COMPUTATIONAL ALGORITHM FOR LONGSHORE ENERGY FLUX INCORPORATING FRICTION

by

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Introduction

The calculation of longshore sand transport on beaches is a significant coastal engineering problem with application to various areas of coastal structure design (i.e., jetties, groins, and offshore breakwaters), and inlet navigation channel design (i.e., studies of required maintenance dredging). Longshore sand transport as a first approximation is linearly related to longshore energy flux (see Bruno, et al. (1981)), hence, this paper simply presents a method for computing longshore energy flux as a means of determining longshore sand transport.

The approach used herein for calculating longshore energy flux includes an analytical method for incorporating frictional wave energy dissipation. The method is simple enough to program on a hand-held programmable calculator. It therefore provides a method by which rapid calculations can be made for a site at which offshore wave data exist. If offshore directional random wave data are available (i.e. directional wave spectra) then more advanced techniques should be used (see Walton and Dean (1981)).

Computation of the longshore energy flux factor $P_{\ell s}$ is in accordance with the Shore Protection Manual (1977) equation (4-28)

$$P_{\ell s} = \frac{\rho g}{16} H_b^2 \, C_{gb} \, \sin 2\alpha_b \tag{1}$$

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in which $P_{\ell s}$ is the longshore energy flux, ρ is the mass density of the fluid, g is the acceleration of gravity, H_b is the breaking wave height, C_{gb} is the wave group velocity at the point of wave breaking, and α_b is the angle the breaking wave crest makes with the shoreline, where the subscript "b" denotes breaking wave conditions. The quantity $P_{\ell s}$ is not truly a longshore energy flux as Longuet-Higgins (1972) has noted, but since this terminology has been widely used (see Shore Protection Manual (1977)), this paper will not deviate from this usage.

Equation 1 can also be written as

$$P_{\ell s} = E_b \, C_{gb} \cos \alpha_b \sin \alpha_b \tag{2}$$

where E_b is the wave energy density at breaking given by,

$$E_b = \frac{\rho g}{8} H_b^2 \tag{3}$$

From conservation of energy considerations, for waves approaching shore over straight and parallel bottom contours,

$$E_i C_{gi} \cos \alpha_i = E_b C_{gb} \cos \alpha_b \tag{4}$$

if no energy dissipation has occurred from a given offshore site (represented by the subscript "i") to the breaker location (See Ippen (1966)). If energy dissipation is included, equation (4) must be modified to the following

$$K_e^2 \, E_i \, C_{gi} \cos \alpha_i = E_b C_{gb} \cos \alpha_b \tag{5}$$

where the factor K_e^2 accounts for energy reduction due to dissipation by bottom friction, percolation, or other dissipative mechanisms between the offshore site and the breaker location.

Equation (1) with frictional dissipation included can be rewritten as,

$$P_{\ell s} = K_e^2 \; (EC_g \cos \alpha)_i \; \sin \alpha_b \tag{6}$$

where offshore wave data can be used directly to evaluate the term in brackets.

The values of K_e and $\sin \alpha_b$ can be found from linear wave theory transformation processes and a breaking wave height to water depth ratio which is dependent on bottom slope and offshore (deep water) wave steepness.

In this paper the energy dissipation is assumed to be due only to bottom friction; percolation is neglected. From a practical standpoint, the importance of percolation in wave energy dissipation is questionable since in many offshore areas sand is underlain by mud and/or rock. Also, the top layer of sand often has mixed within it organic material and very fine silts that reduce the permeability of this layer.

Bretschneider and Reid (1954) developed equations for the friction coefficient K_f (where $K_f = K_e$ in the case of no percolation) for both constant bottom slope and constant depth cases. Their method of estimating wave height decay requires numerical integration for the case of a constant bottom slope. In the absence of refraction, a chart with solutions has been presented for various values of parameters T^2/d and $\frac{fH_o}{md}$; where T is the wave period, d is the water depth, m is the offshore slope, H_o is the deepwater wave height, and f is a friction factor.

The present approach simplifies the constant slope equation by invoking the shallow water assumption and provides an analytical solution. Over much of the range of the parameters T^2/d and $\frac{fH_o}{md}$, the analytical solution provides answers that are within 5% of the more involved numerical integration solution. The analytical solution of the friction coefficient K_f allows computation of

K_f and $P_{\ell s}$ on most hand-held programmable calculators. This simplified solution of K_f assumes straight and parallel offshore bottom contours.

