Wave Forces on Piles of Variable Diameter

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SUMMARY

A method of calculating nonlinear wave induced forces and moments on piles of variable diameter is presented. The method is based on the Morrison equation and the linear wave theory with correction parameters to account for convective inertial effects in the wave field. These corrections are based on the stream function wave theory by Dean (1974). The method permits one to take into account the added wave force due to marine growth in the intertidal zone or due to a protective jacket, and can also be used to calculate forces on braces and an array of piles.

INTRODUCTION

Design of coastal structures such as piers requires calculation of wave induced forces on piles. The basic methodology is based on the application of the Airy theory in the Morrison equation. This has been a very useful tool, but the Airy theory does not account for many of the nonlinear dynamic and kinematic effects which finite amplitude waves exhibit. Several investigators have used various nonlinear wave theories and corrections to linear theory. The stream function wave theory by Dean (1965) has been determined to be one of the most originally accurate theories (LeMehaute and Dean, 1970). It has been used by Dean (1974) to determine wave forces on piles. Dean has also used stream function theory to develop a number of simple graphs which give the total wave force and moment about the mudline on pile of uniform diameter (SPM, 1977).

Many coastal engineering applications require calculation of the wave loading distribution on the pile. On a pier, the top of the pile is in some cases fixed and therefore the moment about the mudline is not a useful parameter, the designer requires the load distribution. Furthermore, the geometry of many marine structures are complicated by

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Photograph 1. Pile of Variable Diameter.

the presence of marine fouling. Barnacles, mussels and other marine organisms can significantly increase the diameter of a pile in the intertidal zone as shown in Photograph 1. Wooden piles are often protected by, or their useful life extended by, a jacket wrapped around the pile. Both marine growth and protective jackets introduce a variable diameter pile near the water surface. Because the diameter is increased near the free surface, wave forces are increased for a given wave condition. Figure 1 schematically shows a pile of three diameters which could represent the situation of marine growth on a protective jacket on a pile.

The objective of this paper is to present a methodology for the coastal engineer to readily calculate the nonlinear, wave-induced loadings on piles of variable diameter. Considering the large number of parameters, it is not possible to present an exact solution to the problem in the form of a few graphs. Therefore an approximate method has been developed in which the velocity and acceleration fields, and forces and moments are initially obtained at the pile location from the linear wave theory. These parameters are integrated from the sea floor into an arbitrary elevation, z_i, which can be the free surface. The free surface is given by the nonlinear wave theory of Dean (1974). Then correction coefficients are introduced to account for the nonlinear convective inertial forces on the velocity and acceleration fields. By so doing the method is, with a few approximations, amenable to description by a limited number of graphs. The graphs reduce by an order of magnitude the interpolations required and permits the engineer to calculate the load distribution to a high degree of accuracy with a minimal effort. Furthermore, the pile can comprise multiple diameters.

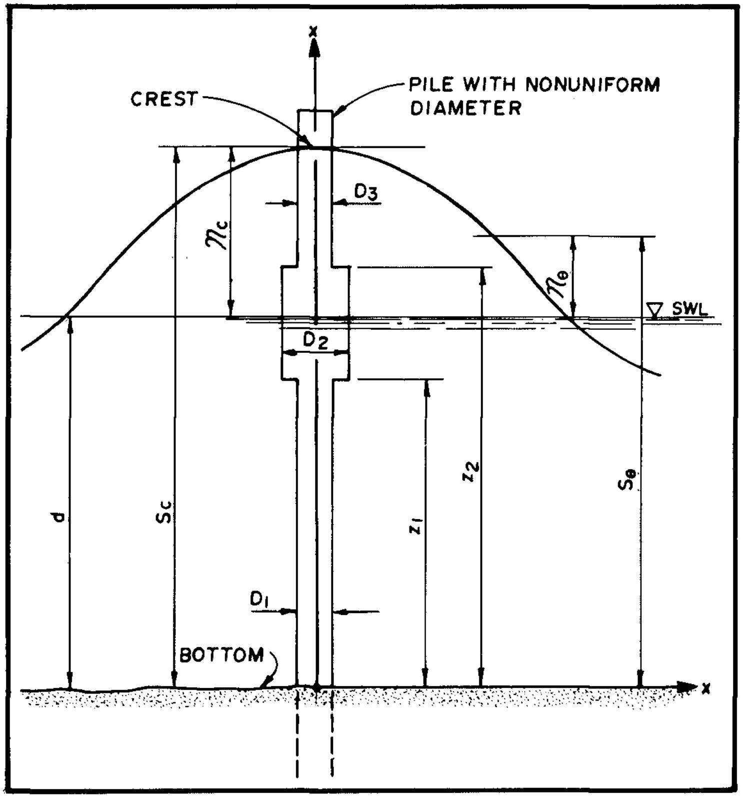

Figure 1. Definition Sketch for a Pile with Three Diameters.

The force on an array of piles can be determined by calculating the forces at various phase angles.

The methodology therefore permits the engineer to calculate wave forces on piles taking into account many of the physical properties of waves observed in nature and the laboratory as well as special circumstances which must be addressed in practical design problems. The force and moment correction factors are primarilly greater than unity and therefore will yield forces and moment which are greater than those calculated by linear theory. Where force and moment correction factors are less than unity, the correction was retained as unity for the sake of conservatism.

