NON-LINEAR WAVE FORCES ON FLOATING BREAKWATERS

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ABSTRACT

A non-linear numerical method for calculating wave forces on floating bodies has been developed by Isaacson (1981). The time stepping procedure is programmed for a computer solution, and an incident wave train is time stepped past a fixed two-dimensional rectangular breakwater. The influence of various input parameters on the accuracy of results is investigated, and optimal values of the parameters are determined. The optimal numerical parameters are used to generate force and transmission coefficient results, which are compared to the results of other published studies. The method is shown to compare favorably with other results, with the non-linear nature of the method being clearly demonstrated by the different force curves produced by varying the wave height.

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1. INTRODUCTION

The forces generated by waves on floating breakwaters or floating bridges are generally predicted on the basis of linearized potential theory. A nonlinear method for calculating the wave forces for the case of a fixed body has been developed by Isaacson (1981) and subsequently extended to floating bodies undergoing motions (Isaacson, 1982). The method employs the second form of Green's theorem, together with the usual governing equations, to time step an incident wave train past the body. In order to test the validity and range of the method, only the fixed body case is tested here. Although the method has been used on three-dimensional bodies, we apply it to the two-dimensional case and exploit that simplification to conduct a study of the incident wave conditions and numerical parameters used in the method. A comparison is made between force and transmission coefficient results generated by the method and those available from previously published studies.

2. GENERAL DESCRIPTION OF METHOD

For the two-dimensional case examined here, a body of rectangular cross-section with beam B and draught D is floating in water of uniform depth d. An x-z coordinate system is defined with x measured horizontally in the direction of incident wave propagation and z measured vertically upward from the still water level. The origin is located at the still water level midway along the beam of the body. Let η denote the free surface elevation above the still water level. A definition sketch is shown in Figure 1.

With the usual assumptions of an incompressible fluid and irrotational flow, the fluid motion is represented by the velocity potential ϕ which must safisfy the Laplace equation within the fluid region,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \ . \tag{1}$$

Assuming an impermeable seabed and body surface, the flow will be subject to the following boundary conditions,

$$\frac{\partial\phi}{\partial z} = 0 \qquad \text{at} \quad z = -d \tag{2}$$

$$\frac{\partial\phi}{\partial n} = 0 \qquad \text{on} \quad S_b \tag{3}$$

$$\frac{\partial\phi}{\partial n} = \frac{\partial\eta}{\partial t} n_z \qquad \text{on} \quad S_f \tag{4}$$

$$\frac{\partial\phi}{\partial t} + g\eta + (\nabla\phi)^2 = \text{constant on} \quad S_f \ . \tag{5}$$

Here, S_f and S_b are the free and body surfaces respectively, n is the direction normal to the surface, t is time, n_z is the direction cosine in the z direction of the vector $\underline{n}$, and g is the acceleration due to gravity. The equations given by (4) and (5)

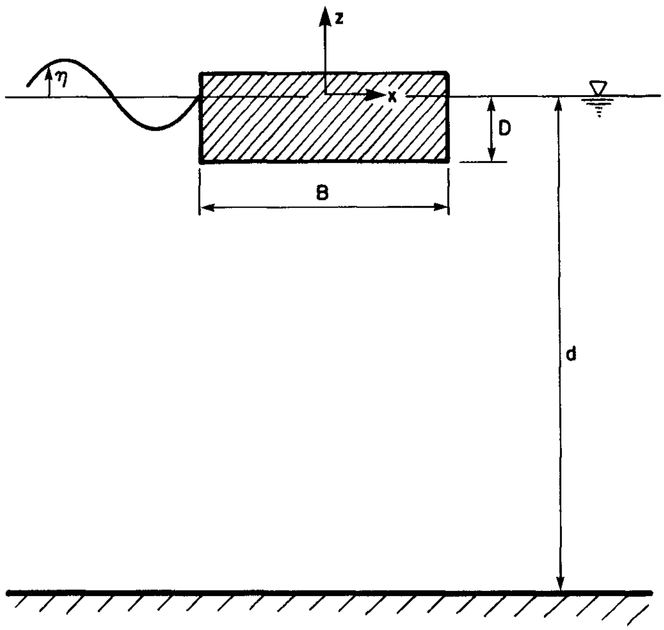

FIG. 1: DEFINITION SKETCH.

are the kinematic and dynamic free surface boundary conditions, with (4) being a form used by Isaacson (1981, 1982).

