CHAPTER TWENTY

A SHALLOW WATER DIRECTIONAL WAVE RECORDER

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ABSTRACT

Shallow water wave theory shows that estimates of directional wave climate may be obtained from the measurement of near-bottom pressure and horizontal velocity components. Recent developments in low powered sensors and high density data loggers, incorporated into a bottom-mounted directional wave recorder, make the collection of directional shallow water wave data possible on a routine basis. The instrument is compact, robust, reliable and easily manageable from small craft, providing deployment capacities of up to four months. Analysis methods have been developed which provide for conversion of the recorded data to useful, convenient information.

1. INTRODUCTION

During recent years, recognition of the importance of directional wave information in many facets of coastal and ocean engineering design has been increasing. Goda et al. (1978) have illustrated the importance of wave directionality for studies of wave diffraction and refraction in harbours; Forristall et al. (1978) have demonstrated the inadequacies of uni-directional wave theories in describing wave kinematics; and Sand et al. (1981) have discussed the applications of directional wave information to the hydrodynamics and design of offshore structures and pipelines.

Advances in remote low powered logger and sensor technology have allowed the development of reliable, compact, robust and manageable water current and pressure measuring devices. In May 1980, Sea Data Corporation were commissioned to modify their 635-11 Wave and Tide Recorder (based on bottom-mounted pressure measurements) to include synchronous horizontal velocity measurements within each "burst" of pressure data. The result was the Sea Data 635-12S Directional Wave and Tide Recorder, the use of which is the subject of this paper. Reference is made to the instrument operation and to the underlying theory on which the data analyses are based. Various analysis techniques are illustrated, and verification of instrument and analysis performance is provided.

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2. INSTRUMENTATION

The instrument measures fluctuations in absolute pressure and horizontal current velocity, at both tide and wind wave frequencies. Instrument temperature is also logged. A complete description of the instrument is given by Aubrey and Hill (1984), and a detailed analysis of sensor specifications is given by Grosskopf et al. (1983).

In operation the 635-12S runs two separate sampling schemes in parallel. These are the mean (tide) and burst (wave) recording modes. Data from each mode are recorded in an interleaved format on a magnetic cassette tape.

In the mean mode, water pressure is integrated continuously over a specified sample interval. To conserve battery power, the mean current is computed only over every eighth mean pressure sample interval. The mean sample interval may be selected in the range 3.75 to 60 minutes. An instantaneous temperature measurement from a thermistor within the instrument housing is sampled after every 8 mean pressure samples. Data recording takes place every 8 sample intervals when an elapsed time, 8 tide measurements, a water temperature measurement and the mean current components are recorded.

In burst mode, the instrument rapidly samples the water pressure and the two current velocity components at a preset interval in the range 0.5 to 4 seconds. The number of samples in each burst may be adjusted in steps between 64 and 2048, or alternatively the instrument may be set to run continuously. The interval between bursts may be varied from 0.5 to 24 hours. The burst samples are recorded as triples of the water pressure, easterly and northerly current components.

3. DEPLOYMENT AND RETRIEVAL

The directional wave recorder is usually deployed in an aluminium tripod which is lowered to the sea bed by winch from a work vessel. The tripod is attached to small surface floats by a buoyant line, to facilitate recovery. Thus, deployment and retrieval operations can be carried out without diver assistance.

Within the tripod, the velocity and pressure sensors are located at 1.5 m above the base. Provided the sea bed is relatively smooth and flat (easily checked using an echo sounder), then the sensors should be located above the bottom boundary layer.

An alternative deployment arrangement is provided by fixing the directional wave recorder to an offshore pile. This arrangement allows the sensors to be located near the water surface, but can only be used when the pile wake effects are not of significance. Deployment and retrieval of the instrument using this arrangement does require diver assistance.
4. UNDERLYING THEORY

To obtain a representation of the surface sea state from subsurface measurements of pressure and velocity, an appropriate theory describing the kinematics of flow beneath surface waves must be applied. An assessment of available wave theories is presented in Forristall et al. (1978).

The simplest approach is to use linear wave theory which may be extended through spectral concepts to model irregular (i.e. aperiodic) sea. The most serious deficiency of linear theory occurs due to the non-linearity of the free surface boundary condition.