FORCES ON CYLINDRICAL PILES: BASIC FORMULATION

A cylindrical vertical pile subjected to a time dependent horizontal velocity, u(t), has a force, f, per unit length of cylinder which is the sum of a drag force, f_D, and an inertia force, f_I:

$$f = f_D + f_I = \tfrac{1}{2} \rho C_D u|u| + \rho C_m \frac{\pi D^2}{4} \frac{du}{dt} \tag{1}$$

where ρ is the density of sea water, D is the pile diameter, u is the particle velocity, $\frac{du}{dt}$ is the particle acceleration, C_D is the drag coefficient, and C_m is the inertia coefficient.

When a cylindrical pile is subjected to a water wave, one considers that these equations hold true, provided u(t) is the horizontal component of the velocity field at the pile location as if the pile did not exist. The deformation of the velocity field by the pile, wave diffraction, the effect of the vertical velocity component, the vertical acceleration component and the elasticity of the pile are neglected. The equation is commonly called "Morrison's equation".

The total force, F_T, on the pile is determined by integrating the unit forces from the sea floor to the water surface, S_θ.

$$F_T = \int_0^{S_\theta} f \, dz \tag{2}$$

where subscript θ refers to phase angle.

Forces due to nonlinear waves over piles of variable diameter, D_i, can be calculated using equations (1) and (2) using expressions for u and du/dt from the linear wave theory and adjusting them according to results obtained from the nonlinear wave theory. Nonlinear corrections are taken from the stream function theory as presented by Dean (1974). Two basic corrections are made: the asymmetric free surface correction and the nonlinear correction to the wave field.

Figure 1 shows a pile of three diameters, D_1, D_2 and D_3. The total force, F_T, acting from the sea bottom (z=0) to the elevation of the free surface ($z=S_\theta$) is given by:

$$F_T = \int_0^{z_1} f(D_1)\, dz + \int_{z_1}^{z_2} f(D_2)\, dz + \int_{z_2}^{S_\theta} f(D_3)\, dz \qquad (3)$$

where F_T = total force on the pile and $f(D_i)$ = total force per unit length acting on pile diameter D_i.

Graphs can be constructed which integrate unit force and moment from the sea floor to an elevation z_i. Then equation (3) can be written.

$$F_T = \int_0^{z_1} f(D_1)\, dz + \int_0^{z_2} f(D_2)\, dz - \int_0^{z_1} f(D_2)\, dz + \int_0^{S_\theta} f(D_3)\, dz - \int_0^{z_2} f(D_3)\, dz \qquad (4)$$

This expression can be written:

$$F_T = F(D_1)|_{z_1} + F(D_2)|_{z_2} - F(D_2)|_{z_1} + F(D_3)|_{S_\theta} - F(D_3)|_{z_2} \qquad (5)$$

where $F(D_i)|_{zi}$ = wave force acting on a pile of diameter D_i from the sea bottom to a level z_i.

Use of equation (5) requires evaluation of the force $F(D_i)|_{zi}$ on a pile of arbitrary diameter, D_i, and elevation above the bottom, z_i. This is done using the linear wave theory with the appropriate correction coefficients in order to account for the effects due to the nonlinear wave theory. This gives:

$$F(D_i) = \tfrac{1}{2}\, \rho\, C_D\, D_i \frac{H^2}{T^2}\, d\, K_D|_{z_i}\, \phi_D + \rho\, C_m \frac{\pi D_i^2}{4}\, d \frac{H}{T^2}\, K_I|_{z_i}\, \phi_I \qquad (6)$$

where H = wave height, T = wave period, and d = water depth. $K_D|_{zi}$, and $K_I|_{zi}$, are dimensionless drag and inertial coefficients respectively, obtained by integration from the mudline to an elevation z_i, and ϕ_D and ϕ_I are correction factors to the forces obtained by the linear theory. ϕ_D is the correction relating to u^2 and ϕ_I is the correction relating to du/dt.

According to linear theory:

$$K_D|_{z_i} = K_{DM}|_{z_i} \cos\theta|\cos\theta| \qquad (7)$$

$$K_I|_{z_i} = K_{IM}|_{z_i} \sin\theta \qquad (8)$$

The maximum values $K_{DM}|_{zi}$ and $K_{IM}|_{zi}$ integrated from the mudline to an elevation z_i are:

$$K_{DM}\big|_{z_i} = \frac{1}{8}\frac{gT^2}{d}\left[1 + \frac{2kz_i}{\sinh 2kz_i}\right]\frac{\sinh 2kz_i}{\sinh 2kd} \qquad (9)$$

$$K_{IM}\,z_i = \tfrac{1}{2}\frac{gT^2}{d}\frac{\sinh 2kz_i}{\cosh kd} \qquad (10)$$

where k = wave number = $2\pi/L$ and L = wavelength. $K_{DM}\big|_{zi}$ and $K_{IM}\big|_{zi}$ are given by figures 2 and 3, respectively, as a function of z/d and d/L_o.

NONLINEAR FREE SURFACE

$F(D_3)\big|_{zi}$ at $z_i = S_\theta$ requires the knowledge of the free surface elevation S_θ as function of phase angle, θ. This is done by applying the free surface given by the stream function wave theory by Dean (1974). The results are presented in the form of S_θ/d as a function of H/H_b, d/L_o and θ, in figure 4. H_b is the limit wave height as defined by Dean (1974).

The use of figure 4 is illustrated by an example. Given d/L_o = .033, θ = 20° and H/H_b = .75 enter figure 4 vertically with a value of d/L_o = .033. As indicated by the arrow in the figure, one proceeds downward until the line of H/H_b = .75 is intersected. A line is then drawn horizontally towards the right until the θ = 20° line is intersected. Now move vertically downward to read S_θ/d = 1.256.

