the longer becomes the computing time. But if a sea wall height is not so low, the expected run of waves in the longest high wave group which may appear during a single storm period is not so large. From the theory of a run of waves[6], the probability distribution of a run of high waves is given as

$$P_2(\ell) = p_{22}^{\ell-1}(1-p_{22}) \tag{14}$$

where p_{22}: probability that consecutive two waves exceed the threshold wave height h_*, which is given by

$$p_{22} = \frac{\int_{h_*}^{\infty}\int_{h_*}^{\infty} P(h_1,h_2)dh_1dh_2}{\int_{h_*}^{\infty} P(h)dh} \tag{15}$$

$P(h)$ and $P(h_1,h_2)$ are given by eqs.(9) and (10) respectively. Therefor, the probability that a run of high waves does not exceed ℓ_*-1 is

$$P_3(\ell_*-1) = \sum_{\ell=1}^{\ell_*-1} p_{22}^{\ell-1}(1-p_{22}) = (1-p_{22}^{\ell_*-1}) \tag{16}$$

Among N sets of independent high wave runs, the probability that no run exceeds ℓ_*-1 is given by $(1-p_{22}{}^{\ell_*-1})^N$. Therefor the probability that at least one run exceeds ℓ_*-1 is given by $1-(1-p_{22}{}^{\ell_*-1})^N$. In the same manner, the probability that at least one run exceeds ℓ_* is $1-(1-p_{22}{}^{\ell_*})^N$. Probability distribution of the maximum run among N sets of high wave runs becomes ℓ_* is given as[8]

$$\begin{aligned} P_4(\ell_*) &= (1-p_{22}^{\ell_*})^N - (1-p_{22}^{\ell_*-1})^N \\ &= \exp[N\cdot\ell n(1-p_{22}^{\ell_*})] - \exp[N\cdot\ell n(1-p_{22}^{\ell_*-1})] \end{aligned} \tag{17}$$

in which ℓn : natural logarithms.
Its expectation is[8]

$$E(\ell_*)_{max} = \sum_{\ell_*=1}^{\infty} \ell_*(1-p_{22}^{\ell_*})^N - \ell_*(1-p_{22}^{\ell_*-1})$$

$$= \sum_{n=1}^{N} \frac{(-1)^{n+1}\ {}_NC_n}{1-p_{22}^n} \tag{18}$$

$$\simeq -[\ell n(N) + 0.5772] / \ell n(p_{22}) + (2p_{22}+1)/(3+3p_{22})$$

Fig.3 shows probability distribution of the maximum run among 1000 sets of high wave runs ,for example, for several threshold wave heights. (the value of ρ in eqs.(10) and (8)' is about 0.25 in case of the Pierson-Moskowitz type random waves). Solid lines in Fig.4 show the relation between $E(\ell_*)_{max}$ and N when the same value of ρ and z as Fig.3 are used. From this figure, the expected maximum run during a single storm is evaluated as follows.
The mean interval of a run of high waves (mean total run) is given as[6]

$$\bar{\ell}_0 = \frac{1}{1-p_{11}} + \frac{1}{1-p_{22}} \tag{19}$$

where

$$p_{11} = \frac{\int_0^{h_*}\int_0^{h_*} P(h_1,h_2)dh_1dh_2}{\int_0^{h_*} P(h)dh} \tag{20}$$

p_{22} is given by eq.(15).
The total number of waves which arrive during a single storm of duration I_e is $N=I_e/(\bar{\ell}_0 T)$ ($\bar{\ell}_0$: mean total run). Substituting this value into eq.(18), the expected value of the longest run can be evaluated. For example, in case of $h_* = h_{1/10} (=H_{1/10}/H=1.80)$, T=10s and I_e=24 hours in the Pierson-Moskowitz type random waves, N is about 860. $E(\ell_*)_{max}$ can be calculated by eq.(18) or read off from Fig.4 as about 4.3. In case of the non-dimensional sea wall height z=5, $E(\ell_*)_{max}$ is about 2.5. For the practical use these values should be raised to the next whole number. n in eq.(12) for above examples are 5 and 3.
Fig.5 shows an example of a cumulative distribution of the short-term overtopping amount q_0 which is given by

