IRREGULAR WAVE TESTS FOR COMPOSITE BREAKWATER FOUNDATIONS

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ABSTRACT

The stability of armor units for the rubble mound foundations of composite breakwaters has been investigated under the action of irregular waves. The tests establish that irregular waves are more destructive than regular waves, when the height of regular waves is set equal to the significant wave height.

The stability number, defined by Hudson, for quarry stones and concrete blocks with simple shapes is formulated on the basis of irregular wave tests. The stability number is expressed by two parameters of $h'/H_{1/3}$ and κ, where h' is the crest depth of the rubble mound foundation, $H_{1/3}$ is the design significant wave height, and κ is a parameter for the combined effects of the relative water depth and the relative berm width of the rubble mound foundation to the wavelength.

The design mass of armor units can be calculated by the stability equation with the stability number. The application of the proposed method to the results of the irregular wave tests demonstrates that the damage percent for the quarry stones is at most 3.5% at the design condition and the damage progresses rather gradually for the action of higher waves. On the other hand, the damage of the concrete blocks almost jumps beyond the design wave height. In particular, the drastic damage is often caused in the case of high rubble mound foundations. The proposed method is confirmed, however, to be applicable for the ordinary low mound foundations with a sufficient safety.

INTRODUCTION

Composite type breakwaters which are composed of upright sections and rubble mound foundations have several advantages over rubble mound breakwaters. They are more stable, faster in construction, and less in wave transmission than rubble mound breakwaters. Because of these advantages, they have been built in great length of extension in Japan without major mishaps in several scores of years in the past.

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Design practices of this type of breakwaters also have been improved continuously through theoretical, laboratory and field studies. In particular, much efforts have been maid to study wave forces on the upright sections. As a result of those studies, a general formula to calculate the design wave forces due to nonbreaking to postbreaking waves has been established by Goda (1974). This formula is being successfully applied to the design of composite breakwaters (Bureau of Ports and Harbours, 1980).

On the other hand, the armor layer of the foundation mounds are being designed on the basis of egineer's experiences, supplemented by informations obtained from laboratory studies: e.g. Brebner and Donnelly (1962). Those previous studies, however, were based on the regular wave tests and little information on the irregular wave attacks is available until now, although a number of studies on the stability of armor units for rubble mound breakwaters have been conducted using irregular waves. Consequently, design engineers are always puzzled how the results by regular wave tests should be applied to the actual action of irregular waves. A possible solution for this question is undoubtedly to investigate the stability of armor units for foundation mounds of composite breakwaters by irregular wave tests.

The authors have carried out the irregular wave tests for the stability of armor units in order to get better knowledges for the design of composite breakwaters. The present paper describes the results of the irregular wave tests and discusses the stability number, defined by Hudson (1959), to formulate it on the basis of test results. The study in the present paper is of two-dimensional and limitted to the action of waves of normal incidence to breakwaters.

STABILITY EQUATION

Figure 1 illustrates the typical cross section of the composite breakwaters and defines the symbols used in the present paper;

h : water depth at the structure site

h' : depth at the crest of rubble mound foundation excluding the armor layer

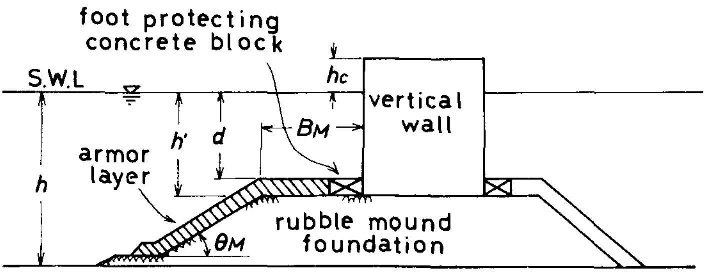

Figure 1 Definition sketch of cross section of composite breakwater

d : depth at the crest of rubble mound foundation
B_M : berm width of rubble mound foundation
θ_M : angle of the front slope measure from the horizontal plane
h_c : crest height of vertical wall above the still water level

Because these dimensions associated with the structure and the water depth as well as wave conditions influence on the stability of armor units for the rubble mound foundation, the problem is very complicated and inherently contains a number of parameters. In the present study, a simplified analytical consideration is made at first in order to find the principal parameters governing the stability of armor units.

In the ordinary conditions, the most critical position of the armor layer being subject to the initial damage may be assumed to be around the top of the slope or the seaward end of the berm crest. For this reason, the composite breakwater is replaced by the vertical breakwater on a hypothetical horizontal bottom having the water depth of h' and the stability of single rubble unit, which is placed at the position with the distance of B_M from the vertical wall, is considered under the action of standing waves as shown in Figure 2.

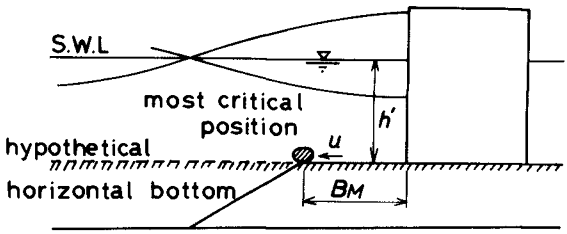

Figure 2 Simplified model for analytical consideration

As to wave forces on the unit, it is assumed that the drag forces predominate over the inertia forces. Then, the horizontal force (drag force, F_D) and the vertical force (lift force, F_L) on the unit are given by the following expressions:

$$F_D = \frac{\rho_w}{2} C_D \, u_{max}{}^2 \, k_A \, a^2 \tag{1}$$

$$F_L = \frac{\rho_w}{2} C_L \, u_{max}{}^2 \, k_A \cdot a^2 \tag{2}$$

where ρ_w is the density of fluid, C_D and C_L are the drag and the lift coefficients, u_{max} is the maximum horizontal velocity, a is the characteristic linear dimension of the unit such that the projected area of the unit perpendicular to the velocity is $k_A a^2$ and the volume of the unit is $k_V a^3$.

