

CHAPTER TWENTY SIX

Shallow Water Waves: A Spectral Approach

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Introduction

Bouws et al. (1983, 1984) have shown that wind sea spectra in finite depth water can be described by a self-similar spectral equation that in the deep water limit is the JONSWAP spectrum (Hasselmann et al. 1973). This paper shows that the spectral parameter α is linked to wave steepness, for wind sea and swell; presents a simple model for wave transformation across the surf zone; and shows that the spectral theory provides data similar to the results of Bretschneider (1958) for shallow water wave growth.

TMA Spectral Spectrum

Examination of the free wave spectrum of wind generated gravity waves in water of finite depth indicates that the shape of the spectrum may be specified by an equation (Bouws et al., 1983) which Bouws et al. termed the TMA-spectrum

$$S(\omega, H) = \alpha g^2 \omega^{-5} \exp[-5/4(\omega/\omega_m)^{-4}] \times \exp[-(\omega - \omega_m)^2 / 2\omega_m^2 \sigma^2] \times \phi(\omega_H) \quad 1$$

with

$$\phi(\omega_H) = \frac{(k^{-3}(\omega, H) \partial k(\omega, H) / \partial \omega)}{(k^{-3}(\omega, \infty) \partial k(\omega, \infty) / \partial \omega)} \quad 2$$

$$\omega_h = 2\pi f(H/g)^{1/2}$$

where the parameters α, γ and σ can be derived by curve fit analysis to a particular spectrum or estimated by prognostic equations. Although the most useful form of the equation is in frequency space, Bouws et al. (1983, 1984) indicate that the spectrum is better expressed in wave number space because depth is included explicitly through wave number. The resulting equation has a simple form in which the spectral density is related to a constant power of wave number, i.e.

$$F(k) = \frac{\alpha}{2} k^{-3} \Psi(\gamma, \omega_m, \omega, \sigma) \quad 3$$

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where Ψ is dimensionless. Equation 1 goes to the JONSWAP equation in deep water.

The prognostic equations for α and γ are derived in terms of the wave number for the peak frequency of the spectrum, gravitational acceleration and wind speed (Table 1). The spectral equation was derived as an extension of the deep water similarity principles used to develop the JONSWAP equation to shallow water using the results of Kitaigorodskii et al. (1975). The results were checked against nearly 3000 wind sea spectra observed off the North Carolina coast and in the North Sea. The bottom slopes ranged from nearly flat to about 1:100, bottom materials ranged from fine to coarse sands, and wind speeds ranged up to 30 meters per second.

TABLE 1

TMA PARAMETRIC RELATIONS

$$\alpha = 0.0078 \kappa^{0.49}$$

$$\gamma = 2.47 \kappa^{0.39}$$

$$\sigma = 0.7 \quad f \leq f_m; \quad \sigma = 0.9 \quad f > f_m$$

where

$$\kappa = \frac{U^2 k_m}{g}$$

U wind speed at 10m elevation, g gravitational acceleration
wave number corresponding to peak frequency

Steepness and Alpha

The general fit of the TMA spectrum to data implies that the Kitaigorodskii et al. (1975) wave number expression

$$F(k) = \alpha k^{-3/2} \quad 4$$

is a reasonable scaling of the spectrum up to the peak wave number (recognizing some modification due to the shape factors near the peak) if α is allowed to vary. An approximation of the total energy in the spectrum may be obtained by integrating equation 4 from k_m to