The second form of Green's theorem provides values of ϕ at any point $\underline{x} = (x,z)$ on the closed boundary in terms of ϕ and its normal derivative on the boundary:

$$\phi(\underline{x}) = -\frac{1}{\pi}\int_S \left[G(\underline{x},\underline{\xi})\,\frac{\partial\phi}{\partial n}(\underline{\xi}) - \phi(\underline{\xi})\,\frac{\partial G}{\partial n}(\underline{x},\underline{\xi})\right] dS \;. \qquad (6)$$

The point $\underline{\xi} = (\xi,\rho)$ is a point on the closed boundary in the x-z plane over which the integration is performed, dS is measured along the closed boundary, and G is an appropriate Green's function.

In order to consider a closed boundary over which the Green's identity can be integrated, vertical control surfaces extending from the free surface to the seabed are set at a chosen distance to either side of the body. The closed surface consists of the body surface, the free surface, and the control surfaces all reflected about the seabed. The control surfaces are set sufficiently distant so that the scattered potential due to the wave interaction with the fixed body will not reach the control surfaces throughout the time stepping procedure.

The closed surface on each side of the seabed is divided into N_b body segments, N_f free surface segments, and N_c control surface segments. This is shown in Figure 2. Values of ϕ and $\partial\phi/\partial n$ are assumed to be constant over each segment, and initial values of ϕ, $\partial\phi/\partial n$, and η are assigned to the midpoint of each segment according to a chosen wave theory defining the incident wave. The initial conditions require zero flow in the immediate vicinity of the body.

At a given time t, time stepping equations obtained from (4) and (5) are used to get N_f values of η and ϕ at $t+\Delta t$. A numerical integration of (6) at each body and free surface segment gives $N = N_f+N_b$ equations in N unknowns (N_b values of ϕ and N_f values of $\partial\phi/\partial n$), which are solved on the computer using conventional matrix solution techniques. Values of ϕ and $\partial\phi/\partial n$ on S_c are given at all times by the chosen incident wave theory. Thus, all values of ϕ, $\partial\phi/\partial n$, and η are obtained at $t+\Delta t$, and the process can be repeated as many times as necessary for the desired flow development.

With the velocity potential ϕ obtained for each segment at each time step, we can obtain the pressure distribution on the body surface using the unsteady Bernoulli equation,

$$p = -\rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right] \;. \qquad (7)$$

A numerical integration of Eq. (7) over the body surface yields the forces on the body at each time step. The numerical procedure described above is readily programmed for computer solution. The flow chart for such a solution is shown in Figure 3. A more complete

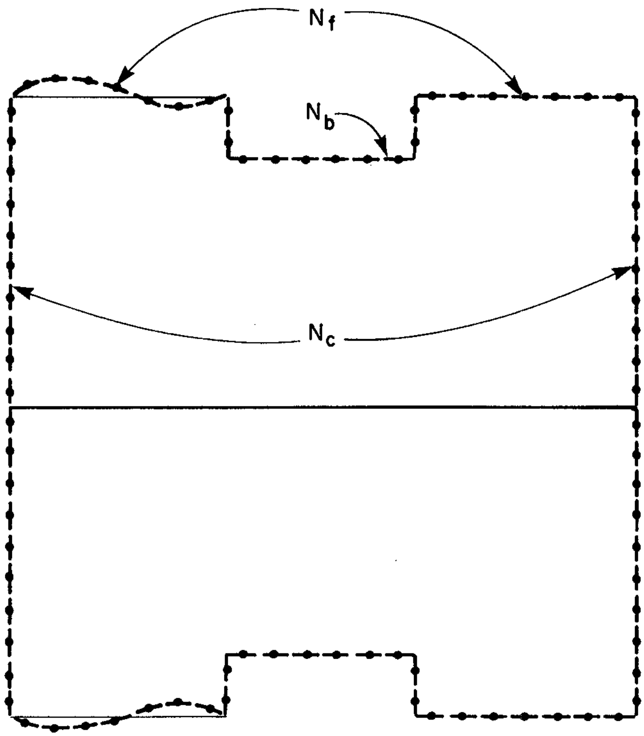

FIG. 2: SEGMENT GRID.