On the other hand, the mass of the unit, W, is expressed

$$W = \rho_r \, k_V \, a^3 \tag{3}$$

where ρ_r is the density of the unit. Then, the condition of limiting equilibrium against the horizontal displacement of the unit is

$$\mu\{g(\rho_r - \rho_w)k_V \, a - F_L\} = F_D \tag{4}$$

where μ is the friction coefficient, g is the acceleration of gravity. When Eqs.(1) and (2) are substituted into the above Eq.(4), the following relation is obtained:

$$a = \frac{1}{2g}\frac{k_A}{k_V}\frac{C_D/\mu + C_L}{S_r - 1} u_{max}{}^2 \tag{5}$$

where

$$S_r = \rho_r/\rho_w \tag{6}$$

Using Eq.(3), Eq.(5) is written as:

$$W = \{\frac{1}{2g}\frac{k_A}{k_V{}^{2/3}} \, (\frac{C_D}{\mu} + C_L)\}^3 \frac{\rho_r}{(S_r - 1)^3} u_{max}{}^6 \tag{7}$$

This relation indicates that the critical mass against the horizontal displacement is proportional to six powers of the maximum horizontal velocity.

When standing waves are formed in front of the vertical wall, the maximum horizontal velocity at the bottom can be expressed by the following equations according to the small amplitude wave theory:

$$u_{max} = \{\kappa \frac{H_I'}{h'} g \, H_I' \}^{1/2} \tag{8}$$

$$\kappa = \frac{4\pi h'/L'}{\sinh\,(4\pi h'/L')} \sin^2\,(2\pi B_M/L') \tag{9}$$

where H_I' is the incident wave height, L' is the wavelength at the depth of h'. The parameter κ represents the combined effects of the relative water depth and the relative distance from the vertical wall on the maximum horizontal velocity at the bottom.

Substituting Eq.(8) to Eq.(7), we get

$$W = \frac{\rho_r}{N_s{}^3 \, (S_r - 1)^3} H_I'^3 \tag{10}$$

$$N_s = \frac{2k_V{}^{2/3}}{k_A \, (C_D/\mu + C_L)} \frac{1}{\kappa} \frac{h'}{H_I'} \tag{11}$$

Eq.(10) is written in the form of the stability equation which was derived by Hudson (1959) for the armor units of rubble mound breakwaters and N_s is called as the stability number. According to Eq.(11), the stability number is in proportion to h'/H_I' and in inverse proportion to κ.

Because there are many assumptions in the above-mentioned derivation, the relation given by Eq.(11) is not directly applicable to the armor units for the rubble mound foundations of composite breakwaters. For example, waves are deformed by the existence of the rubble mound foundation and breaking waves also act on the structure when incident waves are high enough. Therefore, in the present study, h'/H_I and κ are simply considered as principal parameters governing the stability number, although the stability equation in the form of Eq.(10) is adopted as the basic equation. The wave height of progressive waves in the depth of h is applied to the wave height in the stability equation, and for the action of irregular waves the significant wave height, $H_{1/3}$, is selected as a representative wave height. Thus, the stability equation is defined by the following relation:

$$W = \frac{\rho_r}{N_s{}^3(S_r - 1)^3} H_{1/3}{}^3 \qquad (12)$$

and the effect of the irregularity of individual waves becomes to be contained in the evaluation of the stability number.

The stability number may be expressed as:

$$N_s = f\left(\frac{h'}{H_{1/3}}, \kappa ; \begin{array}{l}\text{shape and placing method}\\ \text{of armor units}\end{array} \right) \qquad (13)$$

where f( ) indicates "function of". The parameter κ is calculated for given values of h'/L' and B_M/L'. Figure 3 shows the κ-value against B_M/L' when the value of B_M/h' is fixed. For the irregular waves, the wavelength corresponding to the significant wave period, $T_{1/3}$, is applied to L' in the calculation of κ-value.

TEST CONDITIONS

Tests have been carried out in the wave flume of 163 m long, 1.0 m wide, and 1.5 m deep as sketched in Figure 4. The wave generator consists of a wave paddle of piston type driven by a linear electric motor with low inertia, the movement of which is controlled by an electric signal from a sine-wave generator (regular waves) or a magnetic tape (irregular waves). In front of the wave generator is a wave filter which absorbs, to some extent, waves reflected from the model structure. The bottom of the wave flume is mostly horizontal, but the test section is the sloping bottom formed as a fixed bed. In front of the model structure, a vertically sliding wall is prepared to avoid the unfavorable effect of transient waves which are caused in the wave flume when the motion of wave paddle is stopped. Glass windows are equipped in a side wall at the test site so that visual observation of the stability of armor units can be made from the side.

The water depth, h, at the site of model structure was selected to be 40 to 50 cm and the model of the composite breakwater was set up for four different thicknesses of the rubble mound foundation in the range of h'/h from 0.3 to 0.9. The front slope of the rubble mound was fixed to be constant at cot θ_M = 2 for all the test cases. The relative berm width to the water depth, B_M/h, was fixed mostly to be 0.6, but, in case of h'/h = 0.6, it was changed in the range from 0.4 to 1.4.

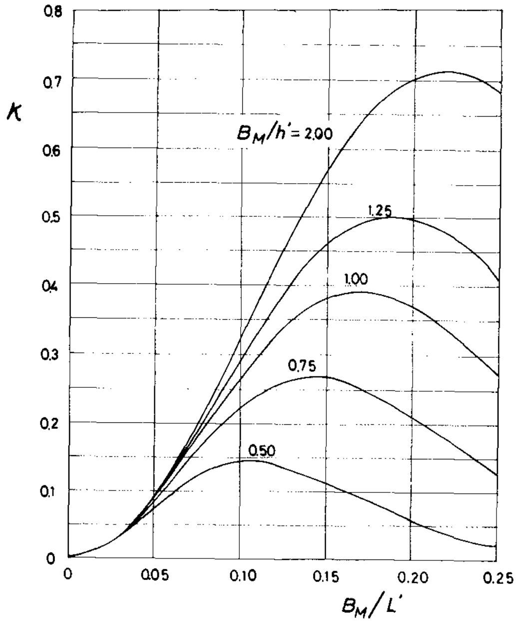

Figure 3 Value of κ

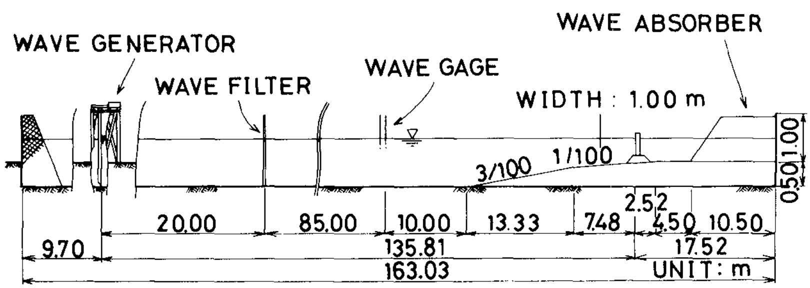

Figure 4 Wave flume

Quarry stones having five different graded sizes as shown Table 1 were tested. They were placed in two layers on the front slope and the crest berm of the rubble mound foundation. The foot of the vertical wall, however, was protected from scouring by placing concrete blocks of the rectangular solid with a sufficient mass.

Table 1 Mass and density of quarry stone

Grade	Mass W (g)		Density ρ_r (g/cm^3)
	average	standard deviation	
I	15.0	2.05	2.60
II	29.9	3.51	2.59
III	57.3	5.99	2.62
IV	105.5	12.5	2.64
V	250.0	27.9	2.75

The concrete block with a vertical hole as shown in Figure 5 was tested as well as the basic shape without a hole. Their masses are from 36.9 to 147.1 g for the basic shape and from 34.1 to 208.7 g for the shape with a hole. The densities are from 2.21 to 2.33 depending on the shape and the size. They were placed regularly in single layer on the rubble mound foundation.

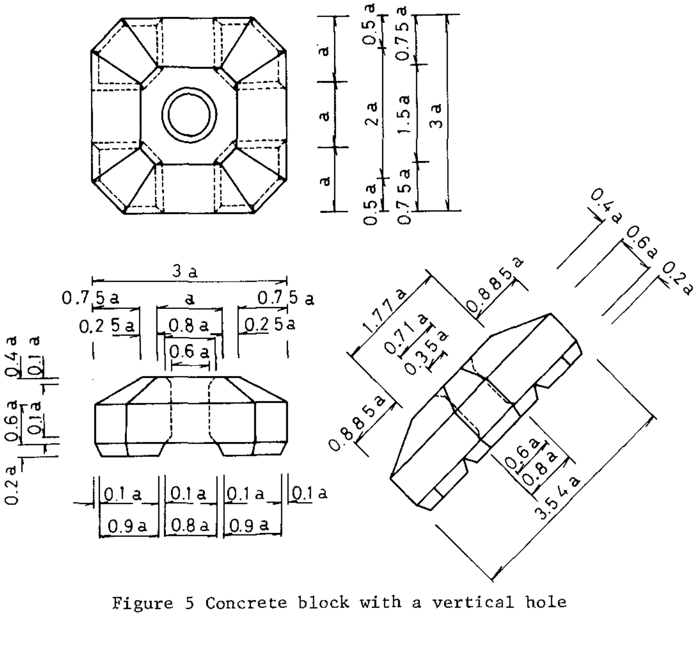

Figure 5 Concrete block with a vertical hole

Irregular waves having the following spectrum of Bretchneider type were simulated:

$$S(f) = 0.257 \ H_{1/3}^{2} \ T_{1/3} \ (T_{1/3}f)^{-5} \ \exp\ [-1.03(T_{1/3}f)^{-6}] \qquad (14)$$

where $S(f)$ is the spectrum density, and f is the frequency. The significant wave period was changed to give the range of relative water depth from 0.075 to 0.143. The effective duration of irregular waves was selected to be 900 seconds in most cases, but for the tests of the concrete blocks having a vertical hole it was determined so that about 500 zero-up cross waves act on the structure. During the tests, waves were recorded at two adjacent locations in the offshore, and the progressive wave height toward the structure was obtained by the resolution technique of incident and reflected waves from the composite waves (see Goda and Suzuki, 1976). Then, the incident wave height at the test section was estimated from the relation between the wave height at the offshore and the wave height at the test section, which had been measured preliminary without the model structure. The test range of relative wave height, $H_{1/3}/h$, is from 0.16 to 0.44 for the quarry stones and from0.14 to 0.61 for the concrete block having a vetical hole.