CORRECTIONS FOR NONLINEAR WAVE KINEMATICS

The second correction to linear theory is due to the nonlinear wave particle velocity and acceleration fields. The correction coefficients ϕ_D and ϕ_I incorporate a number of nonlinear effects, most notably due to the convective acceleration terms. These nonlinear effects are a complex function of relative depth, d/L_o, wave phase angle, θ, the ratio of the local height to breaking height, H/H_b, and the ratio of elevation to water depth, z/d. These are given by the ratio of the forces obtained by using the values obtained by a nonlinear wave theory to the corresponding value given by linear theory. To incorporate all of these variables at each z would require a large number of nomographs. Therefore a conservative approach was adopted where the nonlinear correction factors at the free surface were applied over the water column. While the nonlinear effects actually vary with z/d and θ, the most important corrections are near the free surface. Therefore only a global nonlinear correction is applied over the entire pile length, from the sea bottom to the nonlinear free surface. Also the phase variations given by cosθ and sinθ respectively, are retained in the general equation. The error resulting from this simplification is that forces are generally over-predicted by a few percent at levels below the free surface. Figures 5 and 6 present the nonlinear correction factors ϕ_D and ϕ_I as a function of phase angle, θ, H/H_b and d/L_o. Note that the corrections approach unity for θ = 30° and 50° for drag and inertia respectively. For greater θ, corrections are less than unity, but a conservative design procedure would be to use the correction at unity for θ > 30° for drag forces and θ > 50° for inertia forces.

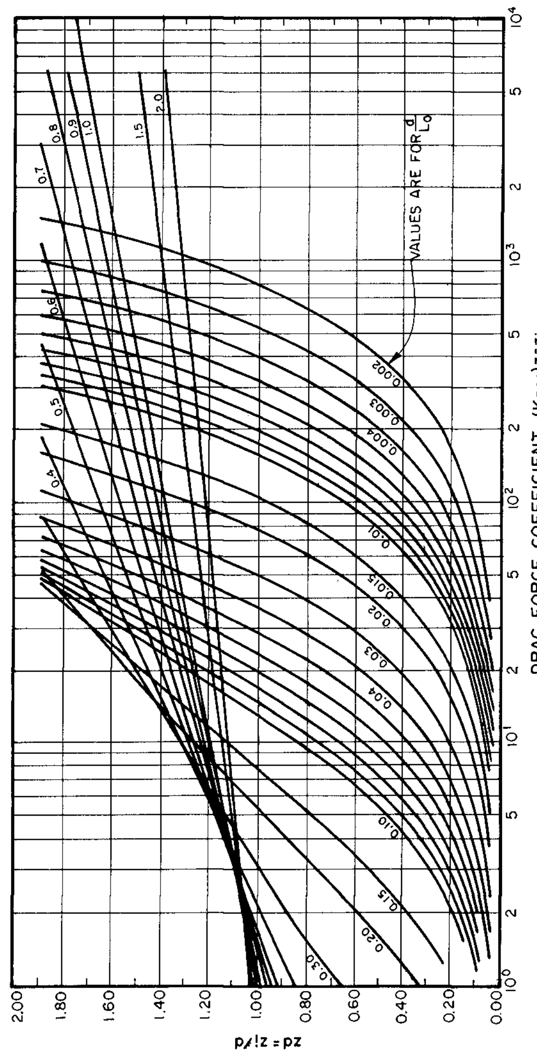

Figure 2. Drag Force Coefficient.

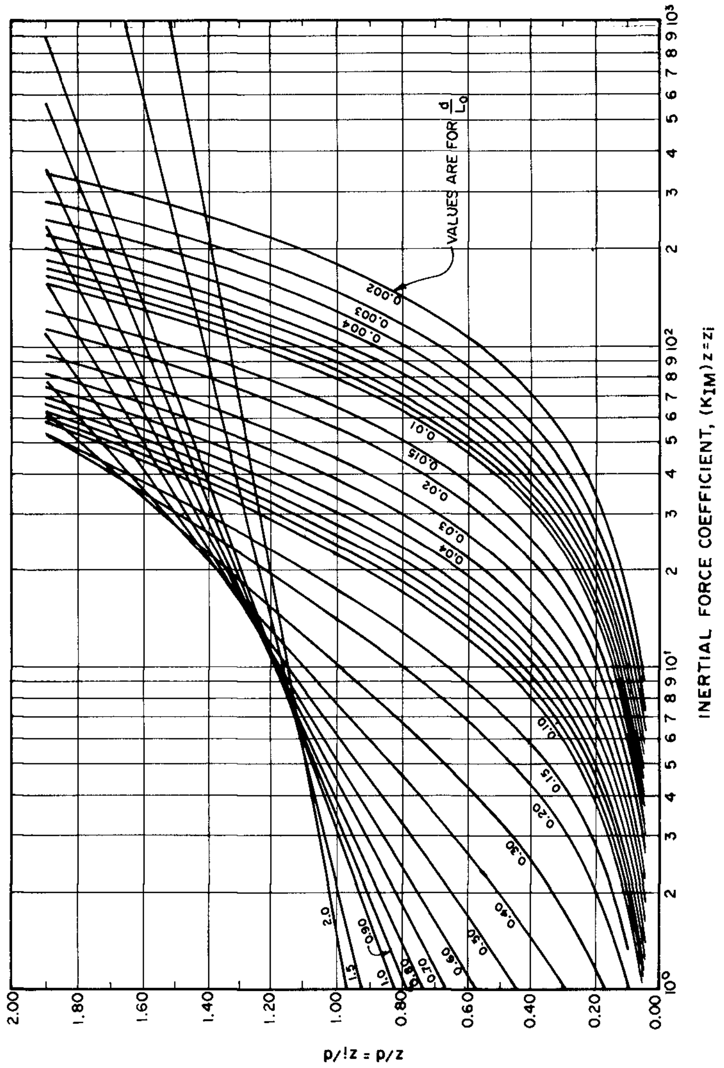

Figure 3. Inertial Force Coefficient.

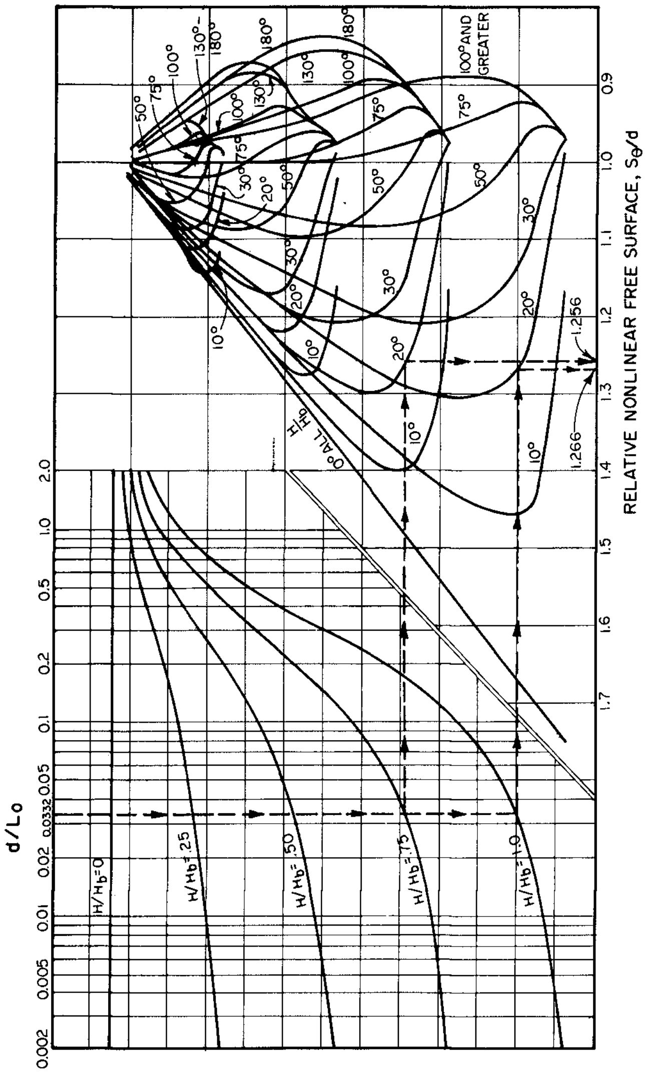

Figure 4. Nonlinear Free Surface.

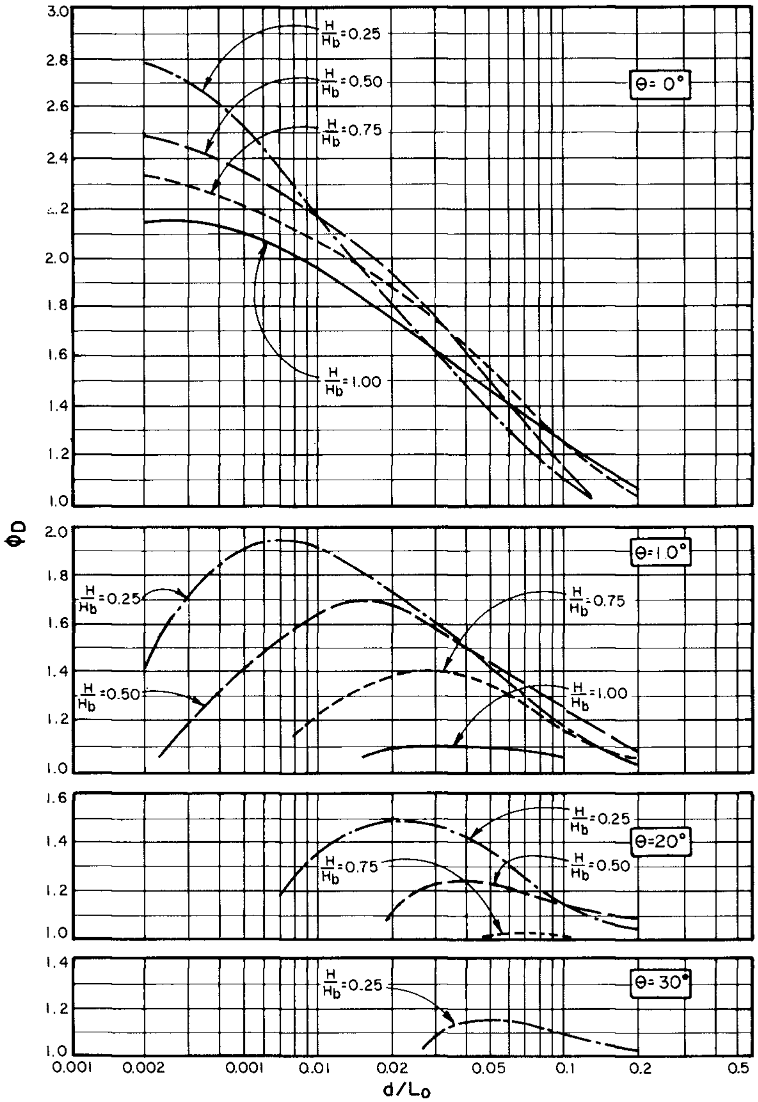

Figure 5. Nonlinear Drag Force Correction Factor.