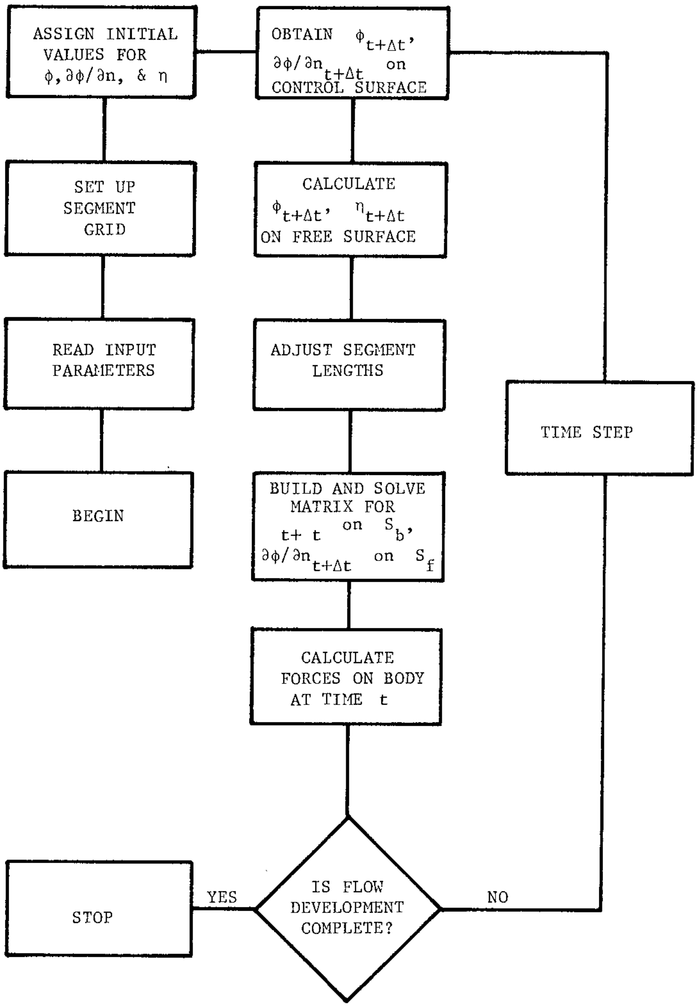

FIG. 3: FLOW CHART FOR COMPUTER SOLUTION.

derivation of the theory and numerical procedure is given by Isaacson (1981, 1982).

3. INCIDENT WAVE CONDITIONS

3.1. Incident Wave Decay Length

A harmonic wave predicted by either linear or Stokes's fifth order wave theory is used to prescribe the incident wave. To satisfy the condition that there initially be zero flow in the vicinity of the body, the incident flow is attenuated over a given decay length, L_d, as shown in Figure 4. When considering computing effort, a short decay length is desirable in order to reduce the length of the free surface and hence the number of segments needed on the free surface. A long decay length is expected to give smoother, more accurate flow development at the expense of much larger computing effort.

The program developed for this study is run with values of L_d varying from 0.25 up to 1.75 wavelengths, and the maximum forces on the body generated by the first fully developed wave to interact with the body are recorded. The results, shown in Figure 5, indicate that a decay length of about 0.75 wavelengths and less yields somewhat unreliable results, while a decay length of 1.25 and greater does not change the results significantly. A value of $L_d = 1.25$ or 1.50 is recommended for accurate results.

