## CHAPTER TEN

# A RANKIN VORTEX NUMBER AS A GUIDE TO THE SELECTION OF A MODEL HURRICANE

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## 1. INTRODUCTION

A model hurricane is defined by a model pressure profile, which is the same in all radial directions from the center of the hurricane. The model describes concentric circles of constant pressure known as isobars. The slope of the pressure profile gives the pressure gradient used in the gradient wind equation, together with other considerations determines the time history moving hurricane wind and pressure fields. The appropriate model hurricane can then be coupled with various other models for the determination of design criteria such as wind, waves, currents, wave forces, storm surge, wave run-up, coastal flooding and inundation limits. Because of the many requirements for accurate output data, there have always been concerns of the proper use of and selection of the appropriate hurricane model for a particular task and location.

The primary purpose of the paper is to begin to build a guide for determining the appropriate model to be used for a particular situation and criteria. When the data pressure profile is available, there is no need for a model since the slope of the data pressure profile gives the pressure gradient, which can be used directly in the gradient wind equation. The data pressure profile can also be fitted to the most appropriate model by various techniques of correlation.

After a sufficient number of data pressure profiles have been determined and correlated with various models, to determine the most appropriate model, then one should be able to extend the guide for better selection of model. One can then make better use of the standard project, maximum probable and actual tabulated hurricane data ($R_G$, $P_O$, $P_W$, $\phi$, $V_F$, etc.) given in the NOAA Report by Schwerdt, Ho and Watkins (1979).

The present guide for selection of a particular model has to do with the hurricane parameters $R_G$, $P_O$, $P_N$, and $\phi$ as related to the cyclostrophic and gradient wind equations. For the convenience, in this paper R is the radius of maximum cyclostrophic wind as distinguished from $R_G$, the radius of maximum wind in the report by Schwerdt, et al. (1979). The parameters determined by data pressure profile analysis are:

1. R = radius of maximum cyclostrophic wind,
2. $P_R$ = pressure from the data pressure profile at R, and
3. MAX [rdP/dr] = related to the maximum cyclostrophic wind and determine the location of R.

## 2. DATA PRESSURE PROFILE

The data pressure profile is determined from the cyclone weather chart, and is an average of eight traverses from the center of the cyclone crossing identical isobars to the last closed or nearly closed isobar. The average distance r is plotted versus the corresponding isobar pressure. This method of analysis eliminates or at least minimizes the distortions in the isobar pattern due to personal judgement in construction of the isobars, possible effects due to forward motion to the cyclone and blocking effects due to adjacent pressure systems or land effects. The net result is a cyclone having concentric circular isobars, the definition of a model cyclone.

The step-by-step procedure is simple and straightforward in this method of analysis. A smooth S-curve is constructed through the data points, defining the data pressure profile. It is not necessary to have a complete data pressure profile, including $P_O$ and $P_N$, which can be calculated by theory depending on choice of the model. The pressure gradient is a smooth profile through points calculated from the slope of the data pressure profile. The cyclostrophic profile is calculated from the smooth pressure gradient profile by multiplying corresponding points of the slope of the pressure profile by the radial distance r. Some fine tuning may be required to increase the accuracy in the range of radius of maximum wind, which can easily be estimated at a distance equal to about twice that of the maximum or peak of the pressure gradient profile, which occurs at the inflection point of the data pressure profile. Three important parameters are then determined as follows:

1. R = radius of maximum cyclostrophic wind at max [r dp/dr],
2. max [r dp/dr] at R, and
3. $P_R$ the pressure from that data pressure profile at R.

It then follows by theory that $P_O$ and $P_N$ can be calculated from the following relations:

$$\Delta P_O = P_N - P_O = 1/C_1 \max [r\ dp/dr] \quad (1)$$
$$P_O = P_R - C_2\ \Delta P_O, \text{ and} \quad (2)$$
$$P_N = P_O + \Delta P_O \quad (3)$$

where $C_1$ and $C_2$ are theoretical constants depending on the choice of the model.

The theoretical maximum cylostrophic wind speed can be determined from:

$$V_c = \sqrt{1/\rho_a \text{ MAX } [rdP/dr]} = K \sqrt{\text{MAX } rdp/dr} \quad (4)$$

where $\rho_a$ = air density of $P_R$ at r = R, radius of maximum wind.

Eq. 4 is independent of choice of model, and $K$ = 18.7 to 19.3 for all pressure profiles.