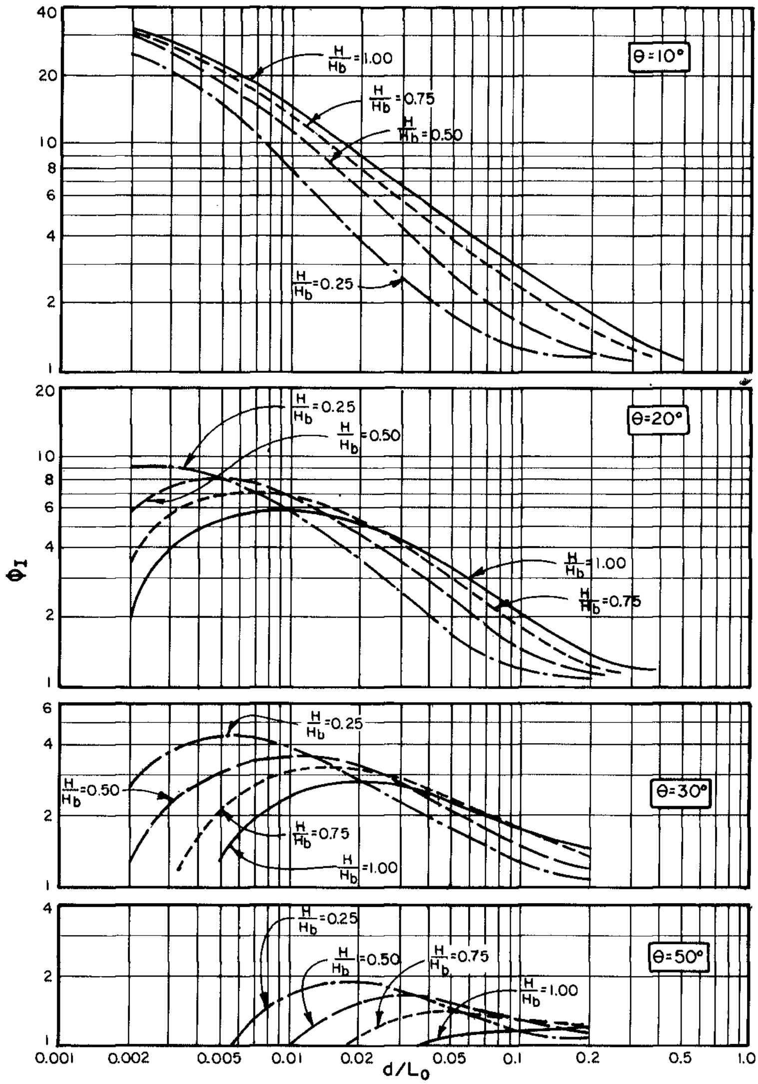

Figure 6. Nonlinear Inertial Force Correction Factor.

TOTAL FORCE ON PILE OF VARIABLE DIAMETER

Based on these assumptions the total force on a pile of three diameters can now be written:

$$F_T = F_D + F_I \tag{11}$$

$$F_D = \tfrac{1}{2}\,\rho\, C_D \frac{H^2}{T^2}\, d\, [K_{DM}|_{z_1} (D_1 - D_2) + K_{DM}|_{z_2} (D_2 - D_3) + K_{DM}|_{S_\theta} (D_3)]\, \phi_D \cos\theta|\cos\theta| \tag{12}$$

$$F_I = \rho\, C_M \frac{\pi d}{4} \frac{H}{T^2}\, [K_{IM}|_{z_1} (D_1^2 - D_2^2) + K_{IM}|_{z_2} (D_2^2 - D_3^2) + K_{IM}|_{S_\theta} (D_3^2)]\, \phi_I \sin\theta \tag{13}$$

MOMENT CALCULATIONS

Expressions analogous to the above force equations are presented for calculation of the wave moment on a pile of diameter, D_i, at an elevation acting about the sea bottom. The moment expression is:

$$M_T = \int_0^{S_\theta} f\, z\, dz \tag{14}$$

Referring to figure 1 and using a similar approach, the total moment, M_T, acting about the sea bottom can be written as:

$$M_T = M(D_1)|_{z_1} + M(D_2)|_{z_2} - M(D_2)|_{z_1} + M(D_3)|_{S_\theta} - M(D_3)|_{z_2} \tag{15}$$

where M_T = total moment acting about the sea bottom; $M\,(D_i)|_{zi}$ = wave moment acting on a pile of diameter D_i about the sea bottom to a level z_i. Equations for $M\,(D_i)|_{zi}$ are given:

$$M(D_i)|_{z_i} = \tfrac{1}{2}\,\rho\, C_D \frac{H^2}{T^2} d^2\, \Gamma_D\, (D_i)|_{z_i}\, \Psi_D + \rho\, C_M \frac{\pi D_i^2}{4} \frac{H}{T^2}\, d^2 \Gamma_I (D_i)|_{z_i}\, \Psi_I \tag{16}$$

where:
$$\Gamma_D(D_i)|_{z_i} = \Gamma_{DM}(D_i)|_{z_i} \cos\theta|\cos\theta| \tag{17}$$

$$\Gamma_I(D_i)|_{z_i} = \Gamma_{IM}(D_i)|_{z_i} \sin\theta \tag{18}$$

$$\Gamma_{DM}(D_i)\Big|_{z_i} = \frac{1}{2}\frac{g^2T^4}{g^2}\frac{1}{(8\pi)^2}\frac{1}{\cosh^2 kd}\,[1 + 2(kz_i)^2$$

$$+ 2kz_i \sinh 2kz_i - \cosh 2kz_i] \qquad (19)$$

$$\Gamma_{IM}(D_i)\Big|_{z_i} = \frac{1}{8\pi^2}\frac{g^2T^4}{d^2}\frac{\sinh kd}{\cosh kd}\,[1 + kz_i \sinh kz_i - \cosh kz_i] \qquad (20)$$