Higher order regular (periodic) and irregular (aperiodic) theories provide a better match to the free surface boundary condition, but their complexity precludes their incorporation in any practical analysis of directionally spread sea states.

From analysis of measurements conducted during Tropical Storm Delia in the Gulf of Mexico, Forristall et al. (1978) demonstrated that using a linear theory which accounted for the directional nature of the measured sea state provided a better representation of wave kinematics than higher-order, uni-directional wave theories.

Linear Wave Theory: A linear representation of the surface elevation $\eta$, above or below the equilibrium water level, of an arbitrary cosine wave component of a random sea is given by,

$$\eta = a \cos(kx \cos \theta + ky \sin \theta - 2\pi ft + \phi)$$

(1)

This formulation has been set in a right-hand, rectangular coordinate system $(x,y,z)$ with $z$ chosen as positive upward from the sea bed where

- $a$ is the wave amplitude;
- $\theta$ is the direction of phase propagation, measured counter-clockwise from the $x$ axis;
- $\phi$ is the wave phase;
- $t$ is time;
- $k$ is the horizontal wave number;
- $f$ is the wave frequency in cycles per second.

The horizontal wave number $k = \frac{2\pi}{\lambda}$, where $\lambda$ is the wave length.

A freely propagating surface gravity wave described by the above formula should satisfy a dispersion relation given by,

$$(2\pi f)^2 = gk \tanh kd$$

(2)

where $g$ is the acceleration due to gravity, and $d$ is the water depth.
At any level \( z \) in the water column below such a wave, the horizontal velocity components are described by,

\[
u = \alpha z \frac{\cosh kz}{\sinh kd} \cos \theta \cos (kx \cos \theta + ky \sin \theta - 2\pi ft + \phi), \tag{3}
\]

and

\[
v = \alpha z \frac{\cosh kz}{\sinh kd} \sin \theta \cos (kx \cos \theta + ky \sin \theta - 2\pi ft + \phi), \tag{4}
\]

the wave-induced pressure at \( z \) is given by,

\[
p = \rho \frac{\cosh kz}{\cosh kd} \cos (kx \cos \theta + ky \sin \theta - 2\pi ft + \phi), \tag{5}
\]

where \( \rho \) is the water density.

A confused sea state may be represented by a summation of linear waves, given by (1), at various frequencies, propagation directions and phases, and may be described by a "random" sum

\[
\eta(x,y,t) = \sum \sum \alpha_{mn} \cos \psi_{mn}, \tag{6}
\]

where the indices \( m, n \) run over selected frequencies and directions respectively, and the composite phase function is given by

\[
\psi_{mn} = \psi_{mn}(x,y,t) = k x \cos \theta_n + k y \sin \theta_n - 2\pi ft + \phi_{mn}, \tag{7}
\]

In the above, \( \phi_{mn} \) is the random phase for the \((m,n)\) wavelet, assumed to be uniformly distributed over the direction interval \((0, 2\pi)\) and independent from wavelet to wavelet.

According to linear theory, the kinematics of a confused sea state described by (6) may be obtained by superposition from equations (3) to (5) as:

\[
u(x,y,z,t) = \sum \sum \alpha_{mn} 2\pi f \frac{\cosh k z}{\sinh k d} \cos \theta_n \cos \psi_{mn}, \tag{8}
\]

\[
v(x,y,z,t) = \sum \sum \alpha_{mn} 2\pi f \frac{\cosh k z}{\sinh k d} \sin \theta_n \cos \psi_{mn}, \tag{9}
\]

and

\[
p(x,y,z,t) = \sum \sum \alpha_{mn} \rho g \frac{\cosh k z}{\cosh k d} \cos \psi_{mn}, \tag{10}
\]

where \( u(x,y,z,t) \) is the \( x \) component of the water particle velocity at space location \((x,y,z)\) at some time \( t \). Similarly, \( v(x,y,z,t) \) is the \( y \) component velocity and \( p(x,y,z,t) \) is the pressure.
Spectral Representation of Wave Properties: In practice, p, u, v measurements are taken over time at the same horizontal location, i.e. setting (x, y) = (0, 0). Further, (u, v) measurements are made at the same height z = z_v above the sea bed. Pressure measurements may be made at a different height z = z_p. It is important that z_p and z_v are chosen to be above the bottom boundary layer.