3.2. Group Velocity

Depending on the order of the time stepping procedure (see 4.2), the initial conditions along the entire boundary are specified at two or more time steps according to a chosen wave theory. The modulation envelope of the incident harmonic wave train travels at the group velocity c_g. For linear wave theory, c_g is given explicitly by

$$c_g = \frac{c}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right] . \qquad (8)$$

However, when using Stokes' fifth order wave theory, a reasonable value for the group velocity must be assumed since no explict expression for c_g exists.

To examine the effect of specifying different values of c_g, the program was run twice with an incident wave given by linear wave theory, the first time letting c_g equal its linear theory predicted value given by (8), and the second time setting c_g equal to the wave speed, c, which is about twice the predicted value of the group velocity. It was expected that specifying a value for c_g higher than that predicted by linear wave theory might lead to quicker flow development. However, the flow development results are found to differ only for the first few time steps, with subsequent flow development and force results being almost identical for both runs.

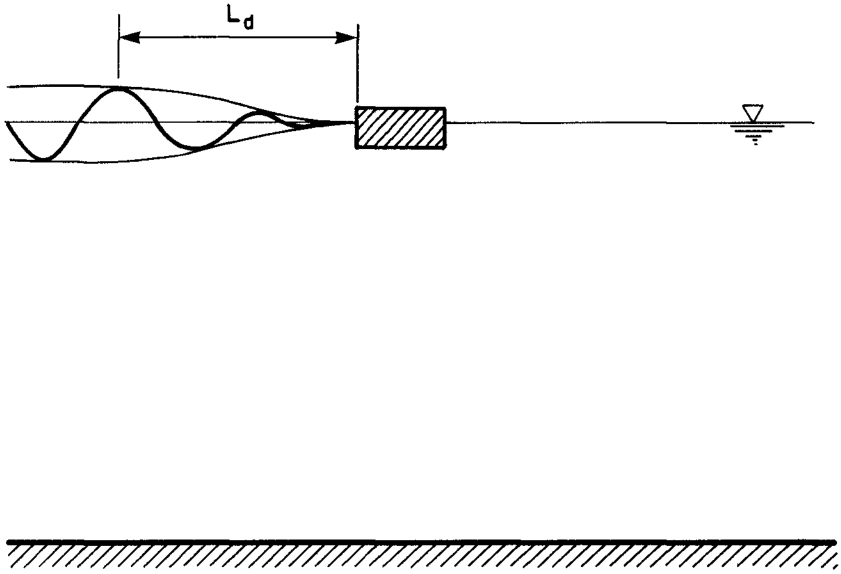

FIG.4: INCIDENT WAVE PROFILE.

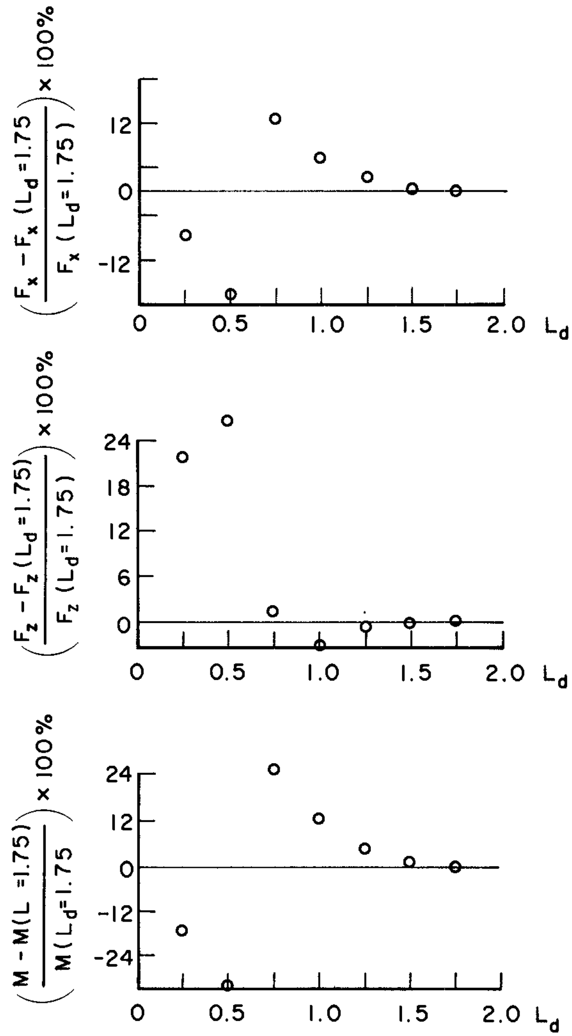

FIG. 5: EFFECT OF VARYING DECAY LENGTH, L_d.