$\Gamma_{DM}(D_i)|_{zi}$ and $\Gamma_{IM}(D_i)|_{zi}$ are given as functions of z/d and d/L_o in figures 7 and 8 respectively and can be evaluated up to the free surface elevation given by figure 4. Ψ_D is the drag moment correction for the velocity field and Ψ_I is the inertial moment correction for the acceleration field. Other variables are as previously defined. The nonlinear moment correction factors, Ψ_D and Ψ_I, are given in figures 9 and 10, respectively, as functions of d/L_o, θ, and H/H_b.

MAXIMUM VALUES - EFFECT OF PHASE ANGLE

The total maximum force and total moment phase angle cannot be readily determined for a pile of variable diameter. The drag force is maximum at $\theta = 0$, and for small diameter piles, the maximum value is near $\theta = 0$; but as the diameter increases the inertial force becomes more prevalent and shifts the location of the maximum total force toward $\theta = 90°$. The maximum inertial force occurs at some unknown angle, but its maximum value can be determined as a function of H/H_b and d/L_o only by application of formula (13) in which one takes $z_3 = d$ and one replaces $\phi_I \sin\theta$ by ϕ_{IM} given by figure 11. A similar method applies to calculate the maximum moment due to inertia. The correction factor, Ψ_{IM}, is then given by figure 12.

CONCLUSION

The preceding outlines the methodology for determining nonlinear wave forces on piles of variable diameters. The methodology greatly simplifies the interpolation required in using stream function theory and permits one to estimate wave forces and force distribution over the pile column. The method presented is for a general case. The US Navy Manual DM26.2 (1982) describes other cases in more detail to simplify the wave force and moment calculation for special cases.

By considering θ as a variable, the time history of the total force and moments on piles of varying diameter can be determined. Therefore the method is amenable to determine the total wave forces on structures supported by a number of piles.

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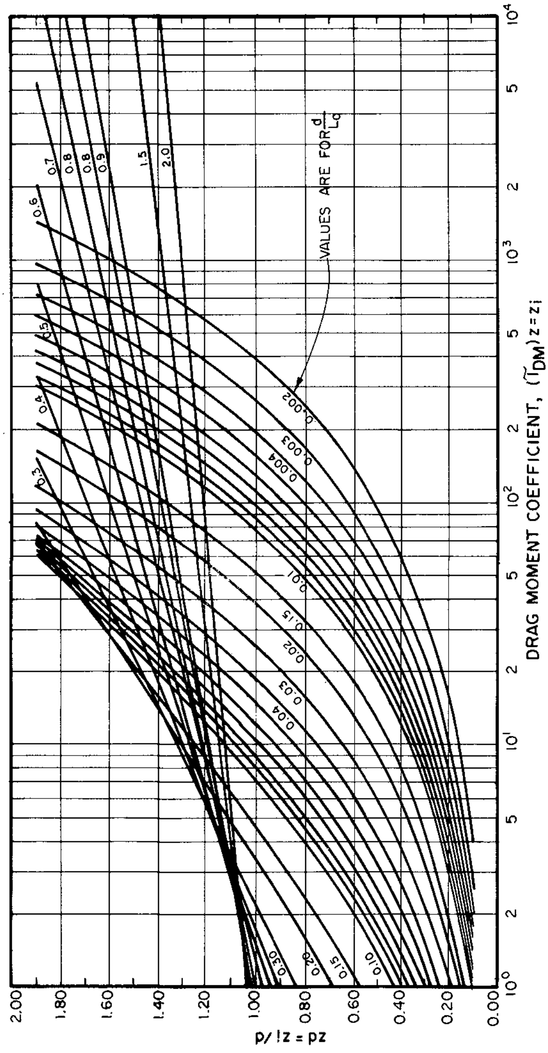

Figure 7. Drag Moment Coefficient.