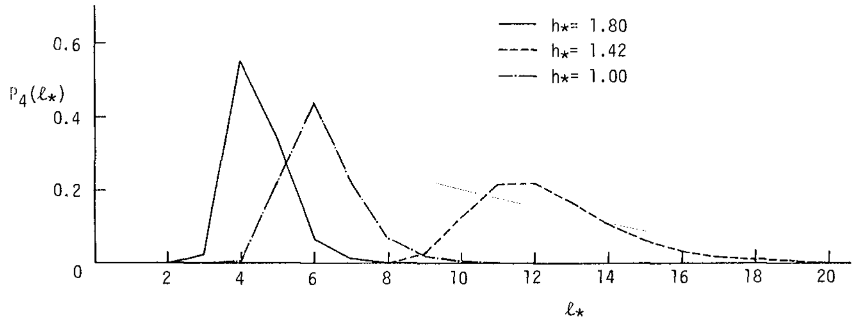

Fig.3 Probability distribution of the maximum run (N=1000)

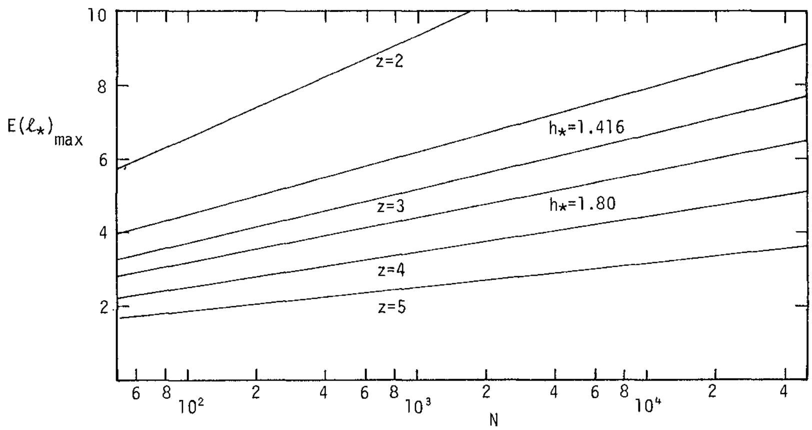

Fig.4 Relation of $E(\ell_*)_{max}$ and N

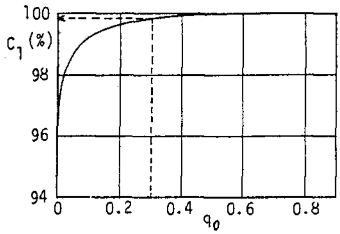

Fig.5 Cumulative distribution of the short-term overtopping amount q_0 (z=3, n=6)

$$C_1 = \int_0^{q_0} P_1(q)\,dq \tag{21}$$

when z=3.0, n=6 are used for the Pierson-Moskowitz type random waves.

The security factor C_1 against a temporal flooding by a single short-term overtopping inside the sea wall can be read off from this Fig.5 in terms of a capacity of the storage reservoir q_c. For example, when a normalized capacity of the reservoir $(q_c/\overline{q})$ is 0.3, C_1 is about 0.998 in this case (dotted line in Fig.5).