$$E_t = \int_{k_m}^{\infty} F(k) = 1/4 \alpha k_m^{-2} \quad 5$$

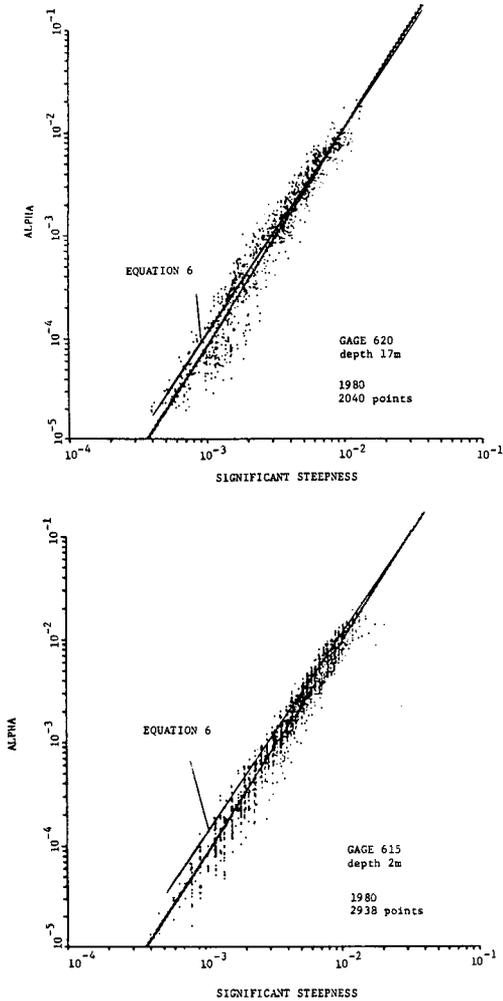


Figure 1. Alpha versus Significant Steepness. Theoretical equation (6) is compared to data taken at two average depths. Unlabelled line is the regression line for the data in each graph.

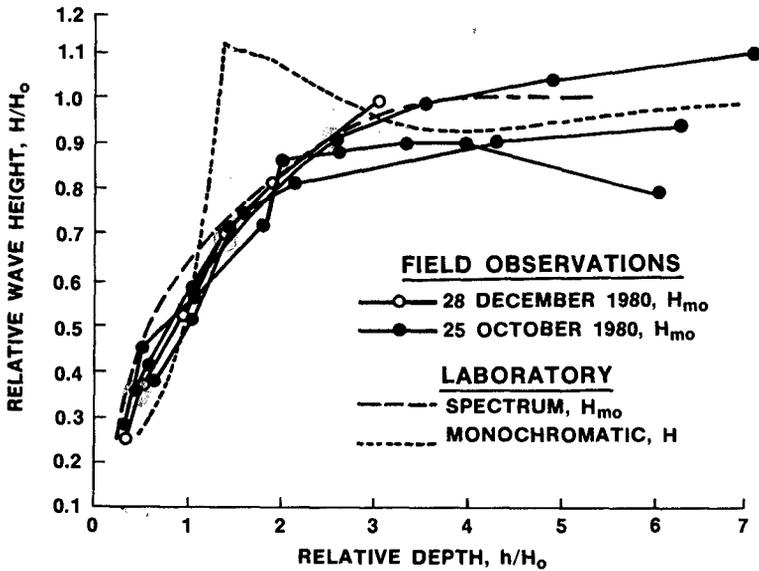


Figure 2. Depth Variation of Zero Moment Wave Height. The zero moment estimate of significant wave height H_{mo} is normalized by the offshore value (ratio denoted as H/H_0) and plotted as a function of depth normalized by the offshore wave height (denoted as h/H_0). Two field data sets and one laboratory data set are plotted. The equivalent variables are plotted for a monochromatic wave from laboratory tests with similar initial steepness.

By inverting equation 5 a relationship between α and a steepness parameter ϵ may be obtained

$$\alpha = 4E_t k_m^2 = 16\pi^2 \epsilon^2 \quad 6$$

where

$$\epsilon = E_t^{1/2} / L_m \quad 7$$

with

$$L_m = 2\pi / k_m \quad 8$$

Huang et al. (1980) had arrived at the same relationship for deep water. Equation 6 is valid for any depth where equation 4 holds. Figure 1 shows a plot of α versus ϵ from field data taken at the Coastal Engineering Research Center's Field Research Facility at Duck, North Carolina, for two average depths: 17m and 2m. Alpha was estimated by fitting equation 6 to both sea and swell spectra. The results indicate an excellent fit at high steepnesses. For low steepnesses there is some divergence. This results from a poor knowledge of depth over time in the surf zone and the irregularity of spectral shape for very low wave height conditions.

Energy Levels in the Surf Zone

Equation 5 was used by Vincent (1983) to derive an upper bound on energy in the surf zone by noting that the zero movement estimate of significant wave height is given by

$$H_{m0} = 4E_t^{1/2} = 2\alpha^{1/2} k_m^{-1} \quad 9$$

For depth limited conditions it is assumed that a linear shallow water dispersion relation

$$k_m = \omega_m (gh)^{-1/2} \quad 10$$

held and equation 9 becomes

$$H_{m0} = \frac{1}{\pi} (\alpha gh)^{1/2} f_m^{-1} \quad 11$$

If α and f_m were constant across the surf zone (which is not generally the case) equation 11 would imply a square root of depth dependence for H_{m0} . Because of some variation of α and f_m , some difference from square root dependence occurs. Figure 2 provides a comparison of field and laboratory data for high energy conditions compared to a monochromatic wave. The