The analytical solution for the friction coefficient, K_f, integrated from deep water to a shallow water depth d is:

$$K_f = \left(1 + \left(\frac{fH_o}{md}\right)(\cos\alpha_o)^{\frac{1}{2}}\ 0.12(k_o d)^{-\frac{3}{4}}\right)^{-1} \qquad (7)$$

The derivation of equation 7 is given in Appendix A. For values of $\frac{L_o}{d} \geq 15$, and $\frac{fH_o}{md} \leq 1.0$, equation 7 estimates K_f with less than 5% error. Since in most practical applications the friction coefficient f is rarely known with any accuracy, this approximate K_f is believed satisfactory for most engineering purposes.

The method for solving sin α_b is as follows: (1) determine explicitly the breaking wave height using linear wave theory, and the ratio of breaker wave height to water depth (dependent on bottom slope m and deepwater wave steepness) by assuming breaking occurs in shallow water; (2) determine the breaking wave depth from breaker height to water depth ratio; and (3) solve for α_b, the breaking wave angle, using Snell's Law of Refraction. This approach is detailed in Appendix B. The equation used to find the breaking wave height is

$$H_b = \left[\left(\frac{\kappa}{g}\right)^{\frac{1}{2}} K_f^2\ H_i^2\ C_{gi}\ \cos\alpha_i\right]^{0.4} \qquad (8)$$

where K_f represents a spatial average friction coefficient between the site where the wave data observations are available and the breaker site. The breaker depth is given by

$$d_b = H_b/\kappa \qquad (9)$$

where

$$\kappa = 1.16\left[m(H_o/L_o)^{-\frac{1}{2}}\right]^{0.22} \tag{10}$$

Equation 10 is from the work of Singamsetti and Wind (1980) who reviewed various equations for the breaking wave height to breaker depth ratio using Battjes (1974) data.

As the friction coefficient K_f of equation 7 depends on the ratio of deep-water wave height to breaking depth $\frac{H_o}{d_b}$, the solution technique used assumes that $H_o \approx H_b$ to a first approximation for directly computing K_f in equation 7.

The friction factor used is that defined by Bretschneider and Reid (1954) in which the bottom shear stress is defined by a shear stress equation given by

$$\tau_b = \rho f\ |U_b| U_b \tag{11}$$

where U_b is the bottom orbital velocity given by linear wave theory. Values of the friction factor f for oscillatory flow have been given by Kamphuis (1975) and Vitale (1979), where f is defined as a function of the relative roughness parameter $\frac{k_e}{\zeta_b}$ and an oscillatory Reynolds number $\frac{U_b\ \zeta_b}{\nu}$, with k_e = the equivalent sand grain size on the bed, ζ_b = the total horizontal excursion of the water particle motion at the bottom in the absence of a boundary layer, and ν = the kinematic viscosity of sea water.

Kamphuis (1975) notes that k_e can be related to the size distribution of the sand on the bottom by

$$k_e = 2d_{90}$$

where d_{90} is the sand grain diameter such that 90% is finer. Using a Moody-Stanton-type diagram (as is used to present pipe friction factors), Kamphuis (1975) has presented his friction factor, f_k, as a function of Reynolds number and relative roughness. Since Kamphuis used an alternative definition for his friction factor, the relationship between the friction factor used in this paper and Kamphuis' f_k is $f = \frac{f_k}{2}$.

Comparison of Measured and Predicted Breaking Wave Angles

The major error in calculating longshore energy flux involves predicting the breaking wave angle. The present algorithm for calculating breaking wave angle was compared to three sets of breaking wave data taken in three-dimensional laboratory wave basins. Two of these data sets (Shay and Johnson (1951), and Vitale (1981)) were obtained with movable bed models. The objective of their studies was to measure longshore sand transport and correlate transport rates with wave properties. The movable bed model tests had a large variation in breaking wave angle due to shoreline adjustment during the testing. Also, in the movable bed models, the breaking wave angle (defined to be the angle between the breaking wave and the shoreline) is more difficult to measure since the shoreline position is dynamic and difficult to define. The third set of data (Galvin (1965)) were from a fixed bed model. Observations of breaking wave angles had minimal variation and were averaged for each test to provide a good measure of the breaking wave angle. The friction factor used in the calculations (for all data) was assumed to be f = 0.1. This value appeared reasonable for the range of wave heights, periods, water depths, and bottom roughness for the laboratory tests. Results of the calculations were not sensitive to friction. Values of f ranging from 0 (no friction) to 0.1 did not change the correlation coefficients relating the calculations to the data by more than 5%, suggesting that frictional effects in the data are negligable relative to the overall scatter of the measurements and other difficulties inherent in measuring breaking wave angle.