3. SECURITY FACTOR AGAINST FLOODING

The total run of high waves is determined by a sum of a pair of one high wave group and the next one low wave group. Therefor if an amount of a short-term overtopping q_0 brought about by a single high wave group is pumped out until the next overtopping starts (within a total run), no flooding inside the sea wall takes place. When a drainage pump the capacity of which per 1 wave period $(1.1\overline{T})$ equals q_c/r, is facilitated inside the sea wall (it takes r times of the wave period to pump out water of volume q_c (Fig.1) from the reservoir) and if the next total run is longer than r+1, no overtopping takes place.
Since the probability distribution of the total run is given as[6])

$$P_5(\ell_0) = \frac{(1-p_{11})(1-p_{22})}{p_{11}-p_{22}} \left(p_{11}^{\ell_0-1} - p_{22}^{\ell_0-1}\right) \tag{22}$$

The probability that a total run exceeds r ($\ell_0 \geq r+1$) is given as

$$C_2(r) = \sum_{\ell_0=r+1}^{\infty} P_5(\ell_0)$$

$$= \frac{(p_{11}-1)p_{22}^r - (p_{22}-1)p_{11}^r}{p_{11} - p_{22}} \tag{23}$$

Since the expected maximum run of high wave of which length equals n is being discussed now, the total run is always longer than n . Therefor

$$C_2(r) = 1 \qquad (r < n) \tag{24}$$

Finally, the security factor during a single storm inside the sea wall is given by $C_1(q_c)C_2(r)$ when the given capacities of the storage reservoir and drainage pump per 1 wave period are q_c and q_c/r, respectively. Fig.6 shows examples of the security factor C_1C_2 when the given q_c can cope with 99% of short-term overtoppings (C_1=0.99) among entire short-term overtoppings brought about by high wave groups of length 6 (Fig.5). The parameter in the figure is the normalized sea wall height z.

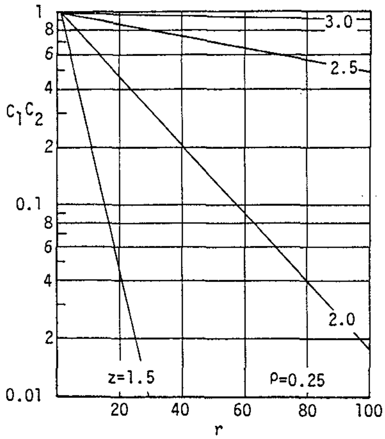

Fig.6 Security factor in the case drainage pump is facilitated (C_1=0.99)

4. SPATIAL DISTRIBUTION OF WAVE HEIGHTS

Short-crestedness of random waves has to be taken into account to cope with the short-term overtopping from a comparatively long sea wall. In this study, such a long sea wall is divided into sections in which the water surface profile along the sea wall can be assumed to be uniform within the individual sections but independent of those of other sections. Overall security inside the sea wall is derived from the synthesis of the securities of all sections. In this respect, a simultaneous spatial correlation coefficient of wave profile along the sea wall may be a good property to determine the above mentioned range along the sea wall.

Short crested random wave profile is usually expressed as[3).

$$\eta(x,y,t) = \sum_{i=1}^{m_f} \sum_{j=1}^{m_\theta} \sqrt{2S(f_i)G(f_i,\theta_j)\Delta f \Delta\theta} \cdot \cos(k_i \cos\theta_j x + k_i \sin\theta_j y - 2\pi f_i t + \varepsilon_{ij}) \tag{25}$$

in which m_f, m_θ : numbers of partitions of the energy spectrum $S(f)$ and directional function $G(f,\theta)$, Δf, $\Delta\theta$: interval of $S(f)$ and $G(f,\theta)$, k_i and θ_j: wave number and direction of propagation, ε_{ij}: initial phase. Directional function used is of Mitsuyasu type[9]. S_{max}=50 at non-dimensional water depth $d/L_{1/3}$=0.1. Fully developed wind wave directional spectra usually take around this value of S_{max} in this water depth range[3]. ($L_{1/3}$: significant wave length), significant wave period is 5s and main direction of wave propagation is normal to the sea wall.

