INTERFACIAL AND BED SHEAR STRESSES IN SALINE WEDGES

by

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ABSTRACT

The shear stresses and the associated friction coefficients at the interface and at the bed of an arrested saline wedge have been studied experimentally together with the detailed flow structure. Interfacial stresses, evaluated from hot film anemometer measurements and actual velocity profiles, agree well with those based on the integration of the equations of motion while the simplified one-dimensional analysis gave considerably higher values. It was found that both the average interfacial friction coefficient, $\bar{f}_I$ and the average bed friction factor, $\bar{f}_b$ are best correlated with the dimensionless number, $ReFr^2$, where $Re$ and $Fr$ are the Reynolds number and the non-densimetric Froude number of the flow respectively, and with the relative density difference, $\Delta \rho/\rho$. The results are presented in two families of curves with $\Delta \rho/\rho$ as a parameter. The scattering of data points is minimal and the agreement with the results of some previous laboratory investigations and field data is good.

1. INTRODUCTION

Shear stresses at the interface and at the bed of arrested or quasi-stationary saline wedges develop as a result of the flow pattern which is generated by the dynamic interaction of fresh and salt water. These stresses, noted by $\tau_I$ and $\tau_b$ respectively, as well as the associated coefficients, $f_I$ and $f_b$, have been extensively studied in the past not only for saline wedges but also for other types of two-layered stratified flows (3). In particular, the frictional resistance at the interface and the related flow dynamics constitute two of the most intricate and most important aspects of stratified flows. A reasonable evaluation of the interfacial friction may lead...
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to a quantitative model, since stratified flow systems are controlled by gravity, friction, and inertia forces; they can, therefore, be readily analyzed to various degrees of approximation provided that suitable expressions for the friction forces at the interface and at the solid boundary are introduced. Two typical examples of such models are the one-dimensional analyses for uniform density underflows (6) and for arrested saline wedges (13).

In an earlier paper the authors reviewed studies on stratified flows prior to 1978 (4, 10). It was found that results obtained through various theories and equations differed enormously, and that the scattering of experimental data points in certain studies was too wide for the formulation of any reliable universal law (7, 8). The discrepancies were attributed to the restricting assumptions and conditions of each study and to the limited range of variation of the governing parameters while the scattering of data points suggested either that an inadequate number of independent variables was taken into consideration or that these variables were not properly separated in the dimensionless correlation parameters for their effect on the friction coefficients to be sufficiently displayed. It, moreover, appears that turbulent exchange through the interface may take place to a degree sufficient for the generation of Reynolds stresses while the flow maintains its stratified appearance (11).

The authors, in their first attempt to derive a universal functional relationship for the interfacial friction coefficient, conducted extensive experimental investigations in a closed rectangular duct where fresh water was flowing over a quasi-stagnant salt water layer at various salinities (4, 10). The analysis of the test data was based on a one-dimensional steady uniform flow model similar to that developed by Schijf and Schoenfeld for open channels (13). Attempts to correlate \( f_1 \) with the Reynolds number, the densimetric Froude number, and the Keulegan number resulted, like in several earlier studies, in wide scattering with no consistent trend of variation. The Keulegan number is defined by the Eq.:

\[
\theta = \frac{1}{V} \left( \frac{\nu}{g \frac{\Delta \rho}{\rho}} \right)^{1/3}
\]

where \( V \) is the local average velocity in the moving layer, \( \rho \) is the density of the fresh water, \( \Delta \rho \) is the density difference between fresh and salt water, \( g \) is the acceleration of gravity and \( \nu \) is the kinematic viscosity assumed to be the same for both fluids.

The scattering was minimized and a consistent trend of variation was achieved when \( f_1 \) was plotted against the parameter \( Re Fr^2 \), where \( Re \) and \( Fr \) are the Reynolds number and the non-densimetric Froude number, with \( \Delta \rho / \rho \) as an independent parameter. The family of curves, thus obtained, agrees reasonably well with reanalyzed data obtained by other investigators and with one data point from the Mississippi river. In the present study, a similar correlation was achieved for both \( f_1 \) and \( f_2 \) for saline wedges. The interfacial shear stresses were evaluated both directly from hot film anemometer measurements and from
an integration of the equation of motion over each layer along the same line used by Keulegan (7, 8).