We may introduce a frequency domain description of the wave field. The spectral density, $S(f_m, \theta_n)$, of the surface elevation is related to the mean (or expectation) of the square of the random amplitude, $E(\alpha_{mn}^2)$, by

$$S(f_m, \theta_n) = \frac{1}{2\Delta f \Delta \theta} E(\alpha_{mn}^2), \quad (11)$$

where $\Delta f$ and $\Delta \theta$ are the increments of frequency and direction "occupied" by the $(m,n)$ wave component.

By taking lagged products of measured quantities (p, u, v) and passing from summation to integration, cross-covariances of p, u and v may be formed (Borgman, 1979). Fourier transformation of the cross-covariances provides the following cospectral expressions which may be used to estimate the direction surface sea state:

$$C_{uu}(f) = A^2(f) \int_0^{2\pi} S(f, \theta) \cos^2 \theta \, d\theta, \quad (12)$$

$$C_{uv}(f) = A^2(f) \int_0^{2\pi} S(f, \theta) \sin \theta \cos \theta \, d\theta, \quad (13)$$

$$C_{vv}(f) = A^2(f) \int_0^{2\pi} S(f, \theta) \sin^2 \theta \, d\theta, \quad (14)$$

$$C_{pp}(f) = B^2(f) \int_0^{2\pi} S(f, \theta) \, d\theta, \quad (15)$$

$$C_{up}(f) = A(f)B(f) \int_0^{2\pi} S(f, \theta) \cos \theta \, d\theta, \quad (16)$$

$$C_{vp}(f) = A(f)B(f) \int_0^{2\pi} S(f, \theta) \sin \theta \, d\theta, \quad (17)$$

where

$$A(f) = 2\pi f \frac{\cosh k z_p}{\sinh k d}$$

$$B(f) = 2\pi f \frac{\cosh k z_v}{\sinh k d}$$

$$C_{pp}(f) = B^2(f) \int_0^{2\pi} S(f, \theta) \, d\theta.$$

$$C_{up}(f) = A(f)B(f) \int_0^{2\pi} S(f, \theta) \cos \theta \, d\theta,$$
and
\[
B(f) = \frac{\cosh kz}{\cosh kd},
\]
represent velocity and pressure attenuation functions, respectively.

The Directional Spreading Function: Progress in the determination of the spectral density function \( S(f, \theta) \) is made by recasting the function as
\[
S(f, \theta) = E(f) H(f, \theta)
\]
with the constraint
\[
\frac{2\pi}{H(f, \theta)} \int_0^{2\pi} H(f, \theta) \, d\theta = 1
\]
In the above, the directional energy spectrum \( S(f, \theta) \) has been separated into a non-directional energy spectrum \( E(f) \), and a directional spreading function \( H(f, \theta) \). \( E(f) \) represents the surface wave energy summed over all propagation directions, i.e.
\[
E(f) = \frac{2\pi}{H(f, \theta)} \int_0^{2\pi} S(f, \theta) \, d\theta
\]
\( H(f, \theta) \) describes the distribution of the total energy \( E(f) \) over all possible propagation directions \( \theta \). Its estimation is the key to obtaining a directional description of a sea state.

Following Longuet-Higgins et al. (1963), a convenient approximation to the directional spreading function may be obtained by expansion of \( H(f, \theta) \) in a truncated Fourier series, such that
\[
H(f, \theta) = \frac{1}{2\pi} + \sum_{m=1}^{M} (a_m \cos m\theta + b_m \sin m\theta)
\]
The \( p, u, v \) measurements which form the basis of this directional wave analysis contain sufficient information for the estimation of the above Fourier series truncated at \( M = 2 \).