All theoretical pressure profiles will be in agreement with the data pressure profile at $P_R$, R, and max [r dp/dr] by the very nature of the analysis of the cyclone weather charts. Furthermore all model pressure profiles will be in very close agreement with the data pressure profile over the range of $0.5\ R<R<1.5\ R$ approximately, but there will be deviations outside this range. A high correlation will always be achieved between data pressure profile and model pressure profile because of the above range in agreement, but the choice of the model will be that model which has the overall best correlation with the data pressure profile, excluding $P_O$ and $P_N$, except when available by measurements. Spot check data points such as $P_O$, $P_N$, $V_{max}$, etc. if available by measurements should also be considered.

Six Indian Seas cyclones have been analyzed by the above method. Regression analysis between the original data of the data pressure profile and the corresponding theoretical pressure points for BRET MODEL-X was made, and the following regression coefficients were obtained: $\rho = 0.9890$, 0.9995, 0.9993, 0.9824 and 0.9996. Figure 1 presents the example theoretical hurricane relations for BRET MODEL-X, and Figures 2 to 7 are the Indian Seas cyclones data pressure profile analysis. Because of the very nature of the method of determination of R, $P_R$ and max [r dp/dr], high regression correlations are expected, and therefore one might reject those cyclones for which $\rho<0.98$ or 0.99.

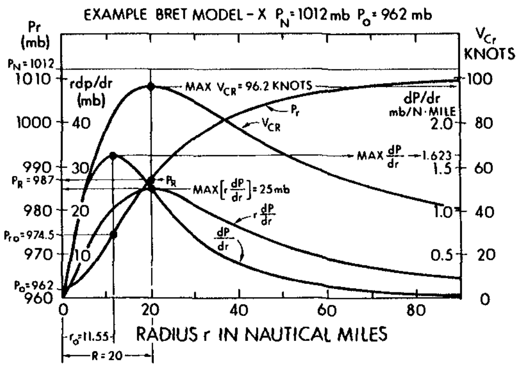


FIG. 1 EXAMPLE THEORETICAL HURRICANE RELATIONS FOR BRET MODEL - X

The choice of the model can also be based in part on previous investigations reported in the literature. By theory the maximum cyclostrophic wind speed for model hurricanes is given by:

$$V_C = K_1 \sqrt{P_N - P_O} \qquad (5)$$

where

$$K_1 = (C_1/\rho_a)^{1/2} \qquad (6)$$

where $\rho_a$ is the air density and generally increases slightly with latitude. $C_1$ is a theoretical constant, depending on choice of model.

Obviously the choice of model depends on $V_C$ of Eq. 5 is equal to $V_C$ of Eq. 4.

It would seem prudent to analyze cyclones, hurricanes and typhoons by the simple straightforward method introduced in this paper, and determine a large number of values for R, $P_R$ and max [r dp/dr]. $P_O$ and $P_N$