2. THEORETICAL BACKGROUND

The interfacial friction coefficients, \( f_I \), have been evaluated by the following four different approaches:

2.1 Direct Measurement Approach

This approach was based on directly measured velocity profiles and Reynolds stresses from which the interfacial shear stress was computed by the equation:

\[
\tau_I = \mu \frac{d u}{d y} - \rho u'v' \tag{2}
\]

In this equation, \( u \) is the local temporal mean velocity, \( y \) is the direction normal to the flow direction, \( x \), \( u' \) and \( v' \) are the instantaneous turbulent velocity components in the \( x \) and \( y \) directions respectively, and \( \mu \) is the dynamic viscosity. From Eq. 2 the local interfacial friction factor, \( f_I \), is defined as:

\[
f_I = \frac{8\tau_I}{\rho V_f^2} \tag{3}
\]

where \( V_f \) is the average fresh water velocity (Fig. 1).

2.2 Integration of the Equations of Motion

In this approach the equations of motion were integrated for each layer in the vertical and in the horizontal directions over the salinity intrusion length.

The general two-dimensional equations of motion and continuity for steady flow have the forms:

\[
\begin{align*}
\frac{3u}{3x} + v \frac{3u}{3y} & = -g \frac{dz}{dx} - \frac{1}{\rho} \frac{3p}{3x} + \frac{1}{\rho} \left( \frac{3\tau_{xk}}{3x} + \frac{3\tau_{yk}}{3y} \right) \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} & = 0
\end{align*}
\tag{4}
\]

and

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}
\]

where, referring to Fig. 1, \( u \) and \( v \) are the local temporal mean velocities in the \( x \) and \( y \) direction respectively, \( z \) is the horizontal direction normal to \( x-y \) plane, \( z_0 \) is the bed elevation and \( \tau_{x} \) and \( \tau_{y} \) are the shear stresses in the direction \( x \), acting on planes normal to \( z \) and \( y \) directions respectively.
A combination of Eqs. 4 and 5 leads to:

\[ \frac{\partial^2 u}{\partial x^2} + \frac{3(uv)}{\partial y} = -g \frac{\partial z}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{3 \tau}{z} \frac{\partial z}{\partial x} + \frac{3 \tau}{y} \frac{\partial z}{\partial y} \right) \]  \hspace{1cm} (6)

The last equation is first integrated over the cross sectional area of each layer separately and subsequently over the length of intrusion. In the second integration two cases had to be considered: 1) the case where the depth near the salt water basin is critical and the salinity intrusion length attains its maximum value, \( L_4 \), and 2) the case where the fresh water depth near the basin is larger than critical, whereas the intrusion length, \( L_0 \), is less than \( L_4 \). In all experiments a layer of fresh water was formed over the salt water basin. As long as the depth of that layer was smaller than the critical depth of fresh water at the channel entrance, it had no effect on the salinity intrusion which attained its maximum length, \( L_4 \); otherwise, the actual intrusion length was equal to the distance from the toe of the wedge to the point where the fresh water depth is equal to that over the basin.

2.2.1. Complete intrusion.

2.2.1.1 Fresh water layer

The pressure distribution in that layer is given by

\[ p = p_0 + pg(h - y) \]  \hspace{1cm} (7)

where \( h = h_0 + h \) (Fig. 1) and where the vertical direction \( y \) is measured from the bottom. Eq. 5 is next integrated over the cross sectional area \( bh_0 \). Setting:
where \( a \) is the momentum flux correction coefficient, neglecting the velocity variation in the \( z \) direction and taking into consideration the Leibnitz rule, integration of the first term of the left hand member yields:

\[
\frac{b}{2} \int_{h}^{b} \frac{2}{h} dy = \frac{a}{2} bh_{1} \frac{dV_{1}}{dx} - bU^{2} \frac{dh}{dx} + bV_{i}^{2} \frac{dh_{s}}{dx}
\]

where \( U \) and \( V_{i} \) are the velocities at the free surface and at the interface, respectively. Likewise:

\[
\frac{b}{2} \int_{h}^{b} \left[ \left( \frac{3}{2} \right) \frac{dV_{1}}{dx} \right] dy = bU^{2} \frac{dh}{dx} - bV_{i}^{2} \frac{dh_{s}}{dx} - bV_{s}
\]

where \( V_{s} \) is the net rate of salt water entrainment. The vertical velocity at the free surface is:

\[
v_{h} = U \frac{dh}{dx}
\]

and the same velocity at the interface was taken as:

\[
v_{hs} = V_{i} \frac{dh}{dx} + V_{s}
\]