Fig.7 shows an example of a simultaneous spatial correlation coefficient along the infinitely long straight sea wall when a bottom slope is uniform and x-axis is set on the sea wall, y-axis is normal to the sea wall.

$$R(x_0,y_0) = \int_{t=0}^{\infty} \eta(x,y,t)\eta(x+x_0,y+y_0,t)dt \tag{26}$$

This correlation coefficient R was approximated with the following function R' (dotted line in Fig.7) in this study.

$$R'(x\ ) = \begin{cases} 1 & ; \quad L_c \le |x_0| \\ 0 & ; \quad \text{otherwise} \end{cases} \tag{27}$$

where L_c was selected so that the integration of

$R(x_0)-R'(x_0)$ from $x_0=0$ to the point where $R(x_0)$ first takes on 0, becomes 0. The long sea wall is divided into sections, the interval of which is $2L_c$. If there are topographical configurations on the sea bottom, the local change of L_c due to the refraction, diffraction and shoaling[3] should be changed locally. To cope with the spatial changes in wave height and L_c due to sea bottom configuration, probability distribution P_1 (or cumulative distribution C_1: Fig.5) for individual sections should be transformed so that they are expressed in terms of the real (not normalized) amount of short-term overtopping by multiplying $2L_c\bar{q}$ to the horizontal axis at individual sections.

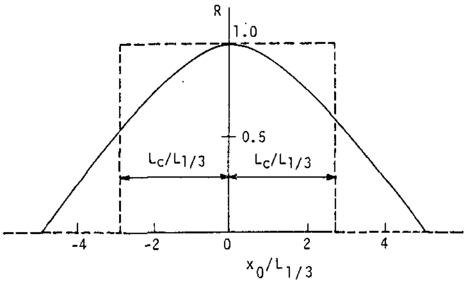

Fig.7 Spatial correlation coefficient of wave profile along the sea wall (S_{max}=50, $d/L_{1/3}$=0.1, $T_{1/3}$=5s)

5. SECURITY FACTOR INSIDE THE COMPARATIVELY LONG SEA WALL AGAINST FLOODING

In case the sea wall is divided into M independent sections and simultaneous amount of short-term overtoppings brought about by a single group of n consecutive waves from individual sections are $q_1, q_2, \ldots, q_M$, respectively, the probability that the overall amount of a short-term overtopping becomes Q is

$$P_6(Q) = \iint\cdots\int_S P_{11}(q_{*1})P_{12}(q_{*2}) \cdots \cdot P_{1M}(q_{*M})dq_{*1}dq_{*2} \cdots dq_{*M} \quad (28)$$

in which $P_{1i}(q_{*i})$ (i=],2, ... ,M): probability that an amount of the short-term overtopping at the section i becomes q_{*i}, S: the region where

$$S \ : \quad Q < \sum_{i=1}^{M} q_{*i} \leq Q+dQ$$

Since P_{1i} are introduced numerically in this study, eq.(28) is rewritten as

$$P_6(Q) = \sum_{i_1=0}^{i_Q} \sum_{i_2=0}^{i_{Q_1}} \sum_{i_{M-1}=0}^{i_{Q_{M-2}}} P_{11}(i_1\Delta q) \cdots \cdot P_{1(M-1)}(i_{M-1}\Delta q) P_{1M}(i_{Q_{M-1}}\Delta q) \tag{29}$$

in which

$$i_m\Delta q = q_m \ , \quad i_Q\Delta q = Q \ , \quad i_{Q_m}\Delta q = i_Q\Delta q - \sum_{j=1}^{m} i_j\Delta q$$

$$(m=1,2, \ \ldots \ ,M-1)$$

When an amount of a single overall short-term overtopping Q is less than the capacity of the storage reservoir Q_0, no flooding inside the long sea wall takes place. This probability is given by

$$P_7(Q_0) = \text{Prob.}[Q \leq Q_0] = \sum_{i_1=0}^{i_{Q_0}} \sum_{i_2=0}^{i_{Q_1}} \cdots \sum_{i_M=0}^{i_{Q_{M-1}}} P_{11}(i_1\Delta q) \cdots P_{1M}(i_M\Delta q) \tag{30}$$