SURF ZONE ALPHA

$$\alpha = 3.55 h / gT_m^2$$

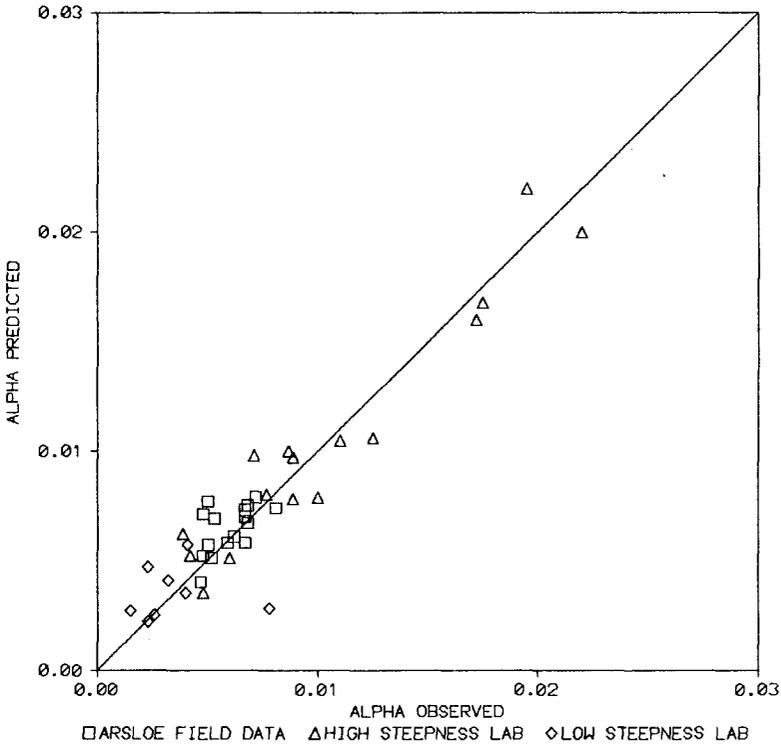


Figure 3. Surf Zone Alpha. Alpha was calculated from a curve fit to the spectra of waves measured in the surf zone in the field and the laboratory and compared to the maximum value of alpha expected from theory.

results show that H_{mo} has a curvilinear form similar to the square root of depth and that its variation with depth is distinctly different than the monochromatic case. In the shallowest water the dependence is not much different from the monochromatic case. Since equation 9 is directly related to equation 6, the data in Figure 1 suggest that the upper bound derived is a good approximation.

Surf Zone ALPHA

The relationship between α and κ given in Table 1 is valid where there is a relative balance between energy gain and loss in the spectrum during wave growth. As the wave field propagates into the surf zone, breaking dominates and the α, κ relation is no longer valid. Mathematically for fixed f_m as $H \rightarrow 0$ κ becomes infinite as does α . Equation 6 provides a link between spectral shape and steepness. In the surf zone, the wave train is largely nondispersive and the largest waves are limited by depth, so it is possible to estimate α by relating the variance in the wave relative to its height and obtain a rough estimate on the maximum alpha for a given depth and peak frequency:

$$\alpha_{max} = 3.55hf_m^2/g \quad 12$$

Figure 3 provides a comparison of measured and estimated α_{max} from field and laboratory data.

Simple Model for Surf Zone Wave Heights

Equation 11 provides an estimate for the upper bound for a wind sea. In order to model the spectrum across a shoaling and breaking region for an arbitrary wind and depth field requires a complex numerical model. For many purposes an assumption of a uniformly varying slope in the along shore direction is not an unreasonable first approximation. A simple one-dimensional numerical code embodying the TMA relationship and extension has been developed to provide an intermediate step between the simplest equation and a full scale model.

The following are the elements of the model:

- (1) Refraction and shoaling is given by the method of Longuet-Higgins (1957).
- (2) The spectrum at any depth is given by equation 1.
- (3) For wind seas α, γ, σ are given in Table 1.