The integrated terms of the right hand member of Eq. 6 are:

\[
\frac{b}{2} \int_{h}^{b} \frac{dx}{h} \int_{h}^{b} \frac{dx}{h} dy = bh_{i} \frac{dS_{0}}{dx}
\]

\[
\frac{1}{\rho} \int_{h}^{b} \frac{dx}{h} \int_{h}^{b} \frac{3g dz}{dx} dy = bh_{i} \frac{dh}{dx}
\]

\[
\frac{1}{\rho} \int_{h}^{b} \frac{dx}{h} \int_{h}^{b} \frac{\frac{3g}{\rho} \frac{x z}{\frac{3g}{\rho}}}{\frac{3g}{\rho}} dz = \frac{2h_{i} \tau_{w}}{\rho}
\]
where \( \tau_w \) is the shear stress at the sidewalls. Eq. 6 obtains thus the form:

\[
\frac{1}{\rho} \int_{-\frac{b}{2}}^{\frac{b}{2}} dz \int_{h_s}^{h} \frac{3\tau_{\text{ex}}}{\rho} dy = -\frac{1}{\rho} b \tau_1
\]  

(16)

where \( \tau_1 \) is the shear stress at the sidewalls. The entrainment velocity is equal to:

\[
V_s = \frac{q_s}{\rho V_1}
\]  

(18)

where \( q_s \) is the rate of salt water entrainment over the saline wedge. The latter has been given by Keulegan (8). In the analysis of data, the term \( V_s \rho V_1/\rho g h_1 \) was found to be of minor importance with a maximum effect of 5%. Because of this reason and of the fact that entrainment takes place not continuously over the interface, but at isolated spots in the form of breaking of interfacial waves, it was neglected in the subsequent analyses. Expressing, next, \( \tau_w \) by:

\[
\tau_w = f \frac{\rho V_1^2}{8}
\]  

(19)

the following equation for \( \tau_1 \) is obtained:

\[
\frac{\tau_1}{\rho} = -g(h - h_s) \frac{dH}{dx} - \frac{a}{2} (h - h_s) \cdot \frac{dV_1}{dx} - \frac{f \rho V_1}{4b} (h - h_s) V_1^2
\]  

(20)

which, integrated for \( -L_1 \) to 0, gives:

\[
\frac{\tau_1}{\rho V_2^2} = -g \frac{h_2}{\rho} \frac{dH}{dx} \tau_1 - \frac{a h_2}{2 L_1} \tau_1 - \frac{f \rho \omega h_2}{4b} \tau_1
\]  

(21)

where \( \tau_1 \) and \( \frac{dH}{dx} \) are the average values of \( \tau_1 \) and of \( \frac{dH}{dx} \) over the wedge and where \( I_1, I_2 \) and \( I_3 \) are the following dimensionless functions:
For the evaluation of these integrals, the following dimensionless equations for the interface, developed by the authors and Mehta and based on Schijf and Schoenfeld's model (11, 13), was used:

\[
\frac{x}{L_1} = \frac{\left(\frac{h_s}{h_o}\right)^3 \left[4 - \frac{2 h_s}{h_o} \left(1 - (F'_o)^{2/3}\right)\right]^2 + \left[4 + \frac{h_s}{h_o} \left(1 - (F'_o)^{2/3}\right)\right]}{6(F'_o)^{4/3} + 3(F'_o)^{2/3} + 1}
\]

\[
- \frac{10\left(\frac{h}{h_{sc}}\right)^2 \left[(F'_o)^{4/3} + (F'_o)^{2/3} + 1\right]}{6(F'_o)^{4/3} + 3(F'_o)^{2/3} + 1} + 1
\]

where \(F'_o = V_0 \sqrt{\frac{g}{(\rho_p/\rho)}} h_o\) is the densimetric Froude number with \(V_o\) and \(h_o\) defined in Fig. 1. The three integrals obtain thus the form:

\[
I_1 = \frac{2 \left[5(F'_o)^2 + 6(F'_o)^{4/3} + (F'_o)^{2/3} + 1\right]}{3 \left[6(F'_o)^{4/3} + 3(F'_o)^{2/3} + 1\right]}
\]

\[
I_2 = 2 \left[(F'_o)^{-2/3} - 1\right]
\]

\[
I_3 = \frac{0.29 \left[5 + 255(F'_o)^{13/6} - 221(F'_o)^{2} - 39(F'_o)^{17/6}\right]}{\left[1 - (F'_o)^{2/3}\right] \cdot \left[6(F'_o)^{4/3} + 3(F'_o)^{2/3} + 1\right]}
\]
2.2.1.2 Salt water layer