Parametric Description of the Directional Spreading Function:
Further improvement to the description of the directional sea state may be obtained by introducing a subjectively selected, parametric approximation to the directional spreading function, \( H(f, \theta) \). A model which has been widely used is that of Longuet-Higgins et al. (1963),
\[
H(f, \theta) = \frac{\Gamma(s+1)}{2\pi} \frac{\cos^{2s}[(\theta - \theta_0)/2]}{\Gamma(s+1/2)}
= \left( \frac{1}{\pi} \right) \left[ 1/2 + \sum_{n=1}^{\infty} (r_n \cos n\theta_0) \cos n\theta \right]
+ \sum_{n=1}^{\infty} (r_n \sin n\theta_0) \sin n\theta
\]
where
\[ r_n = \frac{\Gamma(s+1)}{\Gamma(s+n-1) \Gamma(s-n+1)} \]

In the above, \( s \) is referred to as the "sharpness index", and \( \theta_0 \) is the predominant direction of wave propagation. The multiplying term involving the Gamma function \( \Gamma \) arises from the constraint equation (13). This model should provide a satisfactory description of a unimodal wave field with wave directions dispersed over a narrow sector centred about \( \theta_0 \). A high value of "sharpness index" indicates that all wave directions are concentrated about \( \theta_0 \).

5. ANALYSIS TECHNIQUES

Primary Data Plots: The wave-induced velocity data returned from each "burst" of the Sea Data 635-12S are illustrated by the cross plots of the horizontal current velocity components (figure 1).

![Cross plots of u-v horizontal current velocities](image)

Figure 1: Example cross plots of the u-v horizontal current velocities measured near the sea bed. The elliptical scatter is caused by wave-induced currents, and displaced by tidal currents.
The velocity cross plot shows the elliptical spread of horizontal wave-induced motions. The principle direction of wave propagation is along the major axis of the ellipse. The $180^\circ$ ambiguity about the direction of phase propagation is resolved by examination of the pressure time history. Horizontal wave-induced velocities under wave crests are in the same direction as the phase propagation.

**Frequency Domain Estimation of Directional Parameters**

The directional spectrum of a random sea state may be estimated from Fourier transformations of a set of $N$ triplets of simultaneous $u,v,p$ measurements, consecutively sampled at a time spacing of $\Delta t$. Fourier transformations of the measured $p,u,v$ time series provide estimates of the left-hand sides of equations (12) to (17), the $p,u,v$ cospectra. Using the velocity and pressure attenuation functions (18) and (19) determined from linear wave theory, and the description of the spectral density function given by (20) to (22), the coefficients $a_1, b_1, a_2$ and $b_2$ of the truncated Fourier expansion of the spreading function (23), may be estimated from the $p,u,v$ cospectra.

First order estimates of the directional spectral parameters of the $\cos^2 \theta \left[ \theta - \theta_0 \right] / 2$ model may be obtained by comparing equation (24) with equation (23), from which

$$ s = \frac{r_1}{1-r_1} \quad (25) $$

where

$$ r_1^2 = a_1^2 + b_1^2 \quad , \quad (26) $$

and

$$ \theta_0 = \arctan \left( \frac{b_1}{a_1} \right) \quad , \quad (27) $$

which are obtained from the pressure-velocity cospectra.

An alternative estimate of $s$ and $\theta_0$, known as the second order fit, is also available, as

$$ s = \frac{(1+3r_2) + (1-14r_2 + r_2^2)^{1/2}}{2(1-r_2)} \quad (28) $$

where

$$ r_2^2 = a_2^2 + b_2^2 \quad , \quad (29) $$

and

$$ \theta_0 = \frac{1}{2} \arctan \left( \frac{b_2}{a_2} \right) \quad . \quad (30) $$

which is obtained from velocity-velocity cospectra.
An example of the directional spectral information attainable from each "burst" of p,u,v measurements of the Sea Data 635-12S Directional Wave and Tide Recorder is presented in figure 2. First and second order estimates of wave direction agree very well. First and second order estimates of sharpness index are not theoretically equivalent, but are expected to display similar trends.

Following the example of Forristall et al. (1978) the gain ratio Q(f) has also been calculated. It represents the ratio of the measured pressure to velocity transfer function and that predicted by linear theory. From equations (18) and (19), Q(f) = A(f)/B(f). If Q(f) remains near unity, linear theory is providing a satisfactory description of wave kinematics. Figure 2 shows that Q(f) departs from unity for very high frequencies where ambient turbulence contaminates the data, and for low frequencies where surface wave energy is very low.
Also presented on this plot is a dotted vertical line which marks where the spectral moment computations have been truncated (high frequency cut off), to avoid the increasing noise contamination of the wave signal.