In all laboratory tests the wave generators were in transitional water depths ($1/20 \leq d/L \leq 1/2$) and wave parameters in deep water were calculated using linear wave theory to provide input to the calculations.

The comparison between calculated and observed breaking wave angles is shown in Figure 1, along with the range of the variables and the correlation coefficient for each of the three data sets.

The best data for comparison (i.e., the least scatter for individual tests) was Galvin's (1965). This data set gave a correlation coefficient of $r = 0.97$.

Summary and Conclusions

A technique has been presented for calculating longshore wave energy flux which can be used in areas where offshore wave data are available and the offshore bottom contours are nearly straight and parallel. This method incorporates a simplified analytical technique for computing wave energy dissipation by bottom friction and can be applied with minimal computational effort using a hand-held programmable calculator.

Results for computation of breaking wave angles have been compared to existing laboratory data and found to provide good correlation in one set of tests and reasonable correlation (in view of laboratory data scatter) in two other sets of tests. It is felt that this method of computing longshore energy flux will find many useful applications in view of the simplicity of the algorithm developed.

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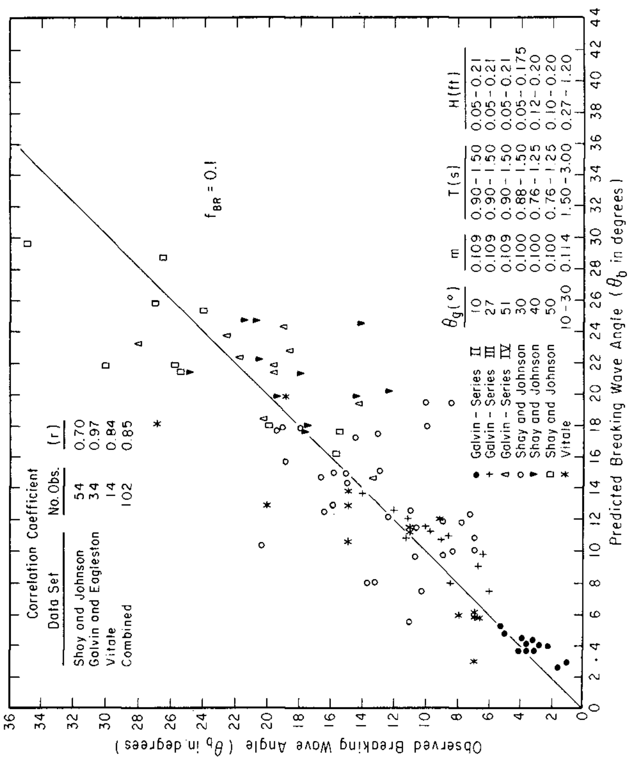

FIGURE 1 - Comparison of Observed and Predicted Breaking Wave Angles

APPENDIX A

Simplified Solution of Friction Coefficient K_f for Constant Bottom Slopes

Bretschneider and Reid (1954) have provided a numerical integration type solution for the friction coefficient K_f as given in equation (B-4a) of BEB TM 45. This solution is as follows:

$$K_f = \left(1 + \Big|\ \frac{f H_o}{m T^2} \int_{\infty}^{d/T^2} \phi_f K_r \ \delta(d/T^2)\ \Big|\right) \qquad \text{(A-1)}$$

where

$$\phi_f = \left(\frac{64\pi^3}{3g^2}\right)\left(\frac{K_s}{\sinh\ kd}\right)^3$$

and

K_r = refraction coefficient

K_s = shoaling coefficient

as normally defined in linear wave theory (see Ippen (1966)).