A similar integration of Eq. 6 over the cross sectional area of the wedge, \( A_s = bh_s \), with the pressure this time described by the equation:

\[
p = p_o + (h - h_s)\rho_g + g(\rho + \Delta \rho)(h_s - y)
\]  

(29)
gives:

\[
\frac{\gamma_1 + \rho_o}{g(\rho + \Delta \rho)h_s} = \frac{\rho}{\rho + \Delta \rho} \cdot \frac{dH}{dx} + \frac{\Delta \rho}{\rho + \Delta \rho} \cdot \frac{d(h_s + z_o)}{dx} + \frac{\beta}{gh_s} \frac{d(V_1h_s)}{dx} + \frac{V_1h_s}{gh_s} = 0
\]

(30)

where \( \beta \) is the momentum flux correction coefficient for the lower layer given in terms of \( V_1 \) by:

\[
gV_1^2 = \int_{A_s} u^2 dA
\]

(31)

and where the sidewall friction has been neglected since, due to the reversal of flow direction, its overall effect is expected to be negligible. Integration of Eq. 30 from \(-L_1 \) to 0 gives:

\[
\frac{T_1}{\rho V_0^2} + \frac{T_o}{\rho V_o^2} = \frac{gh_o}{\rho V_0^2} \frac{dH}{dx} I_4 + \frac{\Delta \rho}{\rho V_o^2} \frac{h_s^2}{2V_o^2 L_4} + \frac{\rho \Delta \rho h_o}{\rho V_o^2} \frac{\frac{dz}{dx} I_4}{V_o} + \beta(1 + \frac{\Delta \rho}{\rho}) \left( \frac{V_1h_s}{V_o} \right)^2 \cdot \frac{h_s}{I_4}
\]

(32)

where \( T_o \) is the average bed shear stress over the length of intrusion, \( \frac{dz}{dx} = -S \) is the average bottom slope, \( h_{sc} = h - h_s \) is the critical depth of the fresh water layer, and \( \epsilon = \frac{V_1h_s}{V_o} \). The term \( \frac{V_1h_s}{gh_s} \) has again been neglected for the aforementioned reasons while the integral \( I_4 \) is:

\[
I_4 = \int_{-L}^{0} \frac{h_s}{h_o} d\left( \frac{x}{L_2} \right)
\]

(33)

which, on the basis of Eq. 25, becomes:
The analysis presented so far is similar to the one used by Keulegan (7, 8) with the following differences: 1) A more detailed analytical equation for the wedge profile has been used instead of one single experimental curve and, 2) the actual measured values of $\alpha$ and $\beta$ for each experiment were used, instead of the constant values of $\alpha = 1.028$ and $\beta = 0.146$ used by Keulegan.

2.2.2 Partial intrusion

In several experiments, the fresh water layer at the downstream basin was larger than $h_f$ - in which case the salt water depth at the downstream end of the channel, $h_0$, was smaller than $h_f$ (Fig. 1). In these cases the following simplified form of Eqs. 21 and 32 was derived and used: First, Eqs. 20 and 30 are written in the form:

$$\tau = \frac{gh_1}{\rho V_o} \frac{dh}{dx} - \frac{ah_1}{V_o} \frac{V_1}{dx} - \frac{f_w h_1 V_1^2}{4bV_o^2}$$

$$\frac{\tau}{\rho V_o} = \frac{gh_1}{\rho V_o} \frac{dh}{dx} + \frac{\Delta p}{\rho \gamma} \frac{gh_1}{\rho V_o} \frac{dh}{dx} + \frac{\beta (1 + \frac{\Delta p}{\rho \gamma})}{\rho \gamma} \frac{2}{V_o} \frac{d(\frac{V^2}{2} h_1)}{dx}$$

where $\frac{dz}{dx}$ in Eq. 32 has been neglected as much smaller in comparison to $\frac{dh}{dx}$ and where the wall friction coefficient $f_w$ has been introduced by the Blasius equation:

$$f_w = \frac{0.316}{Re^{0.25}}$$

where the Reynolds number, $Re$ is:

$$Re = \frac{4V_1 R_h}{\nu}$$

and

$$R_h = \frac{bh_1}{(b + 2h_1)}$$

Eq. 35 obtains thus the form:

$$\frac{\tau}{\rho V_o} = \frac{gh_1}{\rho V_o} \frac{dh}{dx} - \frac{ah_1}{V_o} \frac{V_1}{dx} - \frac{0.0559 h_1^{0.25}}{b^{0.25} \left( \frac{V_1}{V_o} \right)^{b_{0.25}}} \left[ \frac{(b + 2h_1)^{0.25}}{h_1} \right]$$
Averaging of Eqs. 36 and 40 over the actual length of salinity intrusion, \( L \), as in the case of full intrusion, but considering \( h \) and \( h_s \) constant and equal to their average values \( \bar{h} \) and \( \bar{h}_s \), leads to the following equations:

\[
\frac{T_i}{\rho v_o^2} = \frac{g h_i}{h_i} \frac{\partial h}{\partial x} - \frac{h_i}{h_{i0}} \frac{\partial h_s}{\partial x} - 0.0559 \left( \frac{\rho v_o}{\bar{h}} \right)^{0.25} \left[ \frac{h}{h_i} \right]^{0.25} \left( \frac{b + 2h_i}{h_i} \right)^{0.25} \tag{41}
\]

and

\[
\frac{T_o}{\rho v_o^2} = \rho v_o \frac{\partial h_s}{\partial x} + \frac{\Delta h}{\rho} g h_{i0} \left( \frac{\rho v_o}{\bar{h}} \right)^{2} \left( \frac{h_s}{h_{i0}} \right)^{2} + \frac{\beta}{2} \frac{1 + \Delta h}{\rho} \left( \frac{\rho v_o}{\bar{h}} \right)^{2} \left( \frac{h_s}{h_{i0}} \right)^{2} \tag{42}
\]

where bars indicate average values over the length \( L_0 \). For a direct evaluation of the average stresses \( T_i \) and \( T_o \), given either by Eqs. 21 and 32 or by 41 and 42, a reliable measurement of the average slope of the piezometric line, \( \frac{\partial h}{\partial x} \), is required - as was done earlier by Keulegan (7, 8). This has been found to be very difficult, since the head loss over the effective length of the flume is of the order of a few tenths of a millimeter. Attempts to measure that head loss optically through a laser beam required an excessively long time of measurement, due primarily to the slowness of achieving equilibrium in the two static tanks, and to the very long period of oscillation of the water level in these tanks. Moreover, the above equations are quite sensitive to \( \frac{\partial h}{\partial x} \) so that a measurement error of the order of 10% may cause an error in \( T_i \) of the order of 50% (5). This difficulty was bypassed by introducing a linear relationship between \( T_i \) and \( T_o \).

Indeed, the shear stress distribution within the wedge is given by the equation.

\[
\frac{3 \pi}{3 y} = \frac{3}{3 x} \left[ p + g(\rho + \Delta h)z_o + (\rho + \Delta h)\frac{y^2}{2} \right] \tag{43}
\]

The velocity head is very small and can be neglected. Eq. 29 then, for \( p \) gives:

\[
\frac{3 \pi}{3 y} = \rho g \frac{\partial h}{\partial x} + (\Delta h)g \frac{3(h_s + z_o)}{\partial x} \tag{44}
\]

The free surface slope, \( \frac{\partial h}{\partial x} \), is very small, as discussed, and can be considered as constant. Also, apart from two relatively short segments around the wedge toe and the critical section, the interface is very nearly a plane surface, and \( \frac{\partial (h_s + z_o)}{\partial x} \) is also very nearly constant. Therefore, it is reasonable to assume linear variation of \( \tau \) with \( y \) within the wedge. It follows thus from Fig. 1 that:
where \( h_1 \) is the distance from the interface to the point of maximum velocity in the wedge. It was assumed next that \( \lambda \) is constant over the wedge — in which case Eq. 45 holds also for the average stresses \( T_1 \) and \( T_0 \), that is:

\[
\frac{T_1 + T_0}{T_1} = \frac{h_s}{h_{s1}} = \lambda
\]  

(45)