We conclude that the method itself with its physical constraints expressed by the boundary conditions and the Green's identify very strongly defines the flow development and overrides any attempt to force a faster flow development in the vicinity of the body by setting the group velocity to a higher value.

When using Stokes' fifth order wave theory for the incident wave, the group velocity is now set equal to that predicted by linear wave theory.

4. NUMERICAL PARAMETERS

4.1. Time Step Size and Segment Length

Both the time step size and the segment length are required to be as large as possible in order to minimize the computing effort for the time stepping procedure. The time step parameter $\Delta t/T$ determines the number of cycles of the procedure for each wavelength, while the segment length parameter $\Delta S/L$ controls the size of the matrix that is solved at each time step. Here, Δt is the time step size, T is the incident wave period, ΔS is the segment length, and L is the wavelength.

If one considers the cyclical motion of the fluid particles over a wave period, it is readily apparent that the time step Δt must be sufficiently small to ensure that the motion of the particles is small compared to the segment length, and hence that $\Delta t/T$ should be less than $\Delta S/L$ for accurate flow development. Thus, for a given value of $\Delta t/T$, $\Delta S/L$ must be small enough to yield accurate results while being large enough to remain greater than the given $\Delta t/T$.

Since the body dimensions are significantly smaller than the wavelength for waves of longer period, the segment length parameter $\Delta S/L$ found to be appropriate on the free surface may not be very useful on the body surface if only one or two segments are needed to meet the specified global $\Delta S/L$ requirement. Recalling that ϕ and $\partial\phi/\partial n$ (and hence p) are constant over each segment, a minimum number of segments are needed on the body surface to adequately describe the pressure distribution over the body surface. The moment acting on the body is most likely to be sensitive to this consideration. For purposes of the body segments we define a parameter $\Delta S_b/L_b$, where ΔS_b is the body surface segment length, and L_b is the dimension of the body along which the segment lies (in our example, B or D).

Optimal values for $\Delta t/T$, $\Delta S/L$, and $\Delta S_b/L_b$ have been determined by letting each vary in turn while holding all other parameters constant and recording the force results.

It is found that as $\Delta t/T$ is decreased, the force results quickly converge to uniform values. Little increase in accuracy is gained by setting $\Delta t/T$ to values less than 0.05, and for $\Delta t/T = 0.04$ the results obtained are within 3% of the converged values.

The segment length parameter $\Delta S/L$ was set to values ranging from 0.05 to 0.25. The earlier dlscussion concerning upper and lower limits for $\Delta S/L$ is confirmed as the results remain relatively consistent when $\Delta S/L$ ls ln the vlcinlty of 0.1, dlverglng as $\Delta S/L$ ls lncreased or decreased beyond a limited range. When $\Delta S/L$ ls set to a value less than approximately twlce the value of $\Delta t/T$, the results diverge to the point where the computer solution is unable to contlnue. It is concluded that $\Delta S/L$ should be at least two or three times greater than $\Delta t/T$, and that a value of 0.1 for $\Delta S/L$ will yield good results.

As previously discussed, a global limit on $\Delta S/L$ may not provlde enough body segments to calculate the force results accurately. One should also check that the segment lengths on the body surface as deflned by $\Delta S_b/L_b$ fall wlthln the global constralnts on $\Delta S/L$ determined above. Values of $\Delta S_b/L_b$ ranging from 0.1 to 0.25 have been tested. As with the global segment length parameter $\Delta S/L$, $\Delta S_b/L_b$ is found to be constrained by upper and lower bounds. A maximum value of $\Delta S_b/L_b = 0.2$ (five segments along each body dimenslon) is found to be necessary to adequately describe the pressure distribution on the body. More segments on the body, whlle desirable to define the pressure distribution more accurately, would potentlally confllct wlth the requirement that $\Delta S/L$ be larger than $\Delta t/T$. It is recommended that for a particular body shape, a minimum of five segments be used along each body dlmenslon and that the relationship of $\Delta S_b/L_b$ to $\Delta S/L$ and $\Delta t/T$ be checked to ensure that it falls within the required range.