in which

$$i_{Q_0}\Delta q = Q_0$$

When the drainage pump the capacity of which per 1 wave period is Q_u (it takes r_u times of one wave period to pump out water of the volume of Q_0 from the reservoir : $r_u=Q_0/Q_u$) is facilitated, no overtopping takes place as far as the simultaneous total runs at all sections are longer than r_u+1. Since the probability that a total run exceeds r is given by eq.(23), the simultaneous probability that total runs at all sections exceed r_u is given as

$$P_8(r_u) = \prod_{i=1}^{M} C_{2i}(r_u) \tag{31}$$

Suffix i referes to the properties of the section i. Totally, the security factor against flooding inside the sea wall becomes

$$P_7(Q_0)P_8(r_u) \tag{32}$$

Supplemental security can be incorporated into eq.(32) even in the case a certain amount of water is left unpumped from the reservoir. Because if the total amount of unpumped water from the reservoir and that brought about by the next short-term overtopping do not exceed Q_0, no flooding takes place. When the total run of the first high wave group becomes longer than r_u-j at every section, an amount of jQ_u is left unpumped from the reservoir at most. And in case the next short-term overtopping is Q', to pump out the total amount of $Q'+jQ_u$ within the second total run, the second total run should exceed $j+\ell'+1$ ($\ell'=Q'/Q_u$). Therefor the security against flooding in this case becomes

$$P_7(Q_0-jQ_u)P_8(r_u+j) \tag{33}$$

The maximum possible amount carried over to the second total run is $Q_0-(n+1)Q_u$ because the minimum total run is n+1 in the present discussion. The security against flooding when a single carry-over of water to the next total run is permitted, is given as

$$\sum_{j=0}^{n+1} P_7(Q_0-jQ_u)P_8(r_u+j) \tag{34}$$

6. DISCUSSION

The overtopping equation by Kikkawa et al[5]. was used in this study. But to the extent which the three assumptions made in section 2 holds, other overtopping equations which are suitable for various situations considered may be utilized.

The method to divide the long sea wall into independent sections used in this study is found not always appropriate one. Therefor a more effective method needs to be introduced[2].

Some supplemental security factor was discussed in the last part of the section 5. Further supplements are possible, however, in the same way by allowing carry overs of unpumped amounts from the reservoir to occur more than twice. Needed supplements should be determined in accordance with the degree of accuracy of the assumptions made for various situations considered.

REFERENCES

1) Battjes, J.A. (1971): Run-up distributions of waves breaking on slopes, Proc. ASCE, 97, WW-1, pp.91-114.
2) Battjes, J.A. (1982): Effects of short-crestedness on wave loads on long structures, Applied Ocean Res., Vol.4, No.3, pp.165-172.
3) Goda, Y. and Y. Suzuki (1975): Computation of refraction and diffraction of sea waves with Mitsuyasu's directional spectrum, Tech. Note of Port and Harbour Research Institute, No.230, pp.1-45. (in Japanese)
4) Goda, Y. (1978): The observed joint distribution of periods and heights of sea waves, Proc. 16th Conf. on Coastal Engg., pp.227-246.
5) Kikkawa,H, H. Shiigai and T. Kohno (1967): Fundamental study on wave overtopping from sea wall, Proc. 14th Japanese Conf. on Coastal Engg., pp.118-122. (in Japanese)
6) Kimura, A. (1980): Statistical properties of random wave groups, Proc. 17th Conf. on Coastl Engg., pp.2955-2973.
7) Kimura, A. (1981): Joint distribution of the wave heights and periods of random sea waves, Coastal Engg. in Japan, pp.77-92.
8) Kimura, A. (1984): Statistical properties of the maximum random sea wave groups.
9) Mitsuyasu, H. and S. Mizuno (1976): Directional spectra of ocean surface waves, Proc. 15th Conf. on Coastal Engg., pp.329-348.