Eliminating \( \frac{dH}{dx} \) from Eqs. 41 and 42 and using the last relationship, the following equation is obtained:

\[
\frac{T_1}{\rho V_0^2} = \left[ \frac{\left(\frac{\Delta \rho}{\rho}\right) g h_{80} + 2B \left(1 + \frac{\Delta \rho}{\rho}\right) e^{2V_1} h_{10} h_{1} - 2\Delta h V_{80} V_0^2}{2V_0 h_{10} (\lambda h_1 + h_s)} \right] \frac{h_{80}}{L_0} + \\
\frac{0.0559}{b} h_0^2 \left( \frac{V}{V_0 h_0} \right)^{0.25} \cdot \left[ \frac{(b + 2h_1)}{h_1} \right] \cdot \frac{h_s}{(\lambda h_1 + h_s)}
\]  

(47)

All the terms of the right hand member of Eq. 47 are either known or can be directly measured. The maximum measurement error of \( \lambda \) was estimated to be about 15% (5). With \( T_1 \) computed from Eq. 47, \( T_0 \) is readily evaluated from Eq. 46. \( \frac{dH}{dx} \) could be eliminated in the same way from Eqs. 21 and 32, valid for complete intrusion. However, after a few trials for the latter case, it was found that Eqs. 41 and 42 could be used for both partial and complete intrusion. These equations have been used for the analysis of all data.

2.3 One-dimensional Energy Equation

This equation, developed by Schijf and Schoenfeld (13), is based on the energy conservation principle applied separately to each layer and on the assumptions of one-dimensional flow, infinite width, and negligible entrainment through the interface. For zero net flow within the wedge, the average velocity is also zero and, therefore, for the one dimensional approach, \( \tau_0 \) has to be taken also as zero. The final form of the equation is:

\[
\frac{\tau_1}{L_1} = \frac{2h_0}{L_1} \left[ \frac{1}{5(F_0')^2} - 2 + 3(F_0')^{2/3} - \frac{6}{5} (F_0')^{4/3} \right]
\]  

(48)

which gives the average interfacial friction factor if the total intrusion length is measured. If the actual intrusion length, \( L_0' \), is less than \( L_1 \), the latter can be computed graphically from Eq. 25, where \( h_{sc} \) is known and equal to \( h_0 [1 - (F_0')^{2/3}] \) (11),
2.4 Simplified Equations of Motion

This is a combination of the last two approaches and its main objective was to investigate the effect of the assumption of zero bed shear stress and zero wall stress, used in the one-dimensional model, on the interfacial shear. For $\tau_0 = 0$, $\lambda = 1$, $f \omega = 0$ and $\beta = 0$, Eq. 47 reduces to the approximate form:

$$\frac{\tau_1}{\rho \nu_0} \left[ \frac{(\Delta P) \bar{gh}_0 h_{so}}{2 \nu_0^2 h_0} - \frac{\bar{h}_s}{2 h_{io}} \right] = \frac{h_{so}}{l_o} \tag{49}$$

3. BASIC RESEARCH EQUIPMENT

The experiments were conducted in a variable slope plexiglass flume 20 m long, 46 cm wide, and 46 cm deep, constructed at the Coastal Engineering Laboratory of the University of Florida. This flume, outlined in Fig. 2, discharges into a 3 m by 1.53 m tank supported on another 3 x 3 x 0.75 m tank.

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Fig. 2. Outline of basic experimental apparatus.

The channel is connected at its upstream end to a 2.31 m by 0.45 m head tank where the fresh water is pumped into from the fresh water tank and where turbulence dampens out through a honeycombed section. The water circulation system consists of an axial flow pump, of two pipes with either one or both in operation, depending on the required discharge, and of all the necessary orifice meters, valves, gates, pressure taps and manometers. The maximum discharge is about 115 l/sec.

The channel and head tank are supported on a steel frame 1.2 m high, which rests on two hydraulic jacks – one at the center and the other at the upstream end of the flume. Through a gear-chain system, the central jack could be moved up and down at a distance equal to $\frac{1}{2}$ that of the upstream jack. The $\frac{3}{4}$-inch thick plexiglass wall of the flume forms a channel with its flanges, fortified with stiffeners, directed outwards. To this steel frame, rails have been placed on
each of the upper two flanges for the support and smooth movement of
the instrument carriage. The downstream end of the channel is
connected to the salt water tank through a five centimeter long thick
rubber forming a flexible and waterproof hinge. A water-tight
vertical plexiglass gate separates the flume from the salt water
tank. That gate can be raised and lowered mechanically by a vertical
threaded shaft and a circular threaded disc supported on an
independently fixed frame. A second gate at the downstream end of the
salt water tank regulates the water level in the latter. The outlined
system of flume and tanks is supplemented with the necessary pipes and
valves for proper operation and control.

The measuring instruments - that is, the hot film anemometers, a
specially designed salinity probe, and the point gages - are supported
on the aforementioned carriage, which is collapsable sidewise to
permit its movement over the cross bars of the flume without removing
the instruments.

4. EXPERIMENTAL RESULTS

After the necessary preparations and instrument calibration, each
experiment was started and continued to a point where reasonable
steady state conditions were established. Entrainment caused a
gradual erosion of the salt water layer as a result of which salt
water had to be periodically supplied from the preparation tank to the
salt water basin. Space limitations do not permit a description of
the detailed procedures of experimentation and calibration, which can
be found in ref. (5) and in a forthcoming report. The measurements
took place immediately after the steady state was achieved and lasted
for about one-half hour, during which no measurable change in the flow
conditions occurred. The primary measurements consisted of velocity
and density profiles - with points more densely located around the
interface. The instantaneous velocities were recorded on tapes from
which the Reynolds stresses were subsequently computed. A total of 74
experiments were conducted with fully developed turbulent flows. The
relative density difference, \( \Delta \rho / (\rho + \Delta \rho) \), in these experiments ranged
from 0.008 to 0.093. In such flows the velocity distribution in the
neighborhood of the interface was found to be logarithmic and quite
similar to that near solid boundaries. This is in general agreement
with results from earlier studies (2, 9) but with major differences in
certain important aspects. The velocity distribution laws and the
flow structure around the interface is the subject of another
forthcoming paper. It was found that the salinity distribution around
the interface follows closely the law:

\[
\frac{\rho - \rho_s}{\Delta \rho} = \frac{1}{2} \left[ 1 - \tanh \left( \frac{y - h}{8\delta} \right) \right]
\]  (50)

where \( \rho_s \) is the local density, \( y \) is the distance from the bed, \( h \) is
the depth of the density interface, and \( \delta \) is the thickness of the
interfacial zone, as defined in Fig. 3.
As determined in an earlier study by the authors (4, 10), it was again found that the best correlation is obtained if $\varphi_1$, determined by any one of the outlined four methods, is plotted against the dimensionless number $\text{ReFr}^2$ with $\Delta \rho/(\rho + \Delta \rho)$ as an independent parameter, where:

$$\text{Re} = \frac{4V_1 R_h}{\nu}, \quad \text{Fr}^2 = \frac{V_1^2}{gR_h}$$  \hspace{1cm} (51)

and where $R_h$ is the hydraulic radius of the fresh water layer. The parameter $\text{ReFr}^2$ can be written in the form:

$$\text{ReFr}^2 = \frac{4V_1^3}{g \nu} = \frac{4V_1^3}{g \nu} \frac{I_R}{g \nu}$$  \hspace{1cm} (52)

where:

$$I_R = \frac{10}{6(F_0')^{4/3} + 3(F_0')^{2/3} + 1}$$  \hspace{1cm} (53)

In Fig. 4 the results for four of the eight ranges of $\Delta \rho/(\rho + \Delta \rho)$ computed by all four approaches are shown with the average value of $\Delta \rho/(\rho + \Delta \rho)$ for each range indicated at the insert. The results from all four approaches can thus be compared. Here the interfacial friction coefficient is defined either as:

$$\varphi_1 = \frac{8\sigma_1}{\rho V_1^2}$$  \hspace{1cm} (54)
with $T_i$ computed from Eq. 47 for the second approach and from Eq. 49 for the fourth approach. In the third approach $I_i$ was evaluated from Eq. 48. The comparison of these four sets of data leads to the following conclusions:

First, as in the earlier study by the authors (4, 10), the scattering of data points around each curve is minimal for a flow of this type. Secondly, there is a very good agreement between the data obtained from the first two approaches—that is, between the data based on direct shear stress measurements and those based on the integration of the equations of motion. These data constitute the lower family of curves. Thirdly, there is an equally good agreement between the results based on the two approximate approaches and which constitute the upper family of curves (dotted lines). Finally, a comparison of the two families of curves indicates that both approximate approaches give values of $f_i$ considerably higher than the corresponding values obtained either by direct measurements or through the equations of motion. This difference is indicative of the effect of neglecting the bed shear stresses. It follows that $f_i$, as determined from Eq. 48 through calibration, that is, by measuring $I_i$ either in the laboratory or in the field (11), can be used as a convenient virtual friction coefficient for modeling purposes; it is not representative of the true shear stress at the interface, however.