4.2. Time Stepping Equatlon

As mentioned in Sectlon 2, the time stepplng equatlons for η and ϕ on the free surface are obtained from (4) and (5). Applying a central difference approximation to (4), we obtain

$$\eta_{t+\Delta t} = \eta_{t-\Delta t} + 2\Delta t \frac{1}{n_z}\left(\frac{\partial \phi}{\partial t}\right)_t .$$

A similar expresslon can be obtalned for $\phi_{t+\Delta t}$ on S_f using (5).

For better accuracy, higher order time stepping methods are desirable. For our purposes, the Adams-Bashforth multistep methods are useful. These are described by Burden, Faire and Reynolds (1978).

The central dlfference method and the Adams-Bashforth two, three, four, and five step methods have each been tested with the same input parameters, and the forces at each tlme step recorded for each method. The plotted results are shown in Figure 6. It is found that the central difference method produces a slightly uneven plot, particularly for the first few time steps. The two, three, and four step methods give smoother results. The five step method produces highly erratic results which sawtooth about values coincident with the other plots.

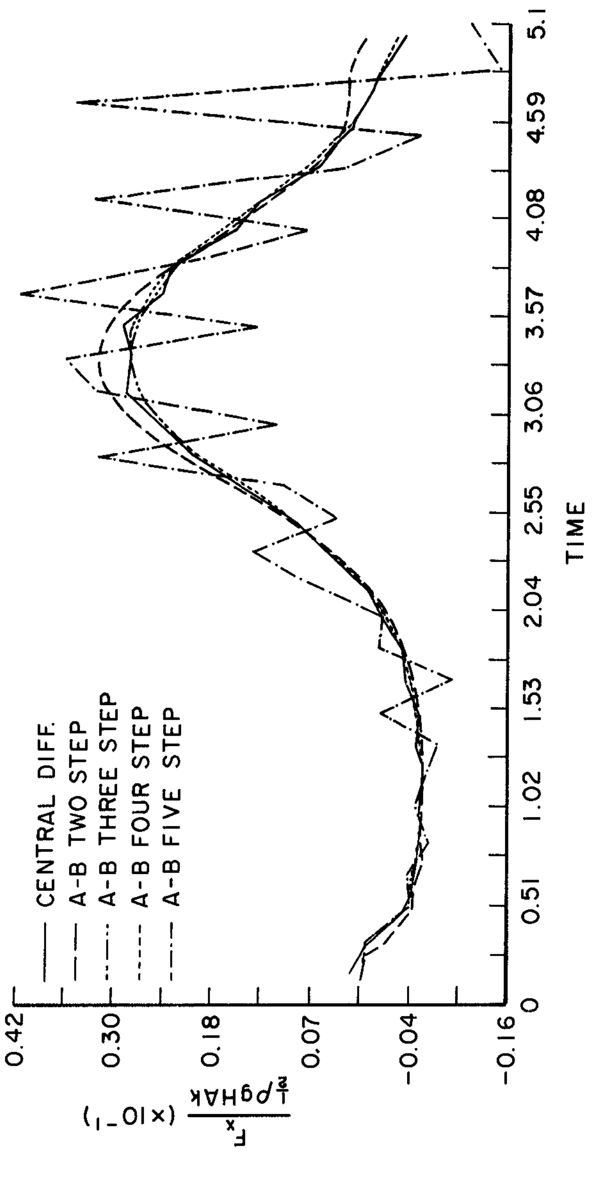

FIG.6: EFFECT OF VARYING TIME STEPPING METHODS.