Equation A-1 can be nondimensionalized in terms of the deep water dimensionless wave number $k_o d$ and the site dimensionless wave number kd to be

$$K_f = \left(1 + \Big|\ \frac{f H_o}{md}\left(\frac{4}{3\pi}\right) k_o d \int_{\infty}^{k_o d} \left(\frac{K_s}{\sinh\ kd}\right)^3 K_r \ \delta(k_o d)\ \Big|\right)^{-1} \qquad \text{(A-2)}$$

Assuming that offshore bottom contours are straight and parallel, Snell's Law of Refraction can be applied from deep water to the site of interest where

$$K_r = (\cos\ \alpha_o / \cos\ \alpha)^{0.5} \qquad \text{(A-3)}$$

which can be reduced to the following for shallow water depth d to

$$K_r = (\cos\ \alpha_o)^{0.5} \left[(1 - (\sin^2\ \alpha_o)\ k_o d\right]^{-0.25} \qquad \text{(A-4)}$$

As $(\sin\alpha_o)^2 k_o d$ is small over most of the wave transformation zone, Equation A-4 is simplified to

$$K_r \approx (\cos\alpha_o)^{0.5} \tag{A-5}$$

Again assuming depth d is in shallow water

$$K_s \approx (2kd)^{-0.5} \tag{A-6}$$

Upon applying the above assumptions to the integral term of equation A-2 the integral becomes

$$\begin{aligned} I_1 &= \int_{\infty}^{k_o d} \frac{X^{-9/4}}{2^{3/2}}\, dX = \frac{1}{2^{3/2}} \left(-\frac{4}{5} X^{-5/4} \right) \Bigg|_{\infty}^{k_o d} \\ &\approx \frac{\sqrt{2}}{5} (k_o d)^{-5/4} \left(1 - (k_o d/\infty)^{5/4}\right) \\ &\approx \frac{\sqrt{2}}{5} (k_o d)^{-5/4} \end{aligned} \tag{A-7}$$

Applying the above integration to equation A-2 the simplified friction coefficient (from deep water to shallow water depth d) becomes

$$K_f = \left[1 + \left(\frac{4\sqrt{2}}{15\pi}\right)\left(\frac{fH_o}{md}\right) (\cos\alpha_o)^{0.5} (k_o d)^{-0.25} \right]^{-1} \tag{A-8}$$

For the case of no refraction ($\alpha_o = 0^o$ or $K_r = 1.0$) the equation becomes

$$K_f = \left[1 + 0.12 \left(\frac{fH_o}{md}\right) (k_o d)^{-0.25} \right]^{-1} \tag{A-9}$$

which can be compared to the values given by the Bretschneider and Reid (1954) complete solution, Equation A-2. The present solution and the percent error between the present solution and that of Bretschneider and Reid are given in Table A-1 and presented in Figure A-1 where the error E is defined as

$$E = \frac{K_f \text{ (Bretschneider-Reid)} - K_f \text{ (eq. A-9)}}{K_f \text{(Bretschneider-Reid)}} \tag{A-10}$$

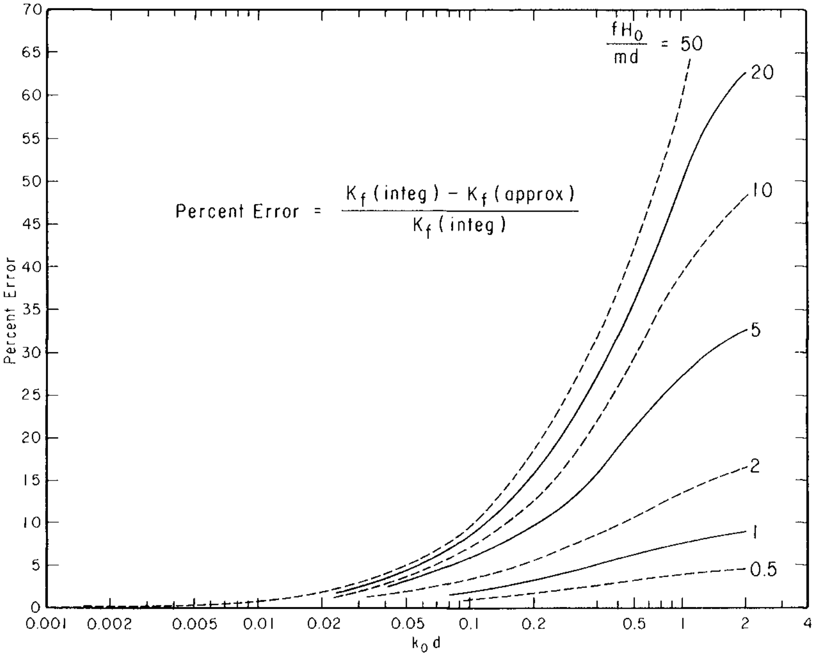

FIGURE A-1 - Percent Error in Approximate Solution

Table A-1

Comparison of Simplified Approximate Wave Height Attenuation by Friction to Asympototic Numerical Solution.