The above conclusions hold also for the case of $\Delta \rho/(\rho + \Delta \rho)$ equal to 0.0225, 0.0355, 0.0485 and 0.082. For this reason the data for these cases are not reproduced in detail; instead, the summary of all eight curves based on the first and second approach is given in Fig. 5.

In Fig. 6 the curve corresponding to $\Delta \rho/(\rho + \Delta \rho) = 0.0225$ has been replotted together with reanalyzed laboratory data by Abraham and Eysink (1) and with one field data point from the Mississippi estuary (11). The agreement is quite good. This suggest that until the curves of Fig. 5 are implemented with field data, they can be tentatively extrapolated following the extrapolation of the 0.0225 curve of Fig. 6.

In Fig. 7, the results for $\Delta \rho/(\rho + \Delta \rho) = 0.0225$ are compared with recent results obtained by Powell for flow of fresh water over a semistagnant deep water layer in an open conduit and for a relative density difference close to 0.024 (12). When plotted against the Reynolds number only, these two sets of data deviate strongly (Fig. 7a); however, they appear to closely agree when they are plotted against $ReFr^2$. In fact, the drop of the curve for low values of $ReFr^2$ agrees with the trend observed in the authors' experiments. Thus, Powell's point for $Re=22000$, which in his original diagram (Fig. 7a) seems to be in error, is in fact in agreement with the general law derived from this study.
Fig. 4. Interfacial friction coefficients obtained from all four approaches.

Fig. 5. Summary of interfacial friction coefficient curves based on approaches (1) and (2).
The bed friction coefficient, $f_o$, defined as:

$$f_o = \frac{8T_o}{\rho V_1^2}$$

has been determined from Eq. 46 where $T_o = \tau_o$ was computed from Eq. 54 for values of $f_o$ taken from Fig. 5. The value of $\lambda$ was determined for each experiment and the results are shown in Fig. 8. $\lambda$ appears to be a function of the number $ReFr^2$ only while for values of the latter smaller than 100, it is essentially constant and equal to about 1.4. The results are plotted in Fig. 9 as a family of curves similar to the
one of Fig. 6. It is observed that, contrary to conclusions of earlier studies (7, 8), \( f_0 \) is clearly smaller than the corresponding \( f_1 \).

![Graph showing \( \lambda \) vs. ReFr\(^2\)]

\[ \gamma = \frac{\tau_1}{\tau_0} \text{ with ReFr}^2 \]

![Graph showing Bed friction coefficients, \( f_0 \)]

5. SUMMARY AND CONCLUSIONS

Laboratory experiments and theoretical analyses led to a new relationship linking the friction coefficients at the interface and at the bed of arrested saline wedges to the pertinent flow parameters and fluid properties. The relationships are presented graphically as a family of curves with ReFr\(^2\) as an abscissa and the relative density difference, \( \Delta \rho / (\rho + \Delta \rho) \), as an independent parameter. The scattering of data points was minimal and much lower than in earlier studies on the subject. In the present correlations the number ReFr\(^2\) incorporates the friction, gravity, and inertia forces, which control the main flow, while the buoyant forces, which are responsible for the
density stratification and the associated flow pattern have been separated and introduced through the parameter \( \Delta p/(\rho + \Delta p) \).

This study indicated that values of \( f^* \) evaluated through an integration of the equations of motion closely agree with values based on measured velocity profiles and Reynolds stresses. Values of \( f^* \) obtained from the simplified and frequently used Schijf-Schoenfeld's one-dimensional analysis were shown to be higher than corresponding values based on the previous two methods.

ACKNOWLEDGEMENTS

This study has been supported by the U.S. National Science Foundation under Grant No. ENG 76-11537-A01. Travel funds were provided to the second author by the same institution under Grant No. CEE - 8214607 for presentation of this paper before the 18th International Conference on Coastal Engineering in Cape Town, South Africa. Both supports are gratefully acknowledged.

REFERENCES