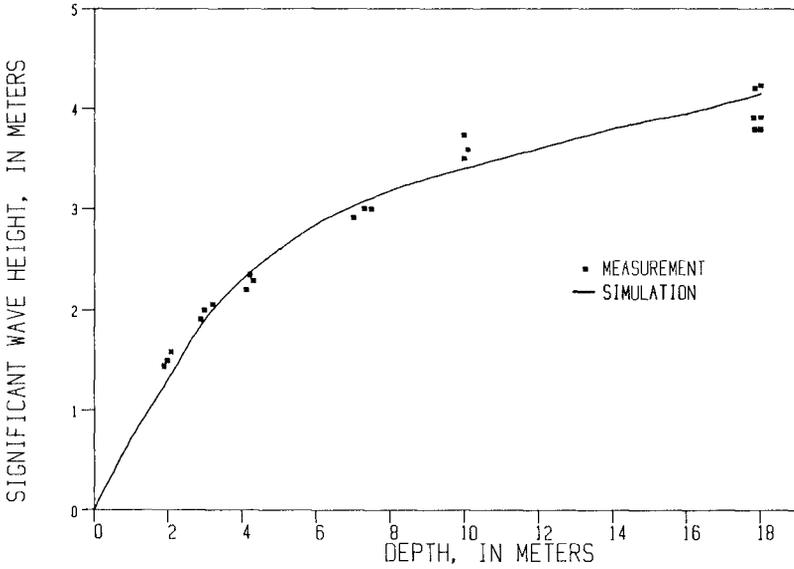


Figure 4. Comparison of Model Predictions to Field Data.

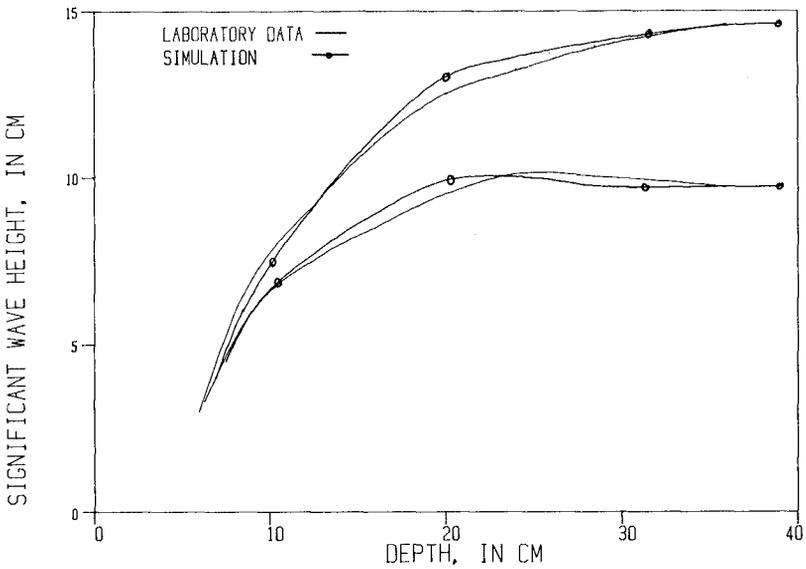


Figure 5. Comparison of Model Predictions to Laboratory Data.

- (4) For swell α is provided by equation 6, with the wave steepness at a new computation estimated by projection of the next upstream wave steepness.
- (5) The maximum value of α in the surf zone is given by equation 12.
- (6) If waves pass over a bar, and the depth then increases α and γ remain fixed at the equilibrium values of the previous shallowest depth until a shallower depth is reached.
- (7) f_m remains fixed.

This model neglects bottom friction and overly simplifies the processes occurring. Its essential assumption is that once the depth becomes a dominant influence on the waves the spectrum rapidly approaches the equilibrium form.

The model allows input of initial TMA parameters and a peak wave direction. The directional spectrum is set to $\cos 4\theta$ spread. Variation of this input may be made by reprogramming.

The model was run for a wind sea case using data from the CERC Field Research Facility Figure 4 and laboratory data from a CERC wave tank Figure 5. In both cases the simple model provided an approximation to energy decay across the shoaling region. Similar results using different field data and a more sophisticated numerical scheme are reported by Hubertz (1984).

Shallow Water Wave Growth Relationships

A discussion of deep water wave growth is required before the TMA spectrum can be related to the Bretschneider wave growth curves. In deep water, wind wave growth appears to stop for wave components whose speed is faster than the wind. In the open ocean for long duration winds the spectrum appears to reach an equilibrium fully developed form (Pierson and Moskowitz, 1964) with a peak frequency f with celerity C satisfying

$$U/C_m = 0.82$$

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The observation that the peak frequency of spectrum occurs at a slightly lower frequency than would be expected from the constraint of zero wind input for $C(f) > U$ has been explained by the nonlinear wave-wave interactions (Masselmann et al., 1976). Although atmospheric input ceases for waves with $U/C < 1$, the wave interactions can still shift wave energy to lower frequencies. If for a given U , f_p is the frequency of waves such that $U=C$,

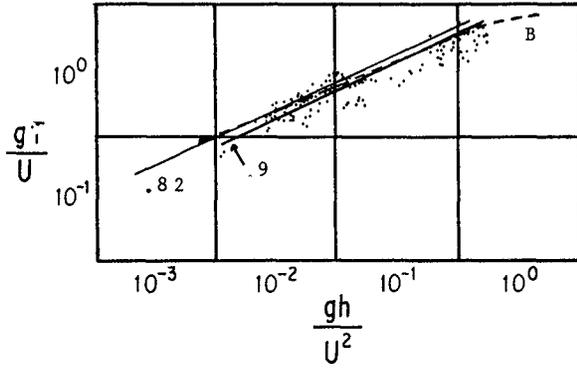


Figure 6. Depth Limited Wave Period. Bretschneider's empirical curve is labelled B; theoretical curves are given for values of A of 0.82 and .9.