The sawtoothed results of the five step method are probably caused by small perturbations from smooth results being magnified by the fitting of a fifth order polynomial to previous points when projecting forward at each time step to values at $t+\Delta t$. Anticipating that the same effect could occur for the four step method, the Adams-Bashforth two and three step methods have been adopted as the preferred time stepping techniques.

The program developed for this investigation was also tested without a body present, using the optimal values of the numerical parameters as determined above. The time stepping technique yielded a wave train progressing along the free surface as expected.

5. RESULTS

5.1. Exciting Forces

Using the method described in Section 2, the exciting forces on a two-dimensional rectangular cross-section were obtained for a range of incident wave angular frequencies ω, and for different values of wave height to water depth ratios, H/d. An incident wave decay length of 1.25 wavelengths was used, with 25 time steps per wave period, 10 segment lengths per wavelength, and 5 segments along each body dimension. The Adams-Bashforth three step method was used as the time stepping technique for the equivalent of (4) and (5).

The force results for a beam to draught ratio, $B/D = 4.0$ are plotted in Figure 7, with Vugts' (1968) deep water experimental results and Fraser's (1979) linear finite element results shown for comparison. The results of Figure 7 clearly demonstrate the non-linearity of the method, with different force curves resulting from different values of H/D. The results show that the method used here has a fairly wide range of application, and that the magnitude of most of the force results compares well to previous experimental and theoretical results. While the results for the horizontal exciting force coincide closely with Fraser's linear results, the non-linear method produces curves for the vertical force and moment that differ significantly in slope from the linear predictions.

5.2. Transmission Coefficient

An interesting byproduct of the method is the capability to monitor the transmission coefficient for the fixed body case. Since the fixed case is used here only as an idealization to calculate exciting forces that can be used for a subsequent analysis of the actual response of a moored breakwater, the observed transmission coefficients are of little or practical value, but do provide a lower limit for the transmission coefficient of a responding body.

The program was run with the same input parameters described in Section 5.1, but for $B/D = 2.0$, 4.0, and 8.0 in turn. The results for the transmission coefficient K_d are shown in Figure 8 together with the results of Nece and Richley (1972) for $B/D = 5.0$. The experimental results are for an incident wave of the same steepness for a given ω as for our results.