**Time Domain Estimation of Directional Parameters**: The mean wave direction and sharpness index estimates may be obtained without recourse to spectral analysis, from the covariance (time domain) equivalents of equations (25), (27), (28) and (30). For each set of $N$ triplets of $p,u,v$ the variances and covariances $C_{\gamma \zeta}$ of the measured quantities are computed by taking the dot products as follows:

$$C_{\gamma \zeta} = \frac{1}{N} \sum_{i=1}^{N} (\gamma_i - \bar{\gamma})(\zeta_i - \bar{\zeta})$$

where the overbars denote sample means, and the subscripts $\gamma$ and $\zeta$ denote $p$, $u$, or $v$.

First order estimates of mean direction $\bar{\theta}$ and sharpness index $\bar{s}$ are obtained explicitly from

$$\bar{\theta} = \arctan \frac{C_{pv}}{C_{pu}}$$

and

$$\bar{s} = \frac{r_1}{1-r_1^2}$$

where

$$r_1^2 = \frac{C_{pu}^2 + C_{pv}^2}{C_{uu} + C_{uv} + C_{vv}} \quad \text{(33)}$$

The second order estimates are obtained from

$$\bar{\theta} = \frac{1}{2} \arctan \frac{2C_{uv}}{C_{uu} - C_{vv}}$$

and

$$\bar{s} = \frac{(1+3r_2^2) + (1-14r_2 + r_2^2)^{1/2}}{2(1-r_2)}$$

where

$$r_2^2 = \frac{(C_{uu} - C_{vv})^2 + 4C_{uv}^2}{C_{uu} + C_{vv}} \quad \text{(36)}$$

It should be noted that these estimates are derived from measurements of wave kinematics which have suffered frequency dependent depth attenuation. As the depth of measurements increases, these estimates become more biased towards longer period waves.
6. **INSTRUMENT PERFORMANCE**

Local deployments of the Sea Data 635-12S Directional Wave and Tide Recorder have now returned in excess of two years of data. Improvements in the design and operation of the instrument, and in the data transcription and processing techniques, now allow better than 98% data recovery.

Experience with deployments at various locations around Australia shows that minimum wave periods of 2, 5 and 8 seconds may be satisfactorily resolved in water depths of 2, 20 and 50 metres, respectively. The directionality of recorded wave fields has been evident in all deployments.

The performance of the 635-12S is illustrated by measurements conducted in 20 metres of water at a location 30 km south of Barrow Island off the north west coast of Australia, during the passage of tropical cyclone "Chloe". The measurement location and tropical cyclone track are shown in figure 3. The storm track passed about 150 km to the south east of the measurement location.

![Figure 3 Location diagram showing directional wave measurement site and the track of tropical cyclone "Chloe". Arrow heads on the track correspond to 1200 hours GMT. Date and storm central pressure (mb) are shown adjacent each mark.](image-url)
Figure 2 shows the directional spectral information obtained from one "burst" of the 635-12 near the time of closest approach of the storm. This includes the presentation of $\theta(f)$ and $s(f)$ as a function of frequency, derived from equations (25) to (30). In this figure, the distinct bimodality of the sea state is evident, with swell waves (periods longer than 8 seconds) propagating from the north west, and short period, locally generated wind waves propagating from the south.

Averages weighted by the non-directional energy spectrum were taken to produce burst mean estimates, denoted by $\theta$ and $s$. Figure 4 shows the variation of the directional wave climate during the passage of tropical cyclone "Chloe", using burst mean parameters.

Figure 4 Description of the temporal variation of the directional climate using mean spectral parameters, for the period of the passage of tropical cyclone "Chloe".
Wind speeds and directions shown are 20 minute mean values recorded on Barrow Island, about 30 km north of the measurement location. The first order estimate of the wave direction $\theta$, initially shows the incidence of westerly to north westerly swell, then tracks the wind direction after the onset of the storm. The sharpness index $s$ peak is observed to coincide with the storm peak, as also noted by Mitsuyasu et al. (1975).

Non-directional spectral parameters $H_g$ (significant wave height) and $T_p$ (period of the spectral peak) illustrate the incidence of low amplitude, long period swell prior to the arrival of the storm, followed by the generation of a high energy, short period sea state.