$k_o d$	fH_o/md	$K_{f(approx)}$	$K_{f(integ)}$	Error(%)
0.00200	0.05000	0.97241	0.97233	0.00869
0.00200	0.10000	0.94630	0.94614	0.01690
0.00200	0.20000	0.89808	0.89779	0.03208
0.00200	0.50000	0.77898	0.77844	0.06956
0.00200	1.00000	0.63798	0.63725	0.11391
0.00200	2.00000	0.46841	0.46763	0.16728
0.00200	5.00000	0.26061	0.26000	0.23268
0.00200	10.00000	0.14983	0.14943	0.26754
0.00200	20.00000	0.08098	0.08075	0.28920
0.00200	50.00000	0.03405	0.03394	0.30396
0.00500	0.05000	0.97793	0.97797	0.00335
0.00500	0.10000	0.95682	0.95688	0.00657
0.00500	0.20000	0.91722	0.91733	0.01259
0.00500	0.50000	0.81590	0.81613	0.02799
0.00500	1.00000	0.68905	0.68938	0.04726
0.00500	2.00000	0.52561	0.52599	0.07212
0.00500	5.00000	0.30709	0.30742	0.10533
0.00500	10.00000	0.18140	0.18162	0.12444
0.00500	20.00000	0.09975	0.09988	0.13686
0.00500	50.00000	0.04244	0.04250	0.14557
0.01000	0.05000	0.98138	0.98153	0.01487
0.01000	0.10000	0.96344	0.96372	0.02917
0.01000	0.20000	0.92946	0.92998	0.05630
0.01000	0.50000	0.84052	0.84159	0.12728
0.01000	1.00000	0.72491	0.72651	0.21955
0.01000	2.00000	0.56852	0.57049	0.34435
0.01000	5.00000	0.34514	0.34695	0.52264
0.01000	10.00000	0.20856	0.20989	0.63165
0.01000	20.00000	0.11642	0.11725	0.70518
0.01000	50.00000	0.05007	0.05045	0.75814
0.02000	0.05000	0.98430	0.98460	0.03101
0.02000	0.10000	0.96908	0.96967	0.06108
0.02000	0.20000	0.94001	0.94112	0.11848
0.02000	0.50000	0.86240	0.86475	0.27176
0.02000	1.00000	0.75809	0.76173	0.47777
0.02000	2.00000	0.61043	0.61516	0.76943
0.02000	5.00000	0.38528	0.39002	1.21412
0.02000	10.00000	0.23861	0.24225	1.50381
0.02000	20.00000	0.13547	0.13782	1.70752
0.02000	50.00000	0.05898	0.06010	1.85859
0.05000	0.05000	0.98747	0.98812	0.06576
0.05000	0.10000	0.97525	0.97652	0.12989

TABLE A-1 (Continued)