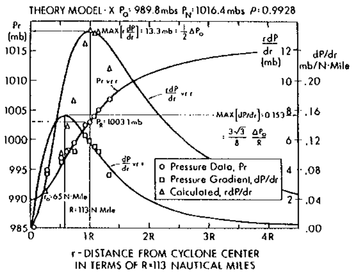


FIG. 2 BAY OF BENGAL CICLONI 5 MAI 1975 2130 IST COMPARED WITH HURRICANE MODEL - X

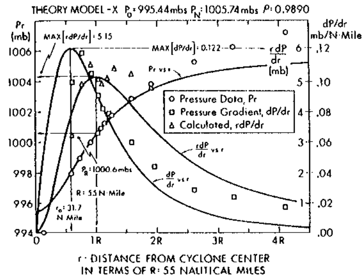


FIG. 3 BAY OF BENGAL CYCLONE 8 DEC. 1965 0830 IST COMPARED WITH HURRICANE MODEL - X

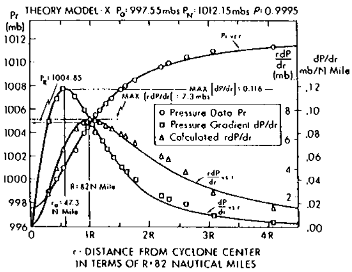


FIG. 4 BAY OF BENGAL CYCLONE 28 OCT. 1977 1800 GMT COMPARED WITH HURRICANE MODEL - X

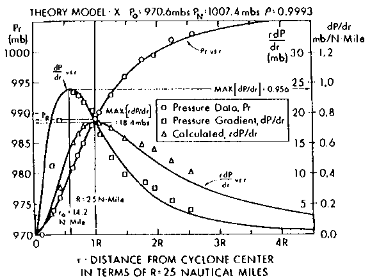


FIG. 5 ARABIAN SEA PORBANDAR CYCLONE 21 OCT. 1975 0200 - 1500 GMT COMPARED WITH HURRICANE MODEL - X

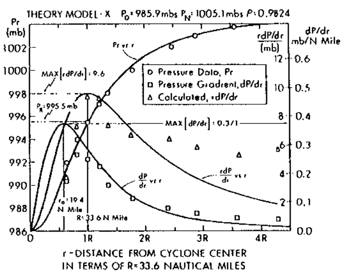


FIG. 6 BAY OF BENGAL CYCLONE 6 MAY 1975 0530 IST COMPARED WITH HURRICANE MODEL - X

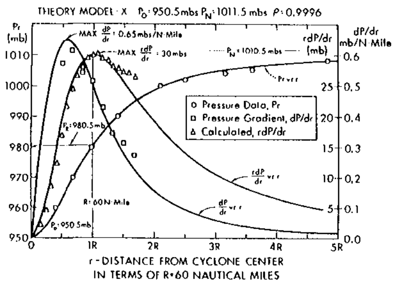


FIG. 7 BAY OF BENGAL ANDHRA CYCLONE 17 NOV. 1977 0830 IST COMPARED WITH HURRICANE MODEL - X

can then be determined for choice of model for correlation purposes.

In summary the uniqueness of this paper is the method of analyzing cyclone weather charts to determine the three parameters R, $P_R$ and max [r dp/dr], the determination of $P_O$ and $P_N$ by theory depending on choice of model, and the correlation between the data pressure profile and the theoretical pressure profile over the range of isobaric pressure data.

3. MODEL PRESSURE PROFILES

Besides, the Hydromet model pressure profile, there are a number of other available model pressure profiles found in the literature. A modification of the Hydromet model was made by Holland (1980) giving a family of pressure profiles, of which one of the pressure profiles reduces to the Rankin Vortex model. This family of pressure profiles has been in use with some success by Rosendal (1982).

A number of other pressure profile models from various sources are given in the NOAA Report by Schwerdt, Ho and Watkins (1979). Fujita (1962) proposed a different model, which Uji (1975) fitted quite well to Typhoon Vera, 22-27 September 1959, a very large western north pacific typhoon. Bretschneider (1982) proposed a new general form of hurricane models, of which Fujita (1962) model is a special case. Jelenianski (1966) used a non-dimensional surface profile corresponding to the BRET MODEL-X non-dimensional cyclostrophic wind profile.

In summary there are two general types of hurricane models: (1) the modified Rankin Vortex model by Holland (1980), of which the Hydromet model is a special case; and, (2) the BRET-General model of which the BRET MODEL-X, the Fujita model and Jelenianski model are all special cases. The mathematical form of the pressure profiles for the two families of models are:

$$\frac{P_r - P_O}{P_\infty - P_O} = A\, e^{-B\,[R/r]} \tag{7}$$

and

$$\frac{P_r - P_O}{P_\infty - P_O} = 1 - \left[1 + a\left(\frac{r}{R}\right)^2\right]^{-b} \tag{8}$$

where

$P_O$ = central pressure of hurricanes
$P_r$ = pressure at radial distance r
$P_\infty$ = pressure at infinite distance r
R = radius of maximum cyclostrophic wind

The constants $A = B^{-1}$ and $a = b^{-1}$ must always hold true to satisfy the mathematics of the cyclostrophic wind equation.

Eq. 7, proposed by Holland (1980), and when A = B = 1, becomes the original Rankin Vortex model after Schloemer (1954) or before. Eq. 8 was proposed by Bretschneider (1981) after pressure profile data analysis.

When $a = b - 1$, Eq. 8 is called BRET MODEL-X. When $a = b^{-1} = 2$, Eq. 8 becomes the same as Fujita model (1962).

Eqs. 7 and 8 above represent families of models that overlap. The deviations between models have to do with (1) the size of the hurricane as governed by the radius of maximum cyclostrophic wind, (2) the wind intensity of the hurricane as governed by the central pressure reduction, and (3) an assumption that there is a latitude affect as governed by the Coriolis parameter. The above assumptions lead to the introduction of a non-dimensional Coriolis or Rankin Vortex number for hurricane classification. Table 1 presents the theoretical constants $K_1$ for four hurricane models -- Hydromet Model, NOAA Model-I, Fujita Model-J, and BRET MODEL-X. Table 2 presents the parameters for the six Indian Seas cyclones data pressure profile analysis.

Table 1: THEORETICAL CONSTANTS FOR FOUR HURRICANE MODELS

| | HYDROMET MODEL-HM | NOAA MODEL-I | FUJITA MODEL-J | BRET MODEL-X |
|---|---|---|---|---|
| $K_1 = \left(\frac{C_1}{\rho_a}\right)^{1/2}$ | $\left(\frac{1}{e\rho_a}\right)^{1/2}$ | $\left(\frac{1}{\pi\rho_a}\right)^{1/2}$ | $\left(\frac{2}{3\sqrt{3}\ \rho_a}\right)^{1/2}$ | $\left(\frac{1}{2\rho_a}\right)^{1/2}$ |
| FOR V = KNOTS and P = INCHES Hg | | | | |
| | 66.0 | 61.39 | 67.51 | 76.74 |
| $K_1$ | 68.0 | 63.25 | 69.56 | 79.28 |
| FOR V = KNOTS and P = millibars | | | | |
| | 11.34 | 10.55 | 11.60 | 13.22 |
| $K_1$ | 11.68 | 10.87 | 11.95 | 13.62 |

## 4. THE GRADIENT WIND EQUATION

The radius of maximum gradient wind and the maximum gradient wind, respectively are not identical to the corresponding radius of maximum cyclostrophic wind and the gradient wind at the radius of maximum cyclostrophic wind. The above was shown to be the case by Bretschneider (1959), using the Hydromet-Rankin Vortex model after Schloemer (1954) and Myers (1954). This will be true for any wind field developed from the pressure gradient by the very nature of the gradient wind equation.

The gradient wind equation for a stationary cyclone can be given as follows:

$$V_g^2 + frV_g = \frac{r}{\rho_a}\frac{dp}{dr} \tag{9}$$

Table 2 Summary Parameters from Six Indian Seas Cyclones

| Cyclone No. - Figure No. | I 6 | II 7 | III 8 | IV 9 | V 10 | VI 11 |
|---|---|---|---|---|---|---|
| Area and Latitude $\phi$ | Bay of Bengal $14.8^\circ$ | Bay of Bengal $10^\circ$ | Bay of Bengal $14^\circ$ | Arabian Sea* $-23^\circ$ | Bay of Bengal $14.3^\circ$ | Bay of Bengal $10^\circ$ |
| Date and Time of Weather Chart | 5 May 1975 2330 IST | 8 Dec 1965 0830 IST | 28 Oct 1977 1800 GMT | 22 Oct 1975 0200-1500 GMT | 6 May 1975 0530 IST | 17 Nov 1977 0830 IST |
| **From Data Pressure Profile and Derivative** | | | | | | |
| R, Radius of Maximum Cyclone Wind Nautical Miles | 113 | 55 | 82 | 25 | 33.6 | 60 |
| $P_R$ Pressure at R (mbs) | 1003.1 | 1000.6 | 1000.85 | 989.0 | 995.5 | 980.5 |
| Max r dP/dr at R (mbs) | 13.3 | 5.15 | 7.3 | 18.4 | 9.6 | 30 |
| $V_{cR}$ (max) Knots | 70.24 | 43.71 | 52.04 | 82.62 | 59.67 | 105.5 |
| **From Theory, Each Model** | | | | | | |
| Hydromet Model - HM | | | | | | |
| $K_1 = 11.68$; $c_1 = 1/e$, $c_2 = 1/e$ | | | | | | |
| $\Delta P_o$ (mbs) | 36.15 | 14.00 | 19.84 | 50.02 | 26.10 | 81.60 |
| $P^o$ (mbs) | 989.8 | 995.44 | 997.55 | 970.60 | 985.90 | 950.50 |
| $P_N^o$ (mbs) | 1026.0 | 1009.44 | 1017.40 | 1020.60 | 1012.00 | 1032.10 |
| NOAA Model - I | | | | | | |
| $K_1 = 10.87$; $c_1 = 1/\pi$, $c_2 = 1/2$ | | | | | | |
| $\Delta P_o$ = (mbs) | 41.78 | 16.17 | 22.92 | 57.77 | 30.13 | 94.20 |
| $P^o$ = (mbs) | 982.2 | 992.52 | 993.40 | 960.10 | 980.40 | 933.40 |
| $P_N^o$ = (mbs) | 1024.0 | 1008.69 | 1016.30 | 1017.90 | 1010.60 | 1027.60 |
| Fujita Model - J | | | | | | |
| $K_1 = 11.95$; $c_1 = 2/3\sqrt{3}$ $c_2 = 1 - \sqrt{3}/3$ | | | | | | |
| $\Delta P_o$ (mbs) | 34.55 | 13.38 | 18.90 | 47.80 | 24.94 | 77.94 |
| $P^o$ (mbs) | 988.50 | 994.94 | 996.90 | 968.80 | 985.00 | 947.56 |
| $P_N^o$ (mbs) | 1023.60 | 1008.32 | 1015.80 | 1016.60 | 1009.90 | 1025.50 |
| Bret Model - X | | | | | | |
| $K_1 = 13.62$; $c_1 = 1/2$, $c_2 = 1/2$ | | | | | | |
| $\Delta P_o$ (mbs) | 26.60 | 10.30 | 14.60 | 36.80 | 19.20 | 60 |
| $P^o$ (mbs) | 989.80 | 995.44 | 997.55 | 970.60 | 985.90 | 950.50 |
| $P_N^o$ (mbs) | 1016.40 | 1005.74 | 1012.15 | 1007.4 | 1005.1 | 1010.50 |
| $\rho$ = correlation coefficient | .9928 | .9890 | .9995 | .9993 | .9824 | .9990 |
| | | | *NOTE: | Time History Pressure Profile Used | | |
| $fR_c$ (knots) | 15.1 | 4.99 | 10.4 | 5.10 | 4.33 | 5.44 |
| $fR_c/V_{R_c}$ | 0.215 | 0.114 | 0.199 | 0.062 | 0.072 | 0.052 |

where

$V_g$ = gradient wind speed at radial distance r from the center of the cyclone
$f$ = Coriolis parameter
$\rho a$ = air density
$dp/dr$ = pressure gradient

In the absence of the Coriolis term fr Eq. 9 becomes the cyclostrophic wind equation as follows:

$$V_r^2 = \frac{r}{\rho_a}\frac{dp}{dr} \tag{10}$$

where $V_r$ is the cyclostrophic wind speed, resulting from the balance between the centripetal force directed toward the center of the cyclone and the force due to the pressure gradient.

Thus Eq. 9 can be written as follows:

$$V_g^2 + frV_g = V_r^2 \tag{11}$$

The maximum cyclostrophic wind velocity can be obtained from

$$V_c^2 = \frac{1}{\rho_a} \text{MAX} \left[\frac{rdp}{dr}\right] \tag{12}$$

where $V_c$ occurs at R radius of maximum cyclostrophic wind. $R_G$ is the radius of maximum gradient wind, and is somewhat smaller than R, depending on the value of $fR/V_c$ or $fR_G/V_c$.

The most accurate evaluation of Eq. 12 would be by use of an accurately determined pressure profile from data, but this is seldom possible because of lack of sufficient data. The procedure is to best fit a pressure profile to the data pressure. Then, an analytical pressure profile or model pressure profile can be selected which best fits all the data including the central pressure, if available. Ideally, it would be excellent to have available the pressure at the radius of maximum cyclostrophic wind.

Once the maximum cyclostrophic wind $V_c$ is obtained, then one can obtain the gradient wind at R as follows:

$$V_{gR}^2 + fR\,V_{gR} = V_c^2 \tag{13}$$

or

$$V_{gR} = \frac{1}{2} - fR + \sqrt{(1/2\ fR)^2 + V_c^2} \tag{14}$$

As can be seen from the works of Bretschneider (1959) Eq. 13 or Eq. 14 does not give the maximum gradient wind $V_G$ and consequently the maximum 10-meter level wind speed $V_s$ at the radius of maximum wind. The radius of maximum gradient wind, $R_G$ for $V_G$ is not the same as the radius of maximum cyclostrophic wind $R = R_c$ for $V_c$.

Actually $R_G = R_c$ and $V_G = V_c$ for the Rankin Vortex model only, which implies that $R_c = R_G$ as $R$-0, $V_G = V_c$ as $V$-$\infty$. Otherwise everything can only be approximate, the approximation depends upon the Rankin or the Coriolis non-dimensional number, which can be defined as follows:

$$N_c = fR/V_c \tag{15}$$

where

$f$ = the well known Coriolis parameter
$R$ = radius of maximum cyclostrophic wind
$V_c$ = maximum cyclostrophic wind

When $N_C$ is very small (0.01 to 0.05) the Rankin Vortex model applies. When $N_C > 0.1$ the Rankin vortex cannot apply. The problem is when does some other model or modification of the Rankin Vortex model become important?

What is required is to establish certain relationships between cyclostrophic wind, gradient wind, and surface wind, usually defined as the 1-minute or the 10-minute average at the 10-meter standard anemometer level.

In sequence of maximum cyclostrophic wind $V_C$, maximum gradient wind $V_G$ and maximum surface wind $V_S$ we have

$$V_G > V_C \quad \text{and} \quad V_{SR_G} = C_f V_G \tag{16}$$

where $C_f$ is the friction reduction factor.

What has not been recognized for hurricane is that the radii of maximum of the above types of winds are not the same. It is very easy to state that:

$$R_S = R_G < R_C \tag{17}$$

In fact one can prove always for a stationary model hurricane that:

$$R_G < R_C \tag{18}$$

However it is not apparent that $R_G$ is not the same as $R_S$ because of different relationships for reduction friction factors. Unlike $R_C$ and $R_G$ which are quite apparent, it is quite probable that $R_G = R_S$.

## 5. AN INVESTIGATION OF MODEL HURRICANE

To begin with, all model hurricane pressure and wind fields are assumed to be stationary models, after which forward speed of translations are applied to change only the winds but not the pressures.

What is required here is to establish relationships for the stationary hurricane between $R_G$, radius of maximum gradient wind, $V_{gR}$ and R, the radius of maximum cyclostrophic wind, $V_C$.

It is important to note here that $R_G$ and not R, the appropriate parameters to establish the radial distance at which $V_G$ applies.

Three cases will be worked out here: (1) the Hydromet-Rankin Vortex model, (2) the BRET MODEL-X and (3) the Fujita model. Similar approaches can be worked out for the other models.

HYDROMET-RANKIN VORTEX MODEL, A=B=1

The pressure profile is given by:

$$\frac{P_r - P_o}{P_N - P_o} = e^{-R/r} \tag{19}$$

The pressure gradient is obtained as follows:

$$\frac{dp}{dr} = \frac{P_N - P_o}{R}\left(\frac{R}{r}\right)^2 e^{-(R/r)} \tag{20}$$

and the cyclostrophic wind equation by:

$$V_r^2 = \frac{1}{\rho a}\frac{rdp}{dr} = \frac{1}{\rho_a}(P_N - P_o)\left(\frac{R}{r}\right) e^{-R/r} \tag{21}$$

The maximum cyclostrophic wind velocity occurs at r = R. Whence

$$V_c^2 = \frac{1}{\rho a} \text{MAX}\left(\frac{rdp}{dr}\right) = \frac{1}{\rho a}(P_N - P_o)\, e^{-1} \tag{22}$$

Dividing Eq. 21 by 22 we obtain

$$V_r^2 = V_c^2\left(\frac{R}{r}\right) e^{(1 - R/r)} \tag{23}$$

Substituting Eq. 23 into Eq. 11 we obtain

$$V_g^2 + fr\,V_g = V_c^2\,\frac{R}{r}\,e^{(1 - R/r)} \tag{24}$$

To find the maximum gradient wind $V_G$ at radius of maximum wind $R_G$, we differentiate Eq. 24 and set the results equal to zero and let $r = R_G$.

$$\frac{V_G}{fR_G} = \left(\frac{V_C}{fR}\right)^2\left(\frac{R}{R_G}\right)^3\left(\frac{R}{R_G} - 1\right) e^{(1 - R/R_G)} \tag{25}$$

For any particular set of conditions

$$\frac{fR}{V_c} = \text{const} = \text{cyclostrophic Coriolis number}$$

when $\frac{R}{R_G} = 1$

$$fR_G = fR = 0$$

$$\frac{V_G}{V_C} = 1$$

The simultaneous solution of Eq. 25 and Eq. 14 will give the proper relationships between R/R and $V_G/V_C$ as functions of the Coriolis number $N_C = fR/V_C$.

FUJITA MODEL, $a=b^{-1}=2$

The cyclostrophic wind relationship for the Fujita model is given by

$$V_r^2 = V_c^2 \frac{3\sqrt{3}\left(\frac{r}{R}\right)^2}{\left[1 + 2\left(\frac{r}{R}\right)^2\right]^{3/2}} \tag{26}$$

Substituting Eq. 26 into Eq. 11, the gradient wind equation becomes

$$V_g^2 + frV_g = V_c^2 \frac{3\sqrt{3}\left(\frac{r}{R}\right)^2}{\left[1 + 2\left(\frac{r}{R}\right)^2\right]^{3/2}} \tag{27}$$

In a similar manner as was done for the Hydromet Rankin Vortex model, differentiate Eq. 27 and set $d\,V_g/dr = 0$ to find the radius, $R_G$ of maximum gradient wind $V_G$

$$\frac{V_G}{fR_G} = 6\sqrt{3}\left(\frac{V_c}{fr}\right)^2 \frac{1 - \left(\frac{R_G}{R}\right)^2}{\left[1 + 2\left(\frac{R_G}{R}\right)^2\right]^{5/2}} \tag{28}$$

and from Eq. 27, let $r = R_g$ it becomes

$$V_G^2 + fR_G V_G = \frac{3\sqrt{3}\,V_c^2\left(\frac{R_G}{R}\right)^2}{\left[1 + 2\left(\frac{R_G}{R}\right)^2\right]^{3/2}} \tag{29}$$

The simultaneous solution of Eqs. 28 and 29 gives relationships between $fR/V_c$ and $R_G/R$.

BRET MODEL-X, $a=b=1$

The cyclostrophic wind relationship for the BRET MODEL-X is given by:

$$V_r^2 = V_c^2 \frac{4(r\ R)^2}{(R^2 + r^2)^2} \tag{30}$$

and the gradient wind equation becomes

$$V_g^2 + frV_g = \frac{4(r\ R)^2}{(R^2 + r^2)^2} V_c^2 \tag{31}$$

In a similar manner as was done for the Hydromet Rankin Vortex model differentiate Eq. 31 and set $d\,V_g/dr = 0$ to find the radius, $R_G$ of maximum gradient wind $V_G$.

$$\frac{V_G}{fR_G} = 8\left(\frac{V_c}{fR}\right)^2 \left(\frac{R}{R_G}\right)^4 \frac{\left(\frac{R}{R_G}\right)^2 - 1}{\left[\left(\frac{R}{R_G}\right)^2 + 1\right]^3} \tag{32}$$

and from Eq. 31 let $r = R_G$, we have

$$V_G + fR_G V_G = \frac{4V_c^2 \left(\frac{R}{R_G}\right)^2}{\left[1 + \left(\frac{R}{R_G}\right)^2\right]^2} \tag{33}$$

The simultaneous solution of Eqs. 32 and 33 gives relationships between $fR/V_c$ and $R_G/R_c$.

Table 3 presents the relationships between the ratio of radius of maximum gradient wind versus radius of maximum cyclostrophic wind and the Rankin Vortex number for Hydromet Rankin Vortex model, Fujita model, and BRET-X model. Table 3 can be used in two ways.

1. Where $R_G$ is given such as for the conditions of previous studies for the Gulf of Mexico, one can calculate the approximate R for the maximum cyclostrophic wind, $V_c$.
2. Where R is obtained by analysis by pressure profiles, then $R_G$ can be used for the maximum gradient wind $V_G$.

## 6. NON-DIMENSIONAL RANKIN VORTEX NUMBER

The non-dimensional Rankin Vortex number, $N_c$ has been defined in Eq. 15. It presents the ratio between the Coriolis velocity (a fictitious velocity) and the cyclostrophic wind velocity (a theoretical velocity) at the radius of the maximum cyclostrophic wind velocity. When $\Delta P_0 = P_N - P_0$ (Eq. 1) is given in millibar, R in nautical miles, and $V_c$ in knots, Eq. 15 becomes

$$N_c = \frac{0.522 \ R \ \mathrm{Sin} \ \phi}{K \sqrt{\Delta P_0}} \tag{34}$$

where $\phi$ is the latitude, K = 11.3 to 11.7 depending on the air density at $P_R$.

Tabulated data from Schwerdt, et al. (1979) for 51 U.S. East Coast and 71 Gulf of Mexico hurricanes were used to calculate values of $N_{cR}$ from Eq. 34 using K = 11.7. There was found a wide scatter of the data with respect to latitude. The average values of $N_{cR}$ increased from 0.05 at lat $\phi = 24°$, to 0.07 at 30° to 0.165 at 41°, where existing models probably do not apply any way. Two lowest values for $N_{cR}$ were 0.01 for Key West (1909) and 0.018 for Camille (1969), both of which can be