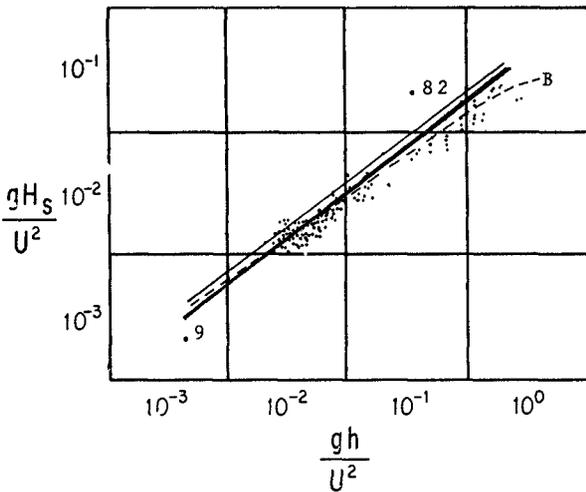


Figure 7. Depth Limited Wave Height. Bretschneider's curve for a friction factor of .01 is labelled B; theoretical curves corresponding to values of a of .082 and .09 for the depth limited period are provided.

once the peak frequency of the wind sea evolves to f_0 , the frequency with maximum input from the wave-wave interactions are 0.8 to 0.9 f_p , or approximately the peak frequency associated with the fully developed spectrum.

In shallow water of depth H the maximum celerity of waves is approximately $(gH)^{1/2}$. If H is sufficiently small, almost any wind speed U will exceed $(gH)^{1/2}$. Hence the mechanism for producing a cut-off to wave growth in deep water does not appear workable in shallow water. Yet the data of Bretschneider (1958) clearly suggest a cut-off value for the wave period. In essence this is a depth limited wave period which would be the shallow water analogy to the fully developed spectrum's peak frequency in deep water.

In the TMA spectrum when ω_H is less than one, the spectrum approaches the f^{-3} saturation dependence suggested by Kitaigorodskii et al. (1975). Thus, a hypothesis for a limit on the peak frequency can be suggested. Assuming a flat bottom, as waves grow and the peak frequency decreases, the waves at the spectral peak ultimately reach a value of $\omega_H < 2$ in which depth begins to influence wave growth. With continued development to lower frequencies ω_H approaches 1 and waves near the peak of the spectrum approach the saturation limit, i.e. cannot grow more without breaking. Thus, allowing for some influence of the wave-wave interactions, the peak frequency should not become less than

$$\omega_{H,m} = 2\pi f_m (H/g)^{1/2} = A \quad 14$$

where A lies between 0.8 and 1.0. Nondimensionalization to include wind speed and algebraic rearrangement provides an equation for a depth limited period

$$T = gT/U = 2\pi(gH/U^2)^{1/2}/A \quad 15$$

with A between 0.8 and 1.0. A plot of equation 15 versus Bretschneider's data (Figure 6) indicates reasonable agreement when the uncertainty in A and the differences in T_m and T_s are considered.

Vincent and Hughes (1985) have taken equations 11 and 15 with the α, κ relationship and calculated an estimate of the H_{mo} for depth limited conditions assuming a shallow water dispersion relation. The results are displayed in Figure 7 for two values of A and are within the general scatter of the data. Pinpointing the correct value of A requires a yet to be performed analysis of the shallow water wave-wave interactions.

Summary

Natural wave trains may be described in several ways: approximation as a monochromatic wave, representation by probability distributions and representation by a spectrum. The TMA spectrum and its extensions discussed here provide a unified wave theory that spans a range of conditions including deep, intermediate and shallow water through a continuous variation of the parameter κ for wind seas. For swell, the parameter ϵ appears equally useful. The theory provides a description of the free wave components of the spectrum even into the surf zone and provides a prediction of energy levels of the same quality as the other methods without a requirement of a site specific dissipation mechanism tuning. Further, the theory provides an explanation of the apparent depth limited wave period which when combined with the spectral shape parameters yields a reasonable approximation to the shallow water wave growth data of Bretschneider (1958).

One useful element of the approach is its direct connection to spectral models for wave forecasts and hindcasts.

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