$k_o d$	$fH_{o/md}$	$K_{f(approx)}$	$K_{f(integ)}$	Error(%)
0.05000	0.20000	0.95170	0.95412	0.25351
0.05000	0.50000	0.88740	0.89268	0.59096
0.05000	1.00000	0.79760	0.80616	1.06232
0.05000	2.00000	0.66333	0.67526	1.76698
0.05000	5.00000	0.44075	0.45408	2.93519
0.05000	10.00000	0.28267	0.29373	3.76489
0.05000	20.00000	0.16460	0.17215	4.38458
0.05000	50.00000	0.07305	0.07679	4.86504
0.10000	0.05000	0.98944	0.99053	0.10929
0.10000	0.10000	0.97911	0.98123	0.21629
0.10000	0.20000	0.95907	0.96315	0.42375
0.10000	0.50000	0.90359	0.91270	0.99807
0.10000	1.00000	0.82413	0.83942	1.82061
0.10000	2.00000	0.70088	0.72327	3.09663
0.10000	5.00000	0.48380	0.51111	5.34385
0.10000	10.00000	0.31909	0.34329	7.04901
0.10000	20.00000	0.18983	0.20721	8.38713
0.10000	50.00000	0.08569	0.09465	9.46519
0.20000	0.05000	0.99111	0.99285	0.17595
0.20000	0.10000	0.98237	0.98581	0.34880
0.20000	0.20000	0.96535	0.97202	0.68551
0.20000	0.50000	0.91767	0.93286	1.62912
0.20000	1.00000	0.84786	0.87417	3.01039
0.20000	2.00000	0.73590	0.77647	5.22572
0.20000	5.00000	0.52709	0.58150	9.35737
0.20000	10.00000	0.35786	0.40994	12.70600
0.20000	20.00000	0.21792	0.25782	15.47490
0.20000	50.00000	0.10028	0.12200	17.80260
0.50000	0.05000	0.99292	0.99600	0.30947
0.50000	0.10000	0.98593	0.99203	0.61457
0.50000	0.20000	0.97225	0.98418	1.21210
0.50000	0.50000	0.93340	0.96137	2.90915
0.50000	1.00000	0.87512	0.92561	5.45497
0.50000	2.00000	0.77796	0.86152	9.69876
0.50000	5.00000	0.58359	0.71334	18.18890
0.50000	10.00000	0.41202	0.55441	25.68320
0.50000	20.00000	0.25946	0.38352	32.34710
0.50000	50.00000	0.12292	0.19926	38.31130
1.00000	0.05000	0.99404	0.99831	0.42771
1.00000	0.10000	0.98814	0.99662	0.85035
1.00000	0.20000	0.97656	0.99326	1.68076
1.00000	0.50000	0.94340	0.98331	4.05919
1.00000	1.00000	0.89286	0.96717	7.68345
1.00000	2.00000	0.80645	0.93643	13.87980
1.00000	5.00000	0.62500	0.85490	26.89210
1.00000	10.00000	0.45455	0.74657	39.11580
1.00000	20.00000	0.29412	0.59563	50.62040
1.00000	50.00000	0.14286	0.37075	61.46770
2.00000	0.05000	0.99498	0.99978	0.47994
2.00000	0.10000	0.99001	0.99956	0.95508

TABLE A-1 (Continued)

$k_o d$	fH_o/md	$K_{f(approx)}$	$K_{f(integ)}$	Error(%)
2.00000	0.20000	0.98022	0.99911	1.89126
2.00000	0.50000	0.95197	0.99779	4.59189
2.00000	1.00000	0.90834	0.99558	8.76290
2.00000	2.00000	0.83207	0.99121	16.05430
2.00000	5.00000	0.66466	0.97830	32.06020
2.00000	10.00000	0.49774	0.95752	48.01780
2.00000	20.00000	0.33133	0.91851	63.92750
2.00000	50.00000	0.16542	0.81846	79.78940

APPENDIX B

Development of Equation for Breaking Wave Angle

The conservation of energy equations from offshore to the breaker location for waves refracting over straight and parallel offshore bottom contours in the case of energy losses to bottom friction can be written as

$$\frac{\rho g}{8} H_i^2 C_{gi} \cos \alpha_i = \frac{\rho g}{8} H_b^2 C_{gb} \cos \alpha_b + \text{losses} \tag{B-1}$$

where the losses can be expressed in terms of a friction coefficient K_f as

$$\text{losses} = (1 - K_f^2) \frac{\rho g}{8} H_i^2 C_{gi} \cos \alpha_i \tag{B-2}$$

Upon combining equations B-1 and B-2, and canceling like terms it can be found that

$$H_b^2 C_{gb} \cos \alpha_b = K_f^2 H_i^2 C_{gi} \cos \alpha_i \tag{B-3}$$

Assuming breaking occurs in shallow water

$$C_{gb} = (g d_b)^{0.5} \tag{B-4}$$

Now assuming $\kappa = \frac{H_b}{d_b}$ is known (see equation 10 in text) equations B-3 and B-4 can be solved as

$$H_b = \left[\left(\frac{\kappa}{g} \right)^{1/2} K_f^2 \; H_i^2 C_{gi} \cos \alpha_i \right]^{0.4} \tag{B-5}$$

Using Snell's Law of Refraction and the shallow water assumption

$$\sin \alpha_b = \left(\frac{g H_b}{\kappa} \right)^{0.5} \left(\frac{\sin \alpha_i}{C_i} \right) \tag{B-6}$$

which can then be solved for α_b.

APPENDIX C - REFERENCES

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