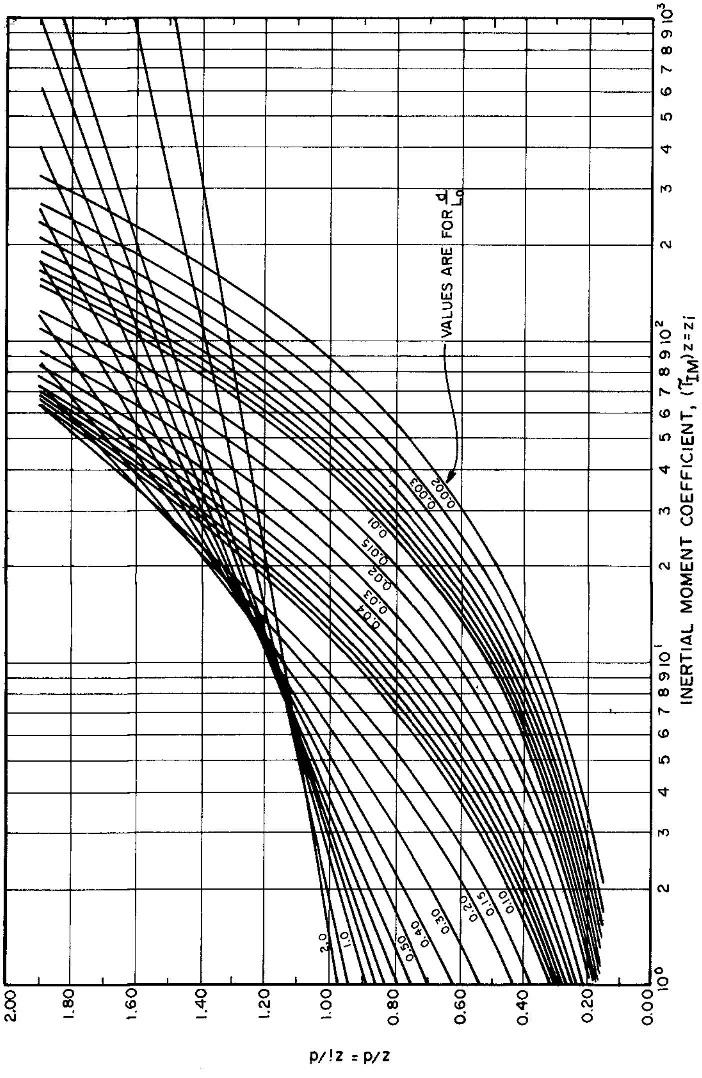

Figure 8. Inertial Moment Coefficient.

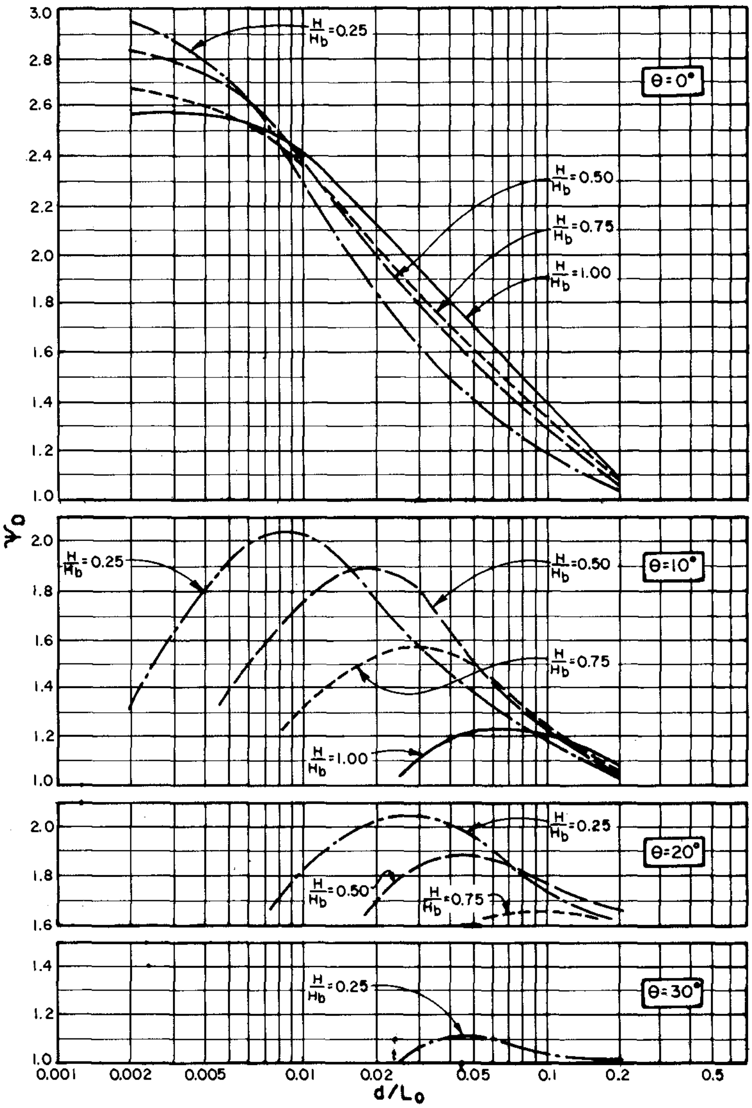

Figure 9. Nonlinear Drag Moment Correction Factor.