The crest height of the vertical wall, h_c, was changed according to the incident wave height so that the relative crest height, $h_c/H_{1/3}$, is kept to be constant value of 0.6. Therefore, the condition is that some extent of wave overtopping is caused for the all test cases.

TEST RESULTS AND DISCUSSIONS

Stability of quarry stones

Armor units of respective masses were tested for given conditions of h'/h, B_M/h and h/L by increasing wave heights, and the relationship between the average mass of armor units and the incident wave height at the equilibrium state between stable and unstable conditions was determined in order to evaluate the stability number. The typical results for the quarry stones are shown in Figure 6. The critical line between stable and unstable conditions is drawn so as to represent one percent damage on the distribution of damage percents which are defined as the percentage of armor units removed from the original position to the total number of armor units in the cover layer. When the critical wave height, Hc, for the respective average mass is determined, the stability number is evaluated by the relation of Eq.(12).

The stability number of quarry stones obtained by the above-mentioned procedure is plotted against $h'/H_{1/3}$ in Figure 7, in which the data are distinguished by the parameters h/L and h'/h, and linked by lines for the same conditions of them. The data indicate that the stability number is increased as $h'/H_{1/3}$ becomes large. It is also seen that the stability number is variated by the other parameters as h/L, and shorter waves are more destructive than longer waves if the value of $h'/H_{1/3}$ is the same. In particular, this tendency is apparently noticed, when the value of $h'/H_{1/3}$ is relatively large. These results establish, as predicted by the analytical considerations, that the stability of armor units for the composite breakwater foundations is greatly effected

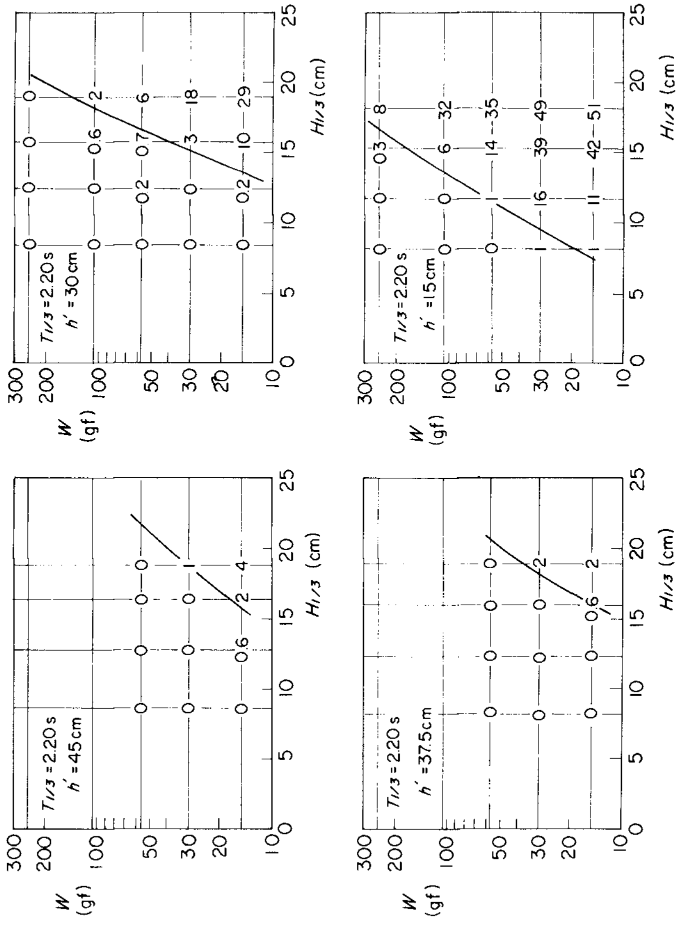

Figure 6 Example of test results for quarry stones
(h = 50 cm, B_M = 30 cm)

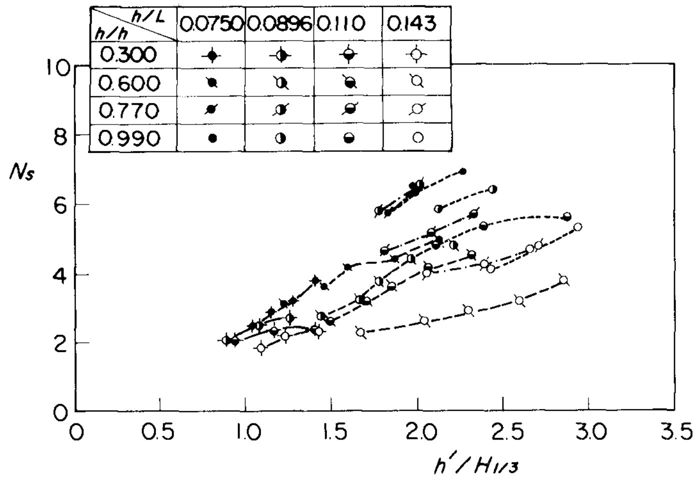

Figure 7 Effect of the crest depth and wave conditions (B_M/h = 0.6)

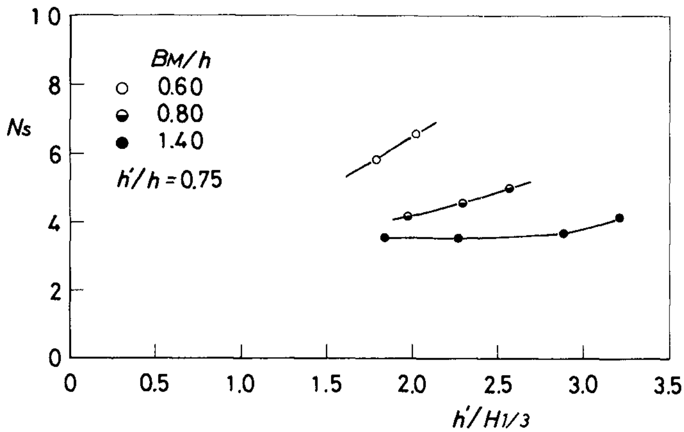

Figure 8 Effect of berm width (h'/h = 0.75)

by the wave height and the period, when the crest depth of the rubble mound becomes deep and standing waves are formed in front of the vertical wall. It is also observed that the stability number has a tendency to converge to a constant value when the value of $h'/H_{1/3}$ becomes small. This constant value must be the stability number of armor units for rubble mound breakwaters, since the extreme state of $h'/H_{1/3} = 0$ corresponds to the rubble mound breakwaters.

On the other hand, Figure 8 shows the stability number when the condition of B_M/h is changed. No data for the condition of $B_M/h = 0.4$ is plotted in the figure, because any damage was not caused at that condition. The results demonstrate that the stability number decreases as the value of B_M/h becomes large. This means that the armor units for the rubble mound foundation with a wide berm width are more subjected to damage than those for the rubble mound foundation with a narrow berm width.

Previous considerations based on the simplified model suggest that these effects of the wave period and the berm width on the stability number may be represented by the parameter κ given by Eq.(9). For this reason, the data classified in the appropriate range of κ-value are replotted in Figure 9. From these results the following remarks are pointed out:

(1) The stability number for the same rank of κ-value increases almost linearly with $h'/H_{1/3}$, although the data are scattered.

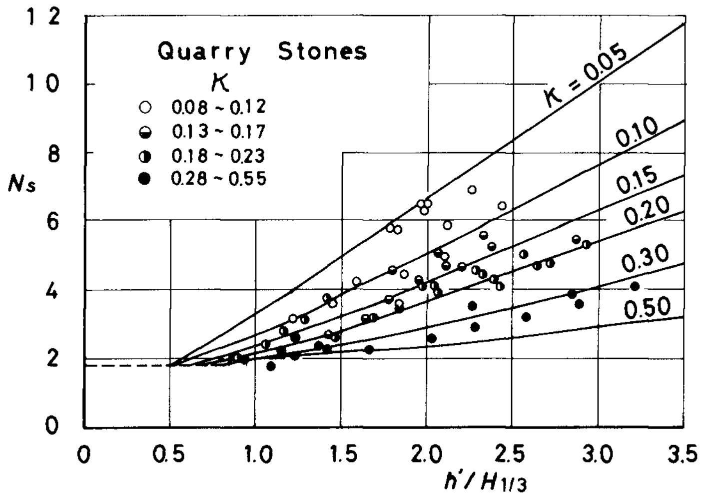

Figure 9 Stability number of quarry stones

(2) As the value of κ becomes large the stability number becomes small, but the difference is not large when the κ-value is larger than 0.3.
(3) When the value of $h'/H_{1/3}$ becomes small the stability number is not much different by the κ-value and has a tendency to converge to a constant value.

Thus, the stability number, N_s, may be formulated by a function of $h'/H_{1/3}$ and κ, so that $N_s = N_{so}$ at $h'/H_{1/3}$ and N_s is increased in proportion to $h'/H_{1/3}$ when it is sufficiently large. In the present study, N_s is expressed as a sum of two terms as shown in Figure 10, the one is the term which increases linearly in proportion to $h'/H_{1/3}$ and the other is the term which decreases from the value of N_{so} to infinitesimal at the large value of $h'/H_{1/3}$. After several trials, the following expression was selected:

$$N_s = A \frac{1-\kappa}{\kappa^m} \frac{h'}{H_{1/3}} + N_{so} \exp \left[-B \frac{(1-\kappa)^2}{\kappa^m} \frac{h'}{H_{1/3}}\right] \qquad (15)$$

The constants of N_{so}, A, B, m in the above equation are determined so that calculated results agree with test results as closely as possible. The following values were determined for the quarry stones:

$N_{so} = 1.8$, $A = 1.3$, $B = 1.5$, $m = 1/3$

The stability number of $N_{so} = 1.8$ corresponds to the K_D-value of 2.9 in the Hudson's formula for rubble mound breakwaters, when the value of $\cot \theta_M$ is set equal to 2.

However, Eq.(15) has a minimum value slightly less than N_{so} at the

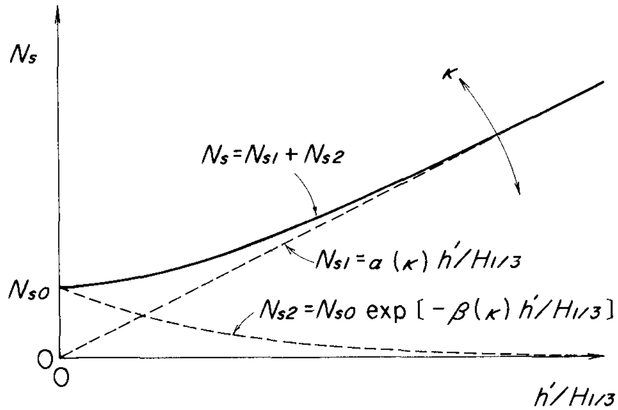

Figure 10 Formulation of stability number

small value of $h'/H_{1/3}$. Finally, the following empirical equation to calculate the stability number of quarry stones is proposed:

$$N_s = \max \{1.8,\ 1.3 \frac{1-\kappa}{\kappa^{1/3}} \frac{h'}{H_{1/3}} + 1.8 \exp [-1.5 \frac{(1-\kappa)^2}{\kappa^{1/3}} \frac{h'}{H_{1/3}}]\} \tag{16}$$

where max $\{a, b\}$ indicates the larger one of a or b. The curves in Figure 9 indicate the relationships between $h'/H_{1/3}$ and N_s which are calculated for the representative κ-values. It is seen that the calculated relations represent considerably well the tendency of test results.

The design mass of quarry stones can be calculated by applying the stability number obtained from Eq.(16) to the stability equation (12). The calculations were made for the all test conditions in order to compare with test results of damage percent. In Figure 11, the data of the damage percent realized in the tests are plotted against W/W_c. Here W is the average mass of quarry stones which were tested and W_c is the mass calculated by the above-mentioned method for the test conditions. In the theory, $W/W_c = 1.0$ indicates the equilibrium state between stable and unstable conditions, and the damage percent should be about one percent. Such ideal results, however, can not be expected, becuase the behaviours of armor units under the action of waves are essentially changeable due to the slight variations of conditions. The results shown in Figure 11 are satisfactory, although they are considerably scattered. It is also pointed out that the damge of quarry stones progresses rather gradually.

Stability of concrete blocks

Figures 12 and 13 show the test results of N_s for the concrete blocks with and without a vertical hole. The stability numbers for these concrete blocks are also formulated by two parameters of $h'/H_{1/3}$ and κ in the same way as the quarry stones. The curves in the figures indicate the relationships calculated for the appropriate κ-values. The constants in the empirical equation are as follows:

For the concrete block without a hole,

$N_{so} = 1.4,\ A = 1.0,\ B = 1.2,\ m = 1/3$

For the concrete block with a hole,

$N_{so} = 1.6,\ A = 0.82,\ B = 0.9,\ m = 1/2$

The stability number of concrete blocks having a vertical hole is generally higher than that of concrete blocks without a hole. Particularly, the advantages of having a vertical hole is noticed when the κ-value is relatively small.

The tests demonstrate that the damage of concrete blocks placed in single layer progresses drastically when the incident wave height exceeds the critical wave height. Therefore, it should be noted that slight overestimation in the stability number sometimes results in the serious damage. In particular, the drastic damage is often caused in the case of high rubble mound foundations. The results presented in Figure 14 show

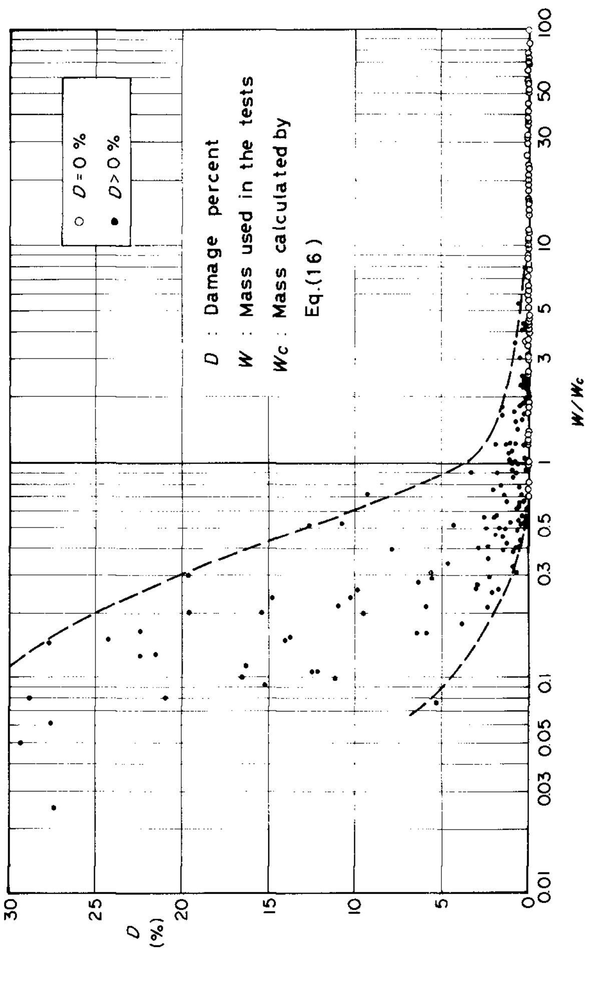

Figure 11 Damage percent of quarry stones

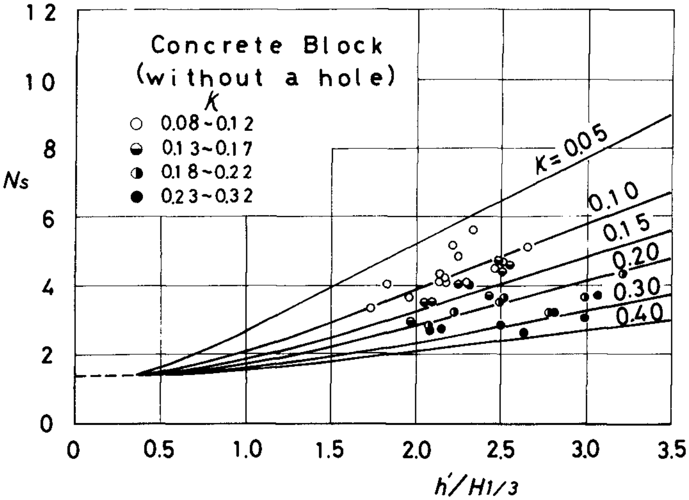

Figure 12 Stability number of concrete block without a hole

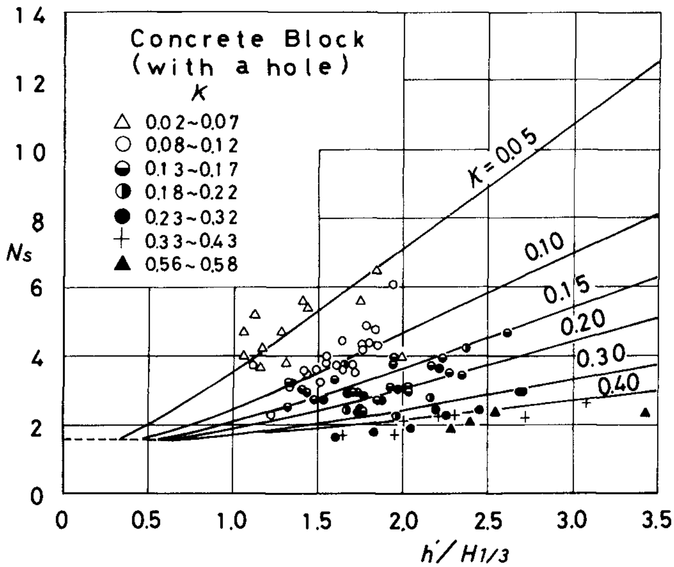

Figure 13 Stability number of concrete block with a vertical hole

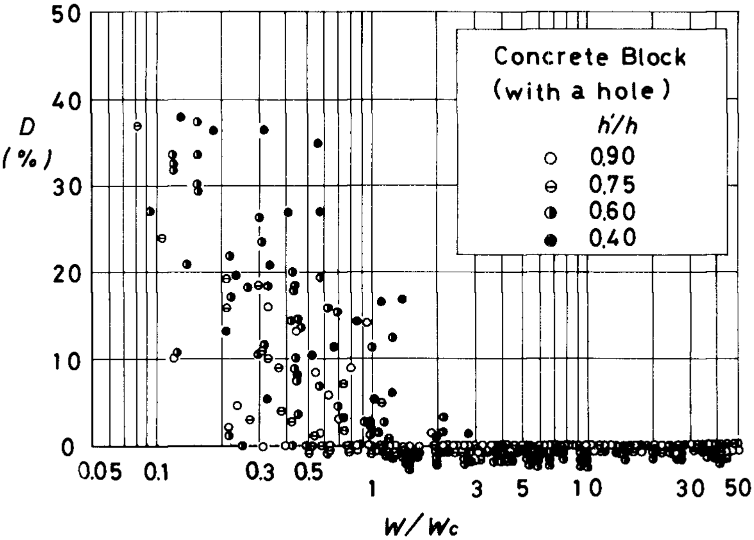

Figure 14 Damage percent of concrete block with a vertical hole

that the damage percent almost jumps beyond the critical conditions and the largest damage percent at W/W_c = 1.0 comes up to about 20 %. However, most data, which indicate the damage in the condition of $W/W_c \geqq 1.0$, are the cases of high rubble mound foundations of $h'/h \leqq 0.6$. For the ordinary low rubble mound foundations, the present method would be applicable with a sufficient safety.

Comparison with results of regular wave tests

Regular wave tests for the concrete block without a hole were carried out in order to make direct comparison with the results by irregular wave tests. The period of regular waves was selected so as to be equal to the significant wave period of irregular waves. The total duration of regular waves with a wave height of certain level, however, was shortened to one third of the duration of irregular waves and it was divided into several short runs so that no re-reflected waves from the wave paddle act on the structure. The tests were carried out by three different conditions of h'/h = 0.60, 0.75, and 0.90, but within the test range of wave heights no damage was caused by the action of regular waves for the case of h'/h = 0.90.

The tests establish that irregular waves are more destructive than regular waves, if the regular wave height is set equal to the significant wave height. Figure 15 shows the relation between the critical wave height of regular waves, H_{creg}, and the critical significant wave height of irregular waves, H_{cirreg}, at the equilibrium state. According to the

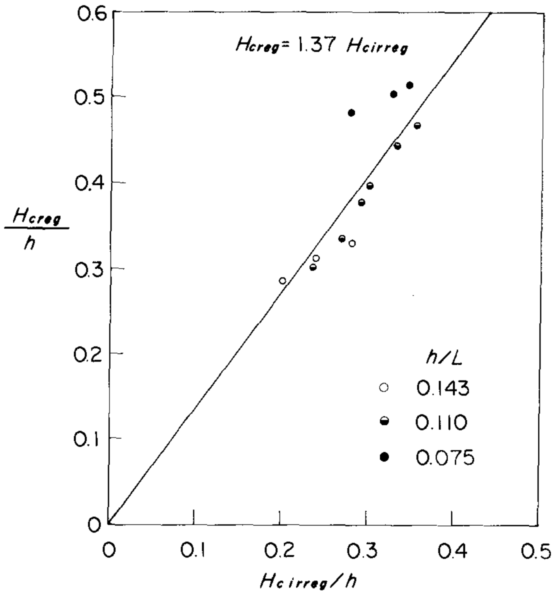

Figure 15 Relation between the critical significant wave height and the critical wave height of regular waves

results, the action of regular waves almost equivalent to that of irregular waves, when the height of regular waves is set equal to 1.37 times of the significant wave height on an average.

Examples of calculations

Examples of calculations of design mass of quarry stones will be given together with the results by two previous studies.

Brebner and Donnelly (1962) proposed the design curve of minimum stability number as a function of d/h. Although thier results were based on regular wave tests, they suggested that the design wave height used in the stability equation should be twice or one and half times of the expected significant wave height depenging on the importance of the structures. In the present calculations, however, the average wave height of the highest one percent of all waves, $H_{1/100}$ (= 1.67 $H_{1/3}$), and the average wave height of the highest of one tenth, $H_{1/10}$ (= 1.27 $H_{1/3}$), are applied according to the recommendation by CERC (1973). The regular wave height equivalent to the action of irregular waves, which was obtained for concrete blocks in the present study, corresponds to the average wave height of highest one sixteenth, $H_{1/16}$, between $H_{1/10}$ and $H_{1/100}$.

The other previous study is made by Inagaki and Katayama (1971). They investigated the field data of damaged and non-damaged cases of armor stones and proposed the following empirical formula:

$$W = 0.08 \frac{1}{5^{d/H_{1/3}}} H_{1/3}{}^{3}$$

The conditions for calculations are as follows:

h = 20 m, h' = 15 m, B_M = 10 m

Wave conditions are selected so as to be $H_{1/3}/L$ = 0.045 for the range of the significant wave period from 8 to 16 seconds. Here, L is the wavelength corresponding to the significant wave period at the water depth of 20 meters. The cover layer for the rubble mound foundations is supposed to have a thickness of two armor units and densities of the rubble unit and the sea water are given to be 2.65 t/m^3 and 1.03 t/m^3, respectively.

The results are very different by the mothods as shown in Figure 16, where the absissa is taken as the significant wave height. Generally, the method by Brebner and Donnelly for the rubble mound as foundation results in a very large mass. On the other hand, the method by Inagaki and Katayama gives the minimum mass. The mass calculated by the authors' method is between them and close to the latter rather than the former. The calculated masses for the rubble mound as toe protection by Brebner and Donnelly are also shown in the figure. In this example, the results are very close to those by authors' method, when $H_{1/10}$ is applied. However, such relative relations by the different methods will be changed by the conditions of water depth and the configuration of the rubble mound foundation, since the parameters involved in the methods are different.

CONCLUDING REMARKS

The results of this study provide the design informations of armor units for rubble mound foundations of composite breakwaters. The stability number of the armor units are greatly influenced by the configuration of rubble mound foundation and wave conditions. The stability numbers for quarry stones and concrete blocks were formulated by two parameters of $h'/H_{1/3}$ and κ on the basis of irregular wave tests. Generally, the damage of quarry stones placed in two armor layers progresses rather gradually. The application of the proposed method to the test results demonstrates that the damage percent of quarry stones is at most 3.5 % at the design condition. On the other hand, the damage of the concrete blocks placed regularly in single layer progresses drastically in the case of high rubble mound foundations. Therefore, it is not recommended to apply the concrete blocks in single layer to high rubble mound foundations.

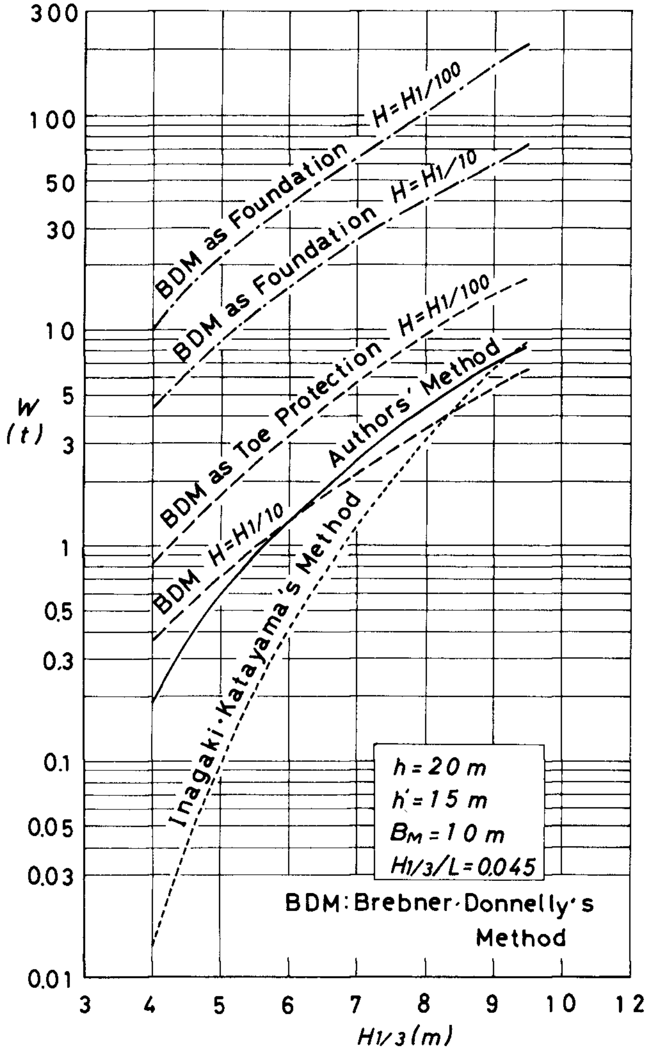

Figure 16 Design mass of quarry stones calculated by several methods

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