The 635-12S may also be used to establish frequency dependent variations in the wave climate. This is achieved by taking averages of $E(f)$, $s(f)$, $\theta(f)$ and $Q(f)$ for each spectral ordinate over entire deployments. Figure 5 illustrates the result of averaging the spectral parameters over a six week deployment period.

Figure 5 Spectral parameter means providing a description of the frequency dependent variation of the directional wave climate, over the period of an entire deployment.
Q(f) determines the limits of applicability of the frequency dependent wave climate description, which holds only while Q(f) is near unity. Within this frequency range, the wave climate is seen to be distinctly bimodal, with swell periods in excess of 10 seconds typically incident from the north west, and shorter period wind waves (6 to 10 seconds) typically incident from the north-north-east. For this deployment, bathymetric constraints limit wave propagation from the east. The sharpness and energy density estimates also reflect the bimodality of the wave climate.

Comparison of Analysis Techniques: Figure 2 shows that first and second order estimates of \( \theta \) are very similar. Estimates of \( s \) vary, but they do display similar trends.

Estimates of \( \theta \) and \( s \) may be obtained without recourse to spectral analysis, using the covariance equations (31) to (36). A comparison of these estimates with their spectral equivalents, \( \theta \) and \( s \), weighted by the energy density) is presented in figure 6, again using data recorded during the passage of tropical cyclone "Chloe".

![Figure 6](image_url)

**Figure 6** Comparison of mean wave directions and sharpness indices derived by time domain (covariance estimates \( \theta \) and \( s \)) and frequency domain (cospectral estimates \( \theta \) and \( s \)) techniques.

First and second order estimates of \( \theta \) in either the time domain (covariances) or frequency domain (cospectra), are almost indistinguishable. However, small differences are evident between time domain and frequency domain estimates, particularly during periods of rapid change in the sea state.
First and second order estimates of $s$ show the expected similar trends, but time domain estimates are generally less than the frequency domain estimates.

Comparison with Surface Wave Measurements: A Datawell Waverider buoy was deployed concurrently with the Sea Data 635-12S, at the same location during the passage of tropical cyclone "Chloe". This allowed comparison of non-directional wave parameter estimates, which are presented in figure 7.

Figure 7 Comparison of non-directional spectral surface wave parameters derived from surface displacement measurements, and near-bottom pressure and velocity measurements.
Estimates of significant wave height derived from both near-bottom pressure and velocity measurements agree very well, except at the storm peak where dynamic pressure effects may have become important. Generally, Waverider estimates of significant wave height exceed those derived from the near-bottom measurements due to the depth attenuation of short period waves (5 seconds or less). At the storm peak, estimates from the Waverider and the near-bottom pressure measurements coincide.

Estimates of average zero upcrossing period and spectral peak period derived from both near-bottom pressure and velocity measurements also agree very well. While Waverider measurements provide good agreement for spectral peak period, average zero upcrossing periods are seen to be much lower than those derived from near-bottom measurements, because of the predominance of short period surface waves.

In accordance with Forristall et al. (1978), comparisons of measured near-bottom wave-induced velocity spectra with those derived from surface wave measurements using non-directional linear wave theory, show the theory to overpredict near-bottom velocities by about 20%.

7. COMMENTS AND CONCLUSIONS

The Sea Data 635-12S Directional Wave and Tide Recorder provides a reliable means of measurement of near-bottom wave-induced kinematics and of shallow water directional wave climate.

Uni-directional linear wave theory is seen to overpredict near-bottom wave-induced velocities derived from surface wave measurements.

A variety of analysis techniques are available, several of which have been applied to the measured data. Analyses undertaken to date have not indicated a preference for first or second order estimates of directional wave parameters. Maximum likelihood techniques which combine both estimates have been successfully applied to simulated data, but small (sensor induced) phase differences between measured pressures and velocities inhibit application to field data. It is hoped to apply these types of data adaptive techniques in the future.

The Sea Data 635-12S Directional Wave and Tide Recorder can routinely provide important information for determination of wave-induced forces on coastal structures and submarine pipelines, for shallow water wave refraction and diffraction studies and for sediment transport studies.

8. REFERENCES


