APPLICATIONS OF A NUMERICAL SHALLOW WATER WAVES MODEL

by

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ABSTRACT

This paper relates three applications of a numerical model of storm waves in shallow waters developed in LNH. The equations are recalled at first and then the applications performed are presented.

The numerical model has been used in the case of the port of Fecamp, on the English Channel coast, on which the results of a scale model were available. The computed results compare well with the scale model measurements.

The second case is the simulation of a tsumami induced by a submarine landslide which appeared in 1979 near Nice ; the model has permitted the simulation of the rising of the wave.

The last applications consisted in simulating breaking waves by introducing a dispersion term in the equations. This simulation has been tested with a one-dimensional model at first. The results show that the numerical model reproduces the elevation of the mean sea surface due to the loss of energy in breakings. Then the longshore current induced by breaking waves coming obliquely over a rectilinear sloping shore has been reproduced with a two dimensional model.

The results show that the model is able to compute with a good accuracy refraction, diffraction and reflection, and that it appears to be very interesting for longshore currents simulation.

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1. INTRODUCTION

This paper follows a previous one presented in the 17th conference, on the numerical model of storm waves in shallow waters developed in LNH. In this previous paper, the different assumption required and the governing equations (more complete than the Boussinesq type equations) were presented. Several indications were also given and the importance of non linear effects in storm waves problems was pointed out.

During the last two years, practical applications have been performed, and new possibilities have been introduced in the model. The applications presented are the computation in the port of Fecamp (English Channel coast) on which the results of a scale model were available, the simulation of a mini-tsunami which appeared near the works of the new airport of Nice in 1979, and the simulation of wave set-up and longshore currents induced by wave breaking over a rectilinear sloped shore.

2. EQUATIONS

By assuming that the vertical velocity linearly increases from the bottom to the surface, instead of assuming a hydrostatic pressure distribution, it is possible to average the Navier - Stokes equations to obtain the Serre type equations :

$$\frac{\partial h}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(\frac{UV}{h}\right) + \frac{\partial}{\partial y}\left(\frac{UV}{h}\right) + \frac{\partial}{\partial x}\left(\left(\frac{g+\beta}{2} + \frac{\alpha}{3}\right)h^2\right) = -\left(g+\beta+\frac{\alpha}{2}\right)h\,\frac{\partial z}{\partial x} - g\,\frac{U\sqrt{U^2+V^2}}{C^2\,h^2}$$

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial x}\left(\frac{UV}{h}\right) + \frac{\partial}{\partial y}\left(\frac{V^2}{h}\right) + \frac{\partial}{\partial y}\left(\left(\frac{g+\beta}{2} + \frac{\alpha}{3}\right)h^2\right) = -\left(g+\beta+\frac{\alpha}{2}\right)h\,\frac{\partial z}{\partial x} - g\,\frac{V\sqrt{U^2+V^2}}{C^2\,h^2}$$

Where h is the water depth, U and V are x - and y - volume fluxes, z is the bed elevation, g the gravitational acceleration.

The new terms α and β come from the new assumption, and characterize the vertical accelerations raised by the steepness of the waves and the slope of the bed :

$$\alpha = \frac{d^2h}{dt^2}, \quad \beta = \frac{d^2z}{dt^2} \quad \left(\text{with } \frac{d}{dt} = \frac{\partial}{\partial t} + \frac{U}{h}\frac{\partial}{\partial x} + \frac{V}{h}\frac{\partial}{\partial y}\right)$$

A condition at the seaward boundary which simulates entering waves in the model at the seaward boundary and allows outgoing waves, