

THE RESPONSE OF SMALL CRAFT TO WAVE ACTION

by

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ABSTRACT

A comprehensive study has been conducted for the Small Craft Harbours Branch, Department of Fisheries and Oceans Canada, to provide improved criteria for acceptable wave climate in small craft harbours (Ref. 3). An important part of the study was directed to model tests of vessel response to waves, and comparisons of these to field measurements and to simplified analytical predictions.

The objective of the present paper is to describe these specific comparisons and present the corresponding results in the context of improving harbour entrance designs. The model tests results and theoretical predictions are adequate to show quantitatively the dependence of a sailboat's response to different wave lengths with sufficient accuracy for wave periods, heights and directions to be selected as variables in formulating the required wave criteria.

INTRODUCTION

A commonly accepted criterion used in the design of small craft harbours is that wave heights within the harbour should not exceed 0.3 m (1 ft). However, it is clear that such a criterion is inadequate to take account of the many variables which give rise to vessel damage within a harbour. Consequently, a comprehensive study has been conducted for the Small Craft Harbours Branch, Department of Fisheries and Oceans Canada, to provide improved criteria for acceptable wave climate in small craft harbours (Ref. 3).

An important part of the study was directed to model tests of vessel response to waves, and comparisons of these to field measurements and to simplified analytical predictions. The model tests were carried out with a 0.8 m long model of a moored, fin-keeel sailboat subjected to head and beam seas. The field measurements were carried out with a 7 m long boat similar to the model. The

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analytical predictions were made for four categories of hull shape using different simplified analytical methods.

The objective of the present paper is to describe these specific aspects of the project and to present the corresponding results. A description of the overall project including the recommended criterion of acceptable wave conditions, is given in a companion paper (Ref. 1).

MODEL TEST PROCEDURE

Tests were carried out in a wave basin with a 0.76 m long model of a high performance fin-keel sailboat. Some of the basic specifications of the model are given in Table 1, and the fundamental configuration of the model is sketched in Fig. 1(a).

The boat was moored to a walkway (dock) which could be either fixed or floating. To arrive at appropriate model characteristics a typical wooden dock, 1.5 m wide and 12.2 m long, was replicated. When the walkway was allowed to float, it was constrained against lateral action by four vertical model piles. For fixed dock tests, the ends of the dock were constrained from vertical motion.

For all tests the model was moored with four lines, bow and stern breast lines set slack and bow and stern spring lines set at a small initial steady state tension. The lines were modelled to represent the elasticity of 1.3 cm braided nylon line commonly used for moorage.

The wave basin at the University of British Columbia used for the tests is approximately 13.7 m long, 4.9 m wide and could take water depths up to 0.6 m. The basin was provided with a wave absorbing beach along the end opposite to the wave generator. The wave generator was a hinged paddle with controls to vary the frequency up to 2 Hz and amplitude of motion up to 0.24 m. Only regular waves could be produced.

The wave height was measured with a capacitance-type wave gauge. A motion transducer was designed to record the three appropriate components of boat motion: heave, surge, pitch for head seas, or heave, sway, roll for beam seas. This comprised of a hinged arm which operates as three parallel systems capable of measuring the three required component motions of the vessel. The weight of the transducer was counterbalanced by a low tension spring or a counterweight in order to eliminate the effective weight of the transducer. In addition, movie film was shot of the vessel's motion with and without the transducer connected in order to establish the importance of the transducer in affecting the vessel motion. The transducer was calibrated over its range of application at the beginning of the tests and checked each day against a single reference point for each motion.

In the series of the tests, measurements were also made of the rise and fall of the dock and of peak hawser forces. Numerous photographs were taken and approximately 300 feet of 16 mm movie film

TABLE 1

PRINCIPAL CHARACTERISTICS OF BOAT HULLS ANALYSED

	Hull #1	Hull #2	Hull #3	Hull #4
	Model	Full Keel	Planing	Non-Planing
	Fin Keel	Sailboat	Powerboat	Powerboat
<u>Basic Specifications</u>				
Length Overall (m)	0.76	12.50	12.80	10.87
Length Waterline (m)	0.67	10.67	11.18	10.16
Beam (m)	0.24	3.96	4.34	3.66
Draft (m)	0.15	1.45	1.07	1.14
Displacement (kg)	2.24	11,000	13,000	8,000
Ballast in Keel (kg)	1.27	4,500	-	-

TABLE 2

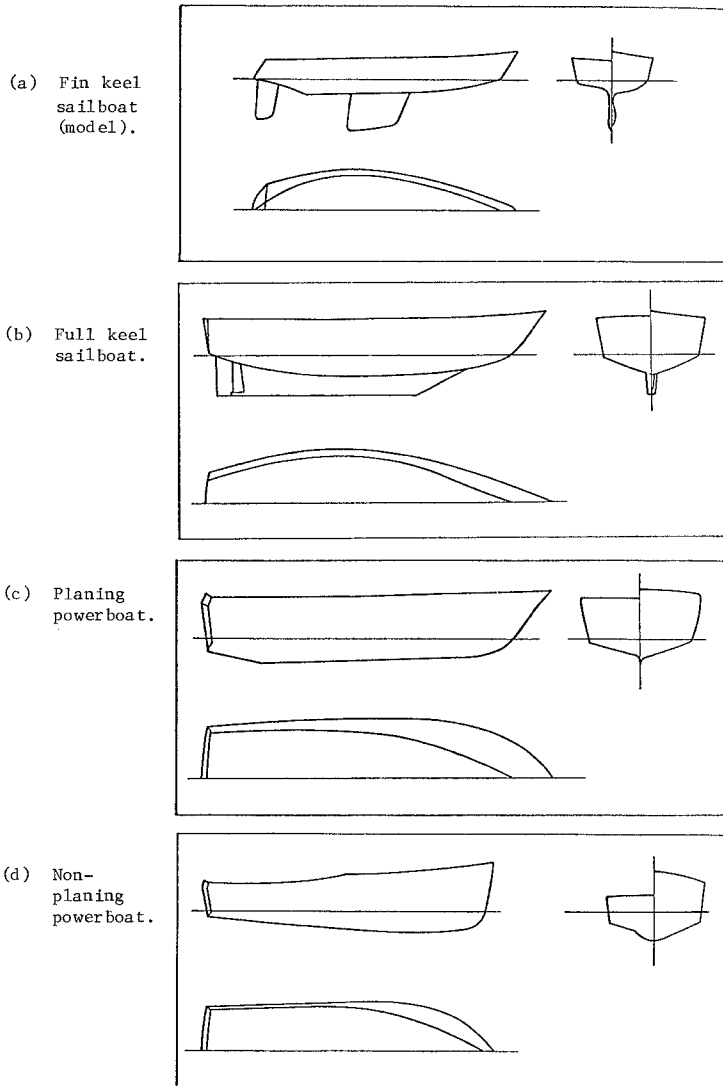
LIST OF CASES TREATED IN ANALYSIS

CASE	WAVE DIRN.	MOORING CONDITION	DEGREES OF FREEDOM	ANALYSIS METHOD	COMMENTS
1	Head	Free Floating	Surge, Heave Pitch	LWA, SBA	
2	Head	Linear Surge Stiffness	Surge, Heave Pitch	LWA, SBA	Heave, pitch as in Case 1
3	Head	Slack/Elastic Surge Spring	Surge, Heave Pitch	SBA	Heave, pitch as in Case 1
4	Head	Stern Hinge Links	Heave, Pitch	LWA, SBA	Heave, pitch as in Case 1
5	Beam	Free Floating	Sway, Heave Roll	LWA, SBA	
6	Beam	Stern Hinge Links	Heave, Roll	LWA, SBA	Heave as in Case 5

LWA: Long Wave Approximation

SBA: Modified Slender Body Approximation

Fig. 1. Hull Configurations Analysed



were shot to provide some documentation of the motions involved.

Preliminary tests were first conducted to measure the vessel's natural periods in roll and pitch, and its static stability characteristics. The curves of pitch and roll angles against applied moment were used to obtain the corresponding hydrostatic stiffnesses and are given in Fig. 2. Subsequently, a series of 148 tests, in which wave period, wave height and wave direction varied, were run and documented. The ranges of conditions for which the tests were carried out were as follows (prototype values for 1:10 scale model are given in parentheses):

water depth:	0.30 and 0.46 m (3.0 and 4.6 m)
water period:	0.5 - 2.0 sec (1.6 - 6.3 sec)
wave height:	0.01 - 0.07 m (0.08 - 0.71 m)
wave direction:	head and beam seas
dock motion:	fixed and floating

FIELD TEST PROCEDURE

The field measurement program, carried out at Fisherman's Cove near Vancouver, developed into several short term activities including:

- (i) Calm water tests mostly on a Swiftsure 24 sailboat to obtain data for comparison with the model.
- (ii) Wave tests on a Swiftsure 24 and a Bayfield 25 for comparison with model response data.

Measurements were first made of the static moment vs. roll or pitch angle relationships for the Swiftsure 24, and the corresponding curves are compared to those obtained with the model in Fig. 2. Natural periods in roll were measured for a number of vessels using the marina, and these ranged from 2.1 sec to 3.7 sec.

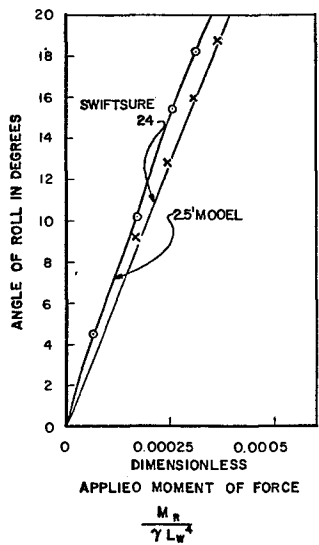
For the wave response tests, the Swiftsure 24 sailboat was moored alongside a gasoline service barge with taut springs and loose breastlines as in the model tests. Wave measurements for the period during testing indicated a significant wave height of 0.37 m and a peak period of 2.4 sec. Measurements of the vessel motions were taken for both head and beam sea conditions, and were carried out with the use of surveying equipment and by photography.

HYDRODYNAMIC ANALYSIS

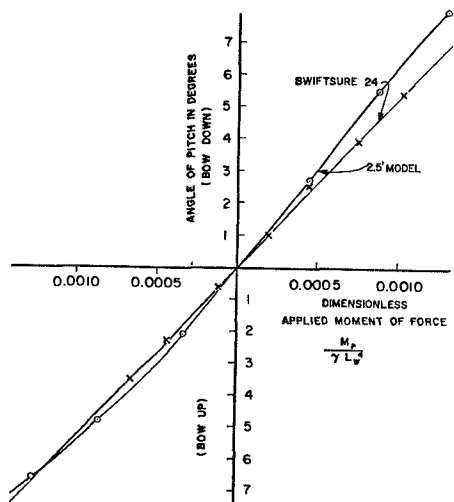
The hydrodynamic analysis of a freely floating vessel responding to wave action is well known and has been reviewed, for example, by Newman (2). In a linear analysis the vessel is taken to oscillate harmonically in six degrees of freedom with displacements given as $\text{Re}\{\xi_j e^{i\omega t}\}$, with $j = 1$ corresponding to surge, $j = 2$ to heave, $j = 3$ to sway, $j = 4$ to roll, $j = 5$ to yaw and $j = 6$ to pitch, and ω is the wave angular frequency.

Fig. 2. Model prototype static test results.

(a) Roll



(b) Pitch



The equations of motion of an unrestrained floating body can be expressed in terms of the complex amplitudes ξ_j by a matrix equation:

$$\{-\omega^2 ([M] + [A]) + i\omega [B] + [C]\} (\xi) = (F) \quad (1)$$

where $[M]$ is the mass matrix, $[A]$ the added-mass coefficient matrix, $[B]$ the damping coefficient matrix, $[C]$ the stiffness matrix, and (F) the exciting force vector. In this notation the components F are the exciting force complex amplitudes, with corresponding time varying forces given as $\text{Re}\{F_j e^{i\omega t}\}$. The mass matrix components and stiffness matrix components are simply derived for a given vessel configuration and weight distribution.

In the case of a moored body, the various terms in the equation of motion, Eq. (1), may be extended to reflect the influence of the moorings on the body's motion. In the usual case, the moorings may be treated as linear springs with constant coefficients and the stiffness matrix can be modified to incorporate these.

In a complete hydrodynamic analysis, the matrices $[A]$ and $[B]$, and the vector (F) are obtained from a solution to the governing radiation/diffraction boundary value problem. This usually derives from the assumptions of a linear motion (small amplitude waves) and an irrotational flow (flow separation effects neglected). An assumption often made in a motion response analysis of a moored vessel is that the mooring system affects the low frequency (drift) oscillations only, but is too light and flexible to affect the vessel oscillations at higher wave frequencies. However, this assumption is unrealistic in the case of light boats with slack moorings.

In the present study, a complete hydrodynamic analysis of a moored vessel with six degrees of freedom has been considered unwarranted because many of the common assumptions generally made are considered unrealistic in relation to the additional effort and cost entailed in a full three-dimensional analysis. For example, for small craft complicating effects may include flow separation, particularly around a keel in beam seas, asymmetric motions for a nominally symmetric condition (e.g. yaw in head seas), nonlinear mooring conditions (e.g. slack/elastic moorings), wave nonlinearities, etc. The intention has been instead to investigate simplified analytical procedures which would adequately predict measured responses over specific ranges of conditions. These analyses are restricted to head seas (with only surge, heave and pitch motions occurring) and to beam seas (with only heave, sway and roll motions occurring).

Long Wave Approximation (LWA)

When the wave length to vessel length ratio is large, a long wave approximation may be made, whereby the exciting force components can be expressed directly in terms of the added-mass the damping

coefficients and the vessel's hydrostatic characteristics. The underlying theory is given by Newman (2) and appropriate expressions for the exciting force complex amplitudes may thereby be derived. These may be written in terms of $F' = 2F/H$, where H is the wave height, as follows:

Head Seas:

$$F'_1 = \frac{ik}{\omega^2} [-\omega^2(m + a_{11}) + i\omega b_{11}] \quad (2a)$$

$$F'_2 = S - \omega^2(m + a_{22}) + i\omega b_{22} + kS_1 \quad (2b)$$

$$F'_6 = S_1 + ikmy_B + ikS_{11} \quad (2c)$$

Beam Seas:

$$F'_2 = S - \omega^2(m + a_{22}) + i\omega b_{33} \quad (2d)$$

$$F'_3 = -\frac{ik}{\omega^2} [-\omega^2(m + a_{33}) + i\omega b_{33}] \quad (2e)$$

$$F'_4 = ik(my_B + a_{34}) + ikS_{33} + \frac{k}{\omega} b_{34} \quad (2f)$$

These equations for the exciting force can be substituted into the RHS of the corresponding equation represented by Eqs. (1) to obtain expressions for peak displacements which can then be solved. Since some terms appear on both sides of the equations considerable simplification arises in many cases.

In the above $k (= 2\pi/\lambda)$ is the wave number, y_B is the vertical ordinate of the centre of buoyancy, S is the waterplane area, and S_1 , S_{11} and S_{33} are the waterplane area moments defined as follows:

$$S_1 = \int x b(x) dx \quad (3a)$$

$$S_{11} = \int x^2 b(x) dx \quad (3b)$$

$$S_{33} = \int \frac{1}{12} b^3(x) dx \quad (3c)$$

where $b(x)$ is the sectional beam of the waterplane profile, the integrals are taken over the waterplane length L_w of the vessel, and x is the horizontal coordinate measured towards the bow.

As part of the approximations carried out in the present analysis, estimates of a_{ij} and b_{ij} (frequency dependent) have been obtained by using published data of the coefficients for related reference configurations. All cross coefficients have been taken equal to zero except a_{34} , b_{34} which couple roll and sway in beam seas. Viscous effects are known to alter the damping coefficients from the predicted potential theory values, particularly for roll motions, and available experimental and theoretical results of drag

coefficients in an oscillatory flow past a flat plate have been used to estimate corresponding values of viscous damping coefficients for vessels (including the model) containing deep keels.

Slender Body Approximation (SBA)

The slender body approximation provides an alternative approximation procedure which is valid for shorter wave lengths of the order of the boat length. This depends on the beam/length ratio being small so that certain terms in the equations of motion which are proportional to higher orders of this ratio may be neglected. In this approximation, the actual Froude-Krylov forces are used in the RHS of Eqs. (1), rather than using Eqs. (2). Simplifications are made by neglecting certain terms in the LHS of Eqs. (1). The method is outlined by Newman (2).

Since mass or stiffness terms are neglected for various modes of motion, resonance behaviour is not predicted for most cases: that is, the resonant frequencies are assumed to occur outside the wave length (frequency) range considered. In order to predict the resonance features found for most modes of motion, an attempt has been made to include additional mass and stiffness terms beyond those used in the formal approximation.

Slack/Elastic Mooring Line Approximation

The non-linear analysis required for a slack/elastic mooring line can be idealized as that pertaining to a spring-mass-dashpot system with non-linear spring characteristics and subjected to a known (exciting) force. The spring constant due to the moorings is approximated to be a constant value (depending on the elasticity of the mooring) for positive vessel displacements and zero otherwise. The corresponding non-linear ordinary differential equation can be approximated as:

$$m\ddot{\xi} + r\dot{\xi} + s\xi = Fe^{i\omega t} \quad (4)$$

where m is the body mass (including added-mass), r is the damping constant, $s = 0$ for $\xi < 0$ and $s = c$ for $\xi > 0$, and c is the elastic constant of the mooring. This non-linear ordinary differential equation can be solved by a Ritz approximation procedure which is particularly simple when extracting the fundamental harmonic of the vessel displacement. The results may be expressed as

$$\xi = X e^{i\omega t} \quad (5a)$$

with the amplitude of the motion given by:

$$|X| = F \left\{ \left(\frac{c}{2} - m\omega^2 \right)^2 + r^2\omega^2 \right\}^{-1/2} \quad (5b)$$

and the phase relative to that of F given by:

$$\text{Arg}(X) = \tan^{-1} \left[\left(\frac{C}{2} - m\omega^2/r\omega \right) \right] \quad (5c)$$

These equations have been used to compute the vessel's surge and sway response to a slack/elastic moorage.

Moorage Conditions Analysed

Application of the different approximations described to different moorage arrangements has been accomplished by considering a series of specific cases, including head and beam seas, a freely floating boat, a linear mooring restraining surge only, a slack/elastic mooring for surge or sway only, and stern hinge links which restrain fore-and-after motions but allow the stern to move vertically. These cases are listed in Table 2. The computations have been carried out for four hull configurations corresponding to a deep fin sailboat, a full keel sailboat, a planing power boat and a displacement type power boat, as indicated in Fig. 1, and their principal characteristics are given in Table 1.

RESULTS

Model Test Results

The model test results are presented in Figs. 3 and 4 showing the response amplitude operators (RAO's) as functions of the wave length to vessel length ratio. The response amplitude operators are defined here as:

$$\text{RAO} = \left\{ \begin{array}{ll} \frac{\text{motion amplitude}}{\text{wave amplitude}} & \text{for surge, heave, sway} \\ \frac{\text{roll amplitude (rads)}}{2 H/B} & \text{for roll} \\ \frac{\text{pitch amplitude (rads)}}{2 H/L_w} & \text{for pitch} \end{array} \right.$$

where H is the wave height, B the beam and L_w the water plane length.

Results for both fixed and floating dock cases are given in Fig. 3 for head seas, and in Fig. 4 for beam seas.

There is some spread of the experimental data points which indicates either non-linear effects, a lack of consistency in the phenomena, or the degree of data taking and instrumentation accuracy. Evidently all three effects were present to some degree.

As mentioned previously, movie footage was taken of several tests with the transducer in place and with it removed. This data has also been plotted in Fig. 3(b) and it fell within the scatter of the transducer data indicating that the transducer did not grossly misrepresent the model motions. A complete discussion of the various trends indicated in the figures is given in Ref. 3.

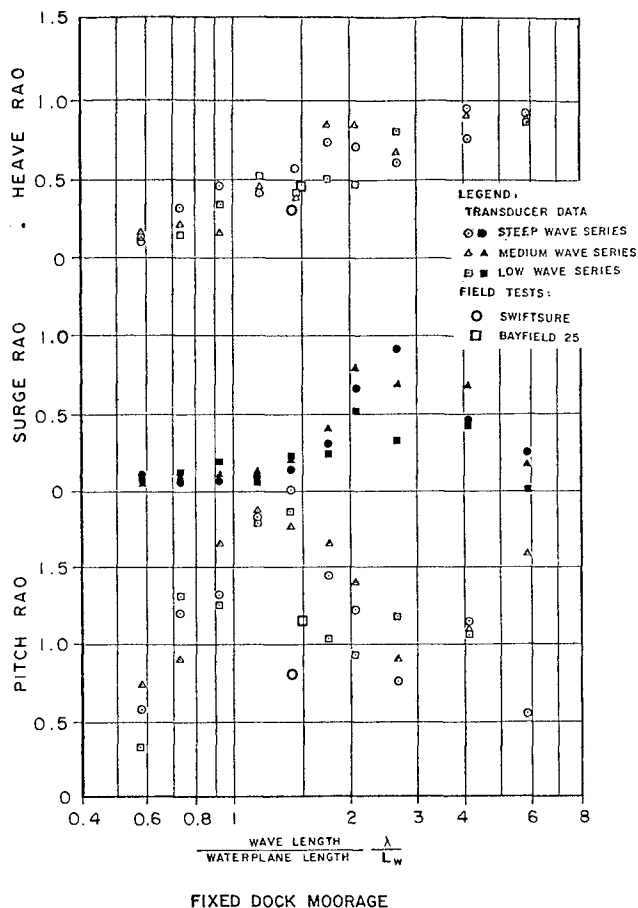


Fig. 3(a). Dimensionless model test results for head seas - Fixed dock moorage.

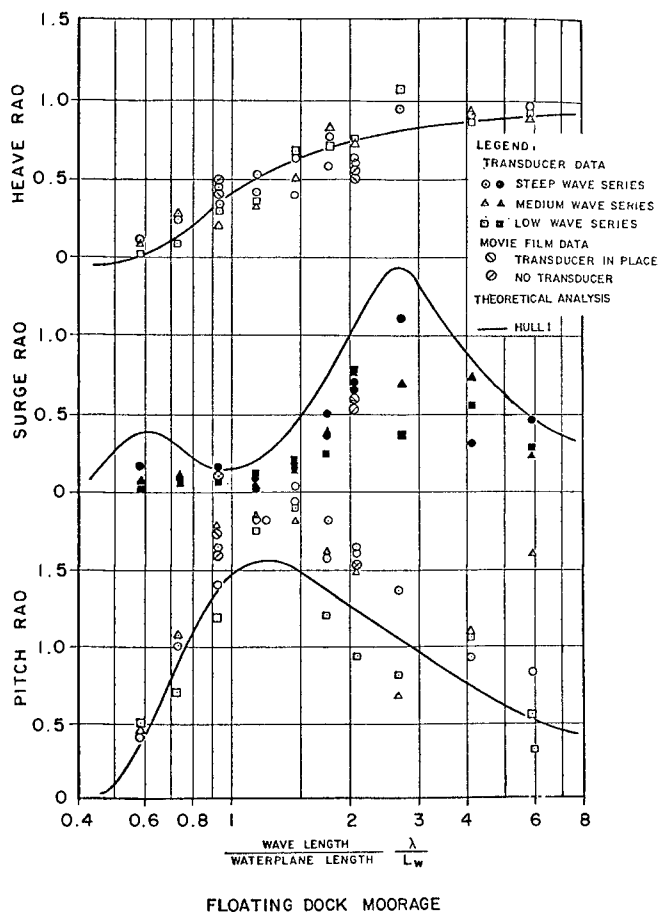


Fig. 3(b). Dimensionless model test results for head seas - Floating dock moorage.

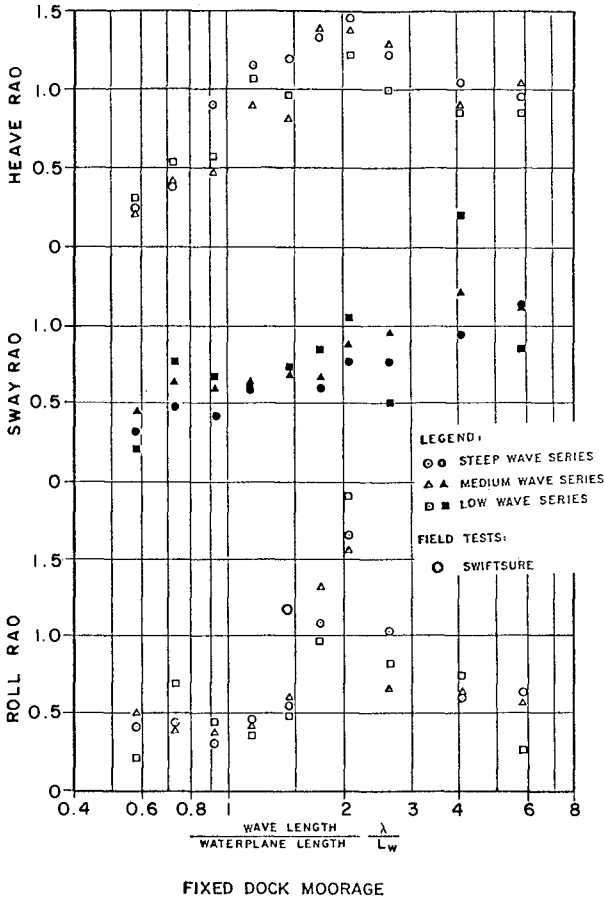


Fig. 4(a). Dimensionless model test results for beam seas - Fixed dock moorage.

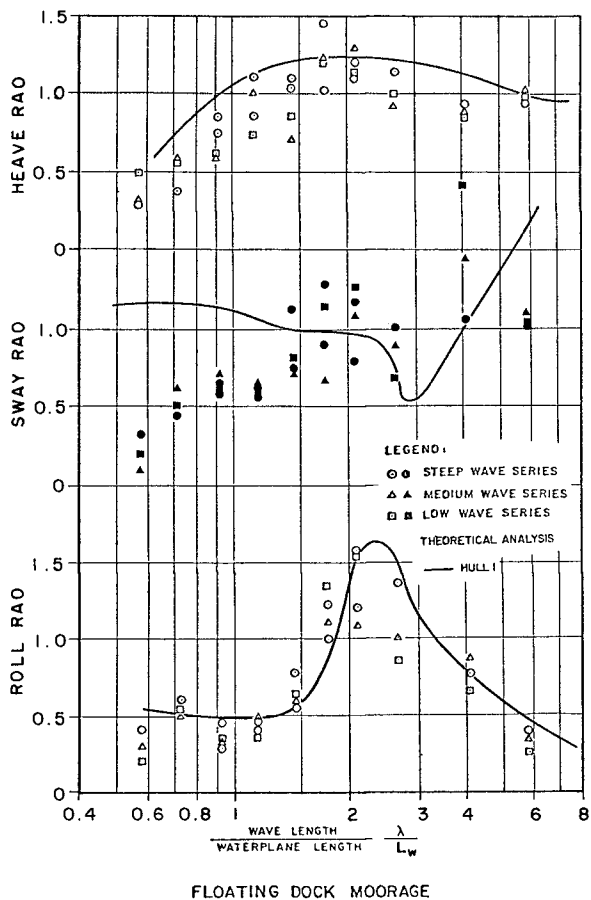


Fig. 4(b). Dimensionless model test results for beam seas - Floating dock moorage.

Field Test Results

The corresponding results for the Swiftsure and Bayfield vessels are superposed with the model test results in Figs. 3(a) and 4(a). The results for pitch are relatively low while those for heave and roll show good agreement with the model data.

Analytical Results

The analytical results for Hull 1 (the model vessel) based on the alternative approximations adopted are given in Fig. 5 for head and beam seas. For head seas, both heave and pitch equations are coupled so that the results are interdependent. For both heave and pitch the LWA produces results close to the SBA for longer waves but they diverge rapidly for shorter waves. In beam seas, sway and roll are also closely coupled and must be treated together. Although moorage constraint is important to sway and roll, the coupling make the constraint difficult to handle so that only free response curves are shown here.

For both head and beam seas, the SBA (modified) is found to provide a better representation of response than the LWA when compared with model results. Thus the SBA analytical results for Hull 1 are compared to the experimental data points in Fig. 3(b) for head seas and in Fig. 4(b) for beam seas. For head seas, the SBA plots will be seen to agree very closely with the measured data over the full range of wave lengths tested. Heave response is shown to be equal to the wave height for longer waves (greater than 5 times boat length) but diminishes for shorter waves until it reaches zero for wave lengths approximately one-half the boat length. Pitch response for longer waves (greater than 4 times the boat length) follows the pitch (slope) of the wave surface. It diminishes for shorter waves approaching zero for wave lengths about one half the boat length. Neither heave or pitch have a resonant condition in head seas under the SBA and this is supported by model data. The response in surge is more complex because surge is heavily constrained by the moorage lines, and free floating response results are expected to be grossly excessive.

For beam seas, the SBA also provides a better representation of response than does the LWA. The roll reaches at least 3 times the equivalent slope of the water surface. The frequency of roll at resonance is very close to the natural period of roll noted in still water. As with surge in head seas, the (free floating) analytical sway results are quite invalid because of the influence of the moorings. Heave response in a beam sea shows a broad resonance condition with boat response as much as 25 percent greater than the wave height. However, as with head seas the heave response ratio approaches unity for long waves and approaches zero for short waves.

Finally, Fig. 6 shows the analytical results based on the SBA for the four hull types indicated previously (Table 1 and Fig. 1). The curves show that responses are very similar except for those

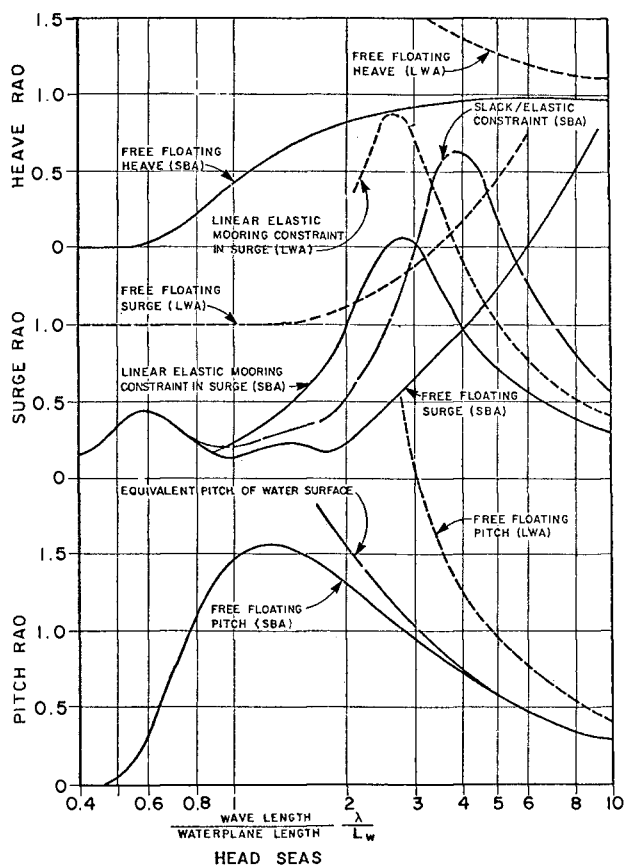


Fig. 5(a). Analytical evaluations of vessel response for Hull 1 - Head Seas.

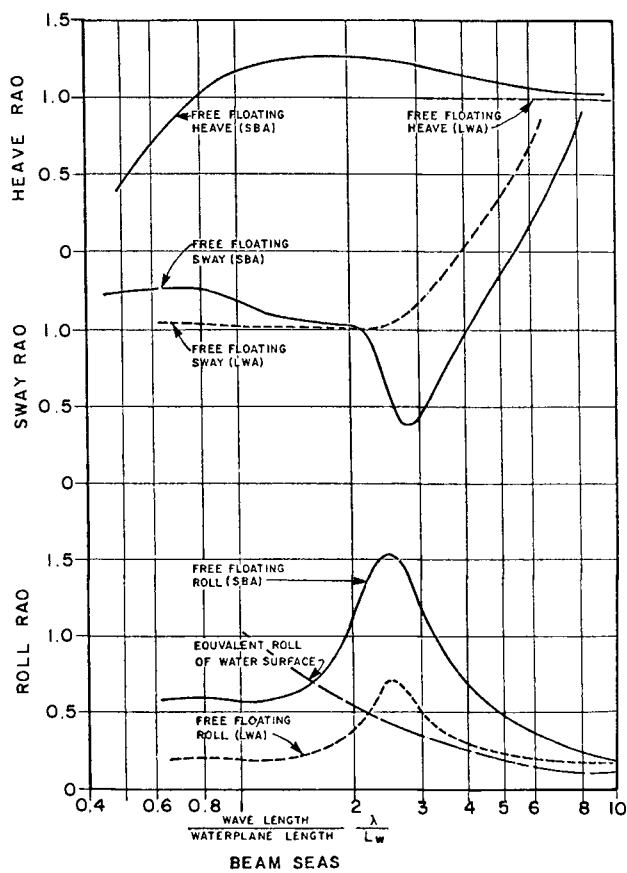


Fig. 5(b). Analytical evaluations of vessel response for Hull 1 - Beam Seas.

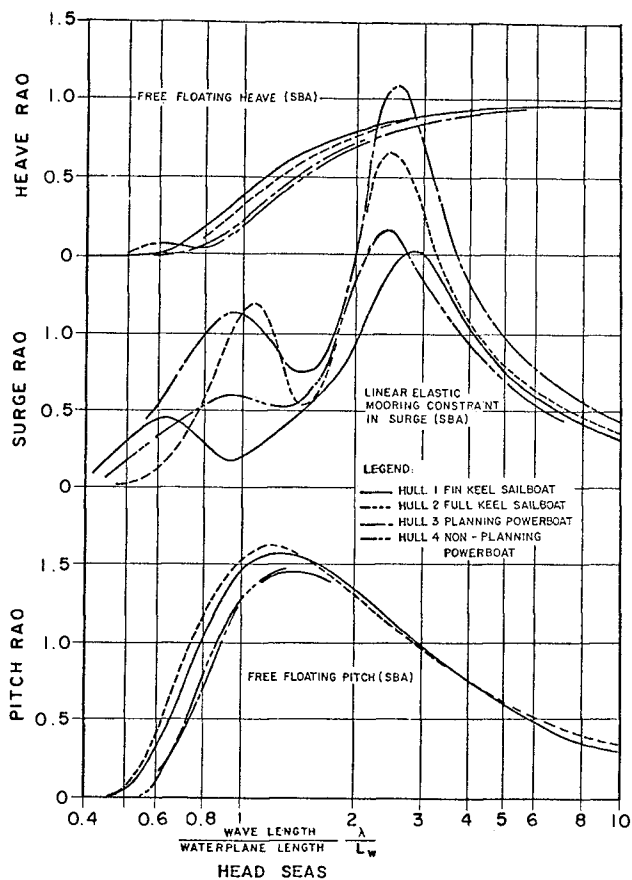


Fig. 6(a). Analytical evaluations of vessel response for four hulls - Head Seas.

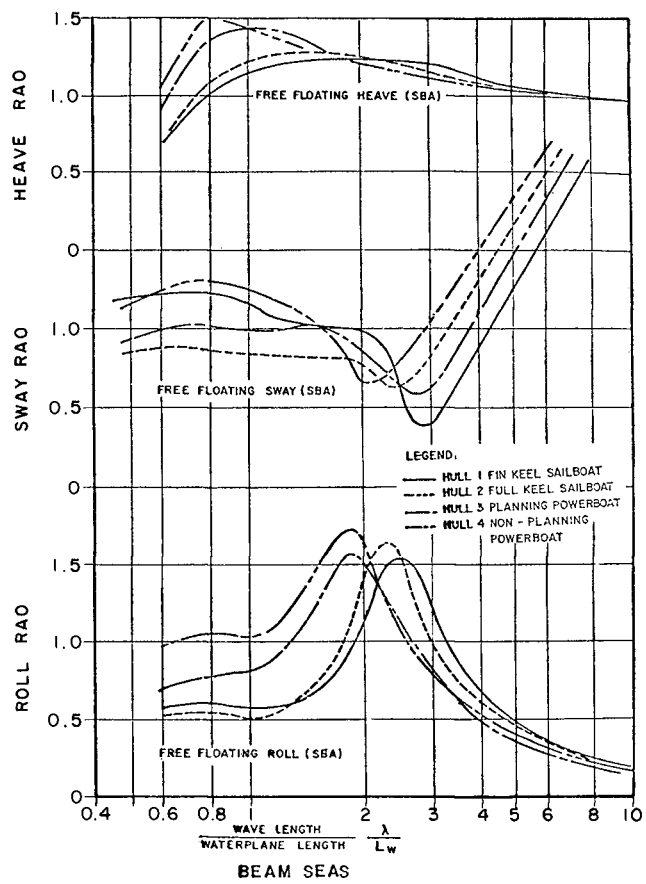


Fig. 6(b). Analytical evaluations of vessel response for four hulls - Beam Seas.

responses involving resonances which include heave in a beam sea and surge with linear elastic constraint. (Sway with moorage constraint also is subject to strong resonances but is not included here.) The resonance in heave is not strong so that the differences between hull types is not large. Nonetheless they are large in surge. Also the response values at the resonant peaks are not too reliable because they vary greatly with amount of damping that is present. As a result, the responses in heave, pitch and roll appear to be reasonably predictable but the responses in surge and sway can only be predicted with a sizeable degree of uncertainty.

CONCLUSIONS

The model test and analytical results have been compared and are generally quite adequate to show quantitatively the dependence of vessel response on wave height, period and direction with sufficient accuracy to enable improved criteria of acceptable wave climate for small craft to be established.

REFERENCES

1. Mercer, A.G., Isaacson, M. and Mulcahy, M.W. (1982). Design Wave Climate in Small Craft Harbours. Proc. 18th International Conference on Coastal Engineering, Cape Town, South Africa.
2. Newman, J.N. (1977). Marine Hydrodynamics, MIT Press.
3. Small Craft Harbours Branch, Fisheries and Oceans Canada (1980). Study to Determine Acceptable Wave Climate in Small Craft Harbours. Prepared by Northwest Hydraulic Consultants Ltd., Vancouver.