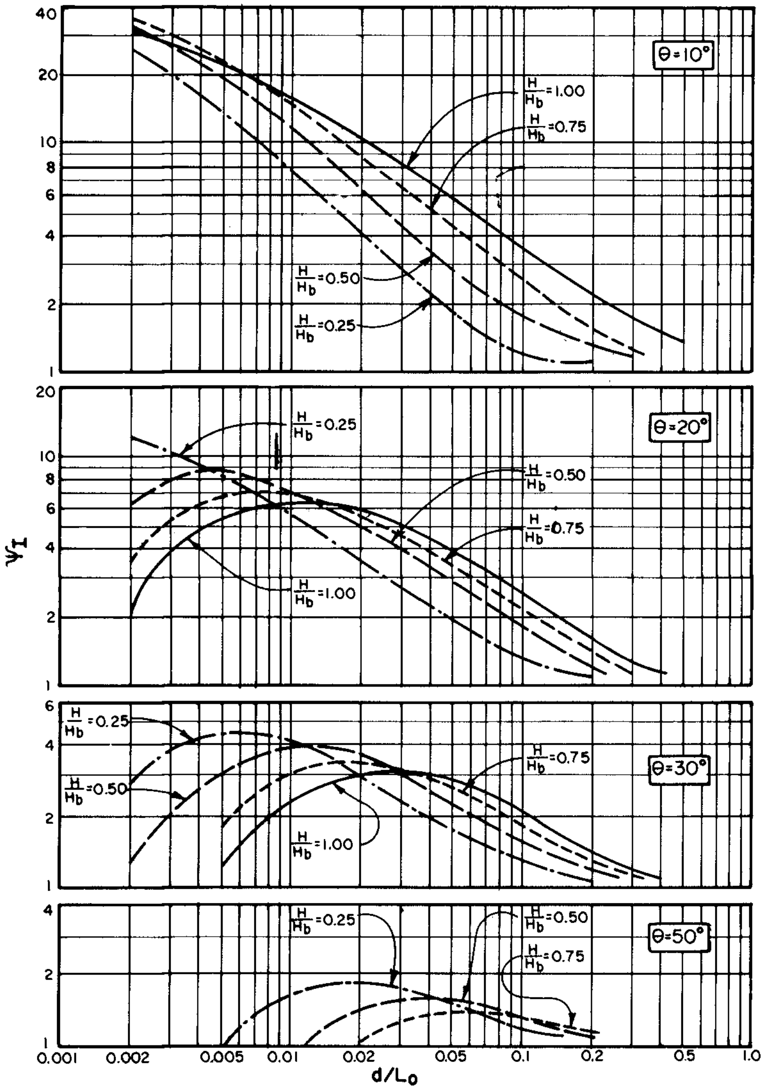

Figure 10. Nonlinear Inertial Moment Correction Factor.

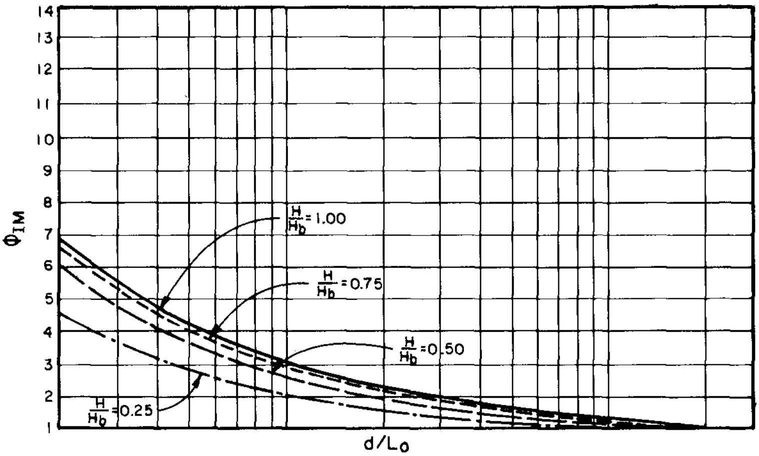

Figure 11. Maximum Nonlinear Inertial Force Correction.

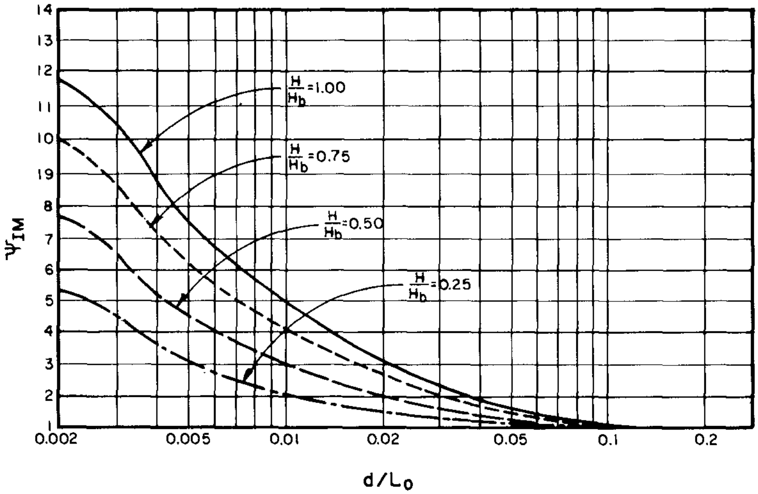

Figure 12. Maximum Nonlinear Inertial Moment Correction.

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NOTATION

C_D = drag coefficient

C_m = inertia coefficient

d = water depth

D_i = pile diameter

$f(D_i)$ = wave force per unit length of pile of diameter D_i

$F(D_i)$ = wave force on a pile of diameter D_i

F_T = total wave force

g = 32.2

H = wave height

k = $2\pi/L$

$K_D|_{z_i}$ = linear drag force coefficient evaluated at an elevation zi above the sea bottom

$K_{DM}|_{z_i}$ = maximum linear drag force coefficient evaluated at an elevation zi above the sea bottom

$K_I|_{z_i}$ = linear inertia force coefficient evaluated at an elevation zi above sea bottom

$K_{IM}|_{z_i}$ = maximum linear inertia force coefficient evaluated at an elevation z_i above the sea bottom

L = wavelength

$M(D_i)$ = wave moment on a pile of diameter D_i

S_θ = free surface elevation at arbitrary wave phase angle θ

T = wave period

z_i = elevation above the bottom

$\Gamma_D|_{z_i}$ = linear drag moment coefficient evaluated at an elevation zi above the sea bottom

$\Gamma_{DM}|_{z_i}$ = maximum linear drag moment coefficient evaluated at an elevation zi above the sea bottom

$\Gamma_I|_{z_i}$ = linear inertia moment coefficient evaluated at an elevation zi above the sea bottom

$\Gamma_{IM}|_{z_i}$ = maximum linear inertia moment coefficient evaluated at an elevation zi above the sea bottom

θ = wave phase angle

ρ = density of water

ϕ_D = nonlinear drag force correction factor

ϕ_I = nonlinear inertia force correction factor

Ψ_D = nonlinear drag moment correction factor

Ψ_I = nonlinear inertia moment correction factor