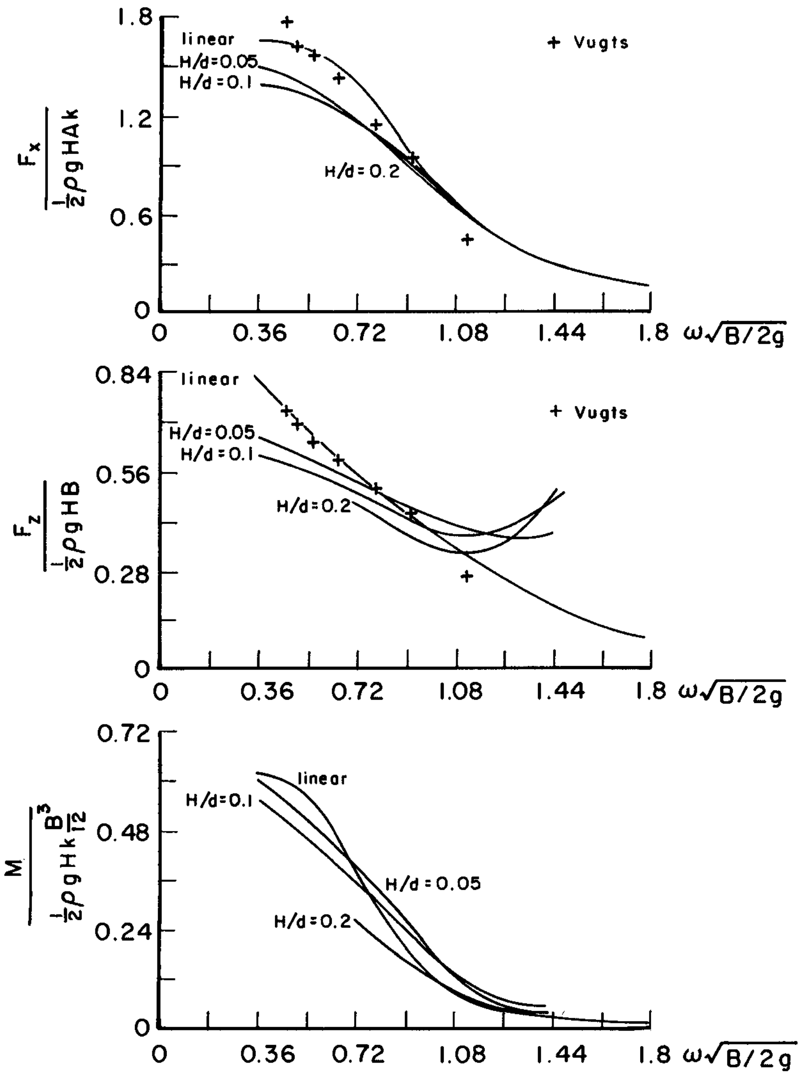

FIG. 7: FORCE RESULTS FOR B/D = 4.0 .

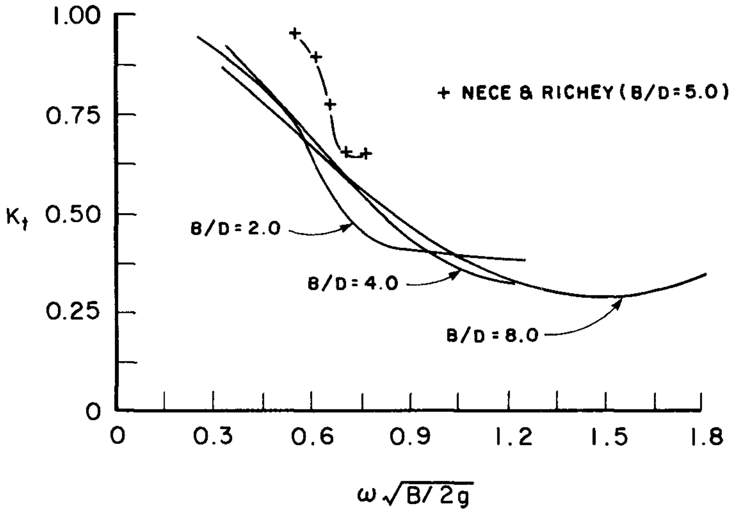

FIG.8: TRANSMISSION COEFFICIENT RESULTS.

The results obtained for the transmission coefficient are found to be relatively independent of water depth over the range tested. As expected, the present results for a fixed body give lower values for K_d than Nece and Richey's experimental results for a responding cable moored body, and thus appear to provide a reliable lower limit for K_d.

6. CONCLUSIONS

By testing a computer program based on the nonlinear numerical method described here, optimal values of the numerical parameters were determined. It was found that 25 or more time steps per wave period and at least 10 segments per wavelength were needed to obtain accurate flow development. There should also be at least 2 to 3 times as many time steps as segment lengths per wavelength. The decay length of the initial incident wave profile should be 1.25 or more wavelengths.

The method itself as defined by the physical conditions and boundary integral determines the group velocity of the flow development, and specifying a different value of the group velocity for the initial time steps changes the flow development of only the first few time steps.

Using a central difference time stepping equation produced a slight sawtooth effect in the force results, while using a four or five step method occasionally produced a divergent instability, probably due to the fitting of a higher order polynomial to the values at previous time steps. The recommended time stepping procedure is a two or three step Adams-Bashforth method.

Force results from the program were plotted in the appropriate dimensionless form over a range of frequencies. The results produced different force curves for different values of the wave height to water depth ratio, thus verifying the nonlinearity of the method. The force curves obtained compared well in magnitude to previous numerical and experimental results, although the slopes of the curves varied significantly. The transmission coefficient for the fixed rectangular breakwater over the range of frequencies tested appeared to provide a reliable lower bound for the transmission coefficient of responding bodies. It is concluded that the nonlinear method used here has a significant potential for improved accuracy in the prediction of forces on floating bodies and their transmission coefficients.

7. REFERENCES

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