Table 3: RADIUS OF MAXIMUM GRADIENT WIND VERSUS RADIUS OF MAXIMUM CYCLOSTROPHIC WIND RELATIONSHIPS FOR HYDROMET RANKIN VORTEX MODEL, FUJITA MODEL AND BRET-X MODEL

| $\frac{fR}{V_c}$ | HYDROMET $\frac{R_g}{R}$ | FUJITA $\frac{R_g}{R}$ | BRET-X $\frac{R_g}{R}$ |
|---|---|---|---|
| 0.0 | 1.0 | 1.0 | 1.0 |
| 0.01 | 0.9903 | 0.9926 | 0.9951 |
| 0.02 | 0.9809 | 0.9856 | 0.9902 |
| 0.03 | 0.9721 | 0.9787 | 0.9855 |
| 0.04 | 0.9636 | 0.9720 | 0.9809 |
| 0.05 | 0.9554 | 0.9656 | 0.9765 |
| 0.06 | 0.9476 | 0.9594 | 0.9721 |
| 0.07 | 0.9401 | 0.9534 | 0.9678 |
| 0.08 | 0.9329 | 0.9476 | 0.9635 |
| 0.09 | 0.9259 | 0.9419 | 0.9594 |
| 0.1 | 0.9192 | 0.9363 | 0.9554 |
| 0.11 | 0.9127 | 0.9310 | 0.9514 |
| 0.12 | 0.9064 | 0.9258 | 0.9476 |
| 0.13 | 0.9003 | 0.9207 | 0.9438 |
| 0.14 | 0.8945 | 0.9158 | 0.9401 |
| 0.15 | 0.8888 | 0.9109 | 0.9364 |
| 0.16 | 0.8832 | 0.9062 | 0.9328 |
| 0.17 | 0.8779 | 0.9016 | 0.9293 |
| 0.18 | 0.8727 | 0.8971 | 0.9259 |
| 0.19 | 0.8676 | 0.8928 | 0.9225 |
| 0.2 | 0.8625 | 0.8884 | 0.9191 |

considered as classical examples of the Rankin Vortex model. Maximum values for the Gulf were between 0.12 and 0.15, and for the East Coast 0.15 to 0.30. $N_{CR}$ = 0.15 for Western Pacific Typhoon Vera (1959), and for Hawaii Hurricane Iwa, $N_{CR}$ = 0.15 to 0.22. $N_{CR}$ = 0.052 to 0.22 for the Indian Seas cyclones. Data pressure profiles for Vera (1959), Iwa (1982) and Indian Seas cyclones, and also mean, minimum and maximum values of $N_{CR}$ for the U.S. East and Gulf Coast hurricanes were used to suggest a guide for selection of model pressure profiles, given in Table 4.

Table 4: A SUGGESTED GUIDE FOR SELECTION OF MODEL

| | |
|---|---|
| HYDROMET RANKIN VORTEX MODEL (Eq. 7) | |
| A=B=1 | $0.0 < N_C < 0.05$ |
| A=B=5/4 (approx. est.) | $0.03 < N_{CR} < 0.08$ |
| BRET MODELS (Eq. 8) | |
| Fujita (b = 1/2) | $0.03 < N_{CR} < 0.08$ |
| BRET-X (b = 1) | $0.06 < N_{CR} < 0.15$ |

NOTE: The above table is only suggested. Revisions will be in order after sufficient hurricane analyses.

## 7. SUMMARY

The radius of maximum gradient wind and the maximum gradient wind to the corresponding radius of maximum cyclostrophic wind and the maximum cyclostrophic wind have been carefully studied with various theoretical hurricane models. The accuracy of the predicted hurricane wind field heavily depends on the choice of the hurricane model. In the current study, it found that the selection of the hurricane model are determined by the range of the non-dimensional Rankin Vortex number. This number presents the ratio between the Coriolis velocity and the cyclostrophic wind velocity at the radius of the maximum cyclostrophic wind velocity. Table 4 gives a suggested guidance for the selection of the hurricane model. But it is only a general guide. Additional data pressure profiles need to be analyzed for various $P_R$, hurricane intensive $V_{CR}$, and for different latitude $\phi$, as well as regional locations.

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LIST OF SYMBOLS

$R = R_C$ = Radius of maximum cyclostrophic wind
$R_G$ = Radius of maximum gradient wind (corresponding to radius of maximum wind of published data on $R = R_G$)
$r$ = Distance from center of hurricane
$V_C$ = Maximum cyclostrophic wind at R
$V_r$ = Cyclostrophic wind at radial distance r
$V_G$ = Maximum gradient wind at $R_G$
$V_g$ = Gradient wind at radial distance r
$P_R$ = Atmospheric pressure at radius of maximum cyclostrophic wind
$P_r$ = Atmospheric pressure at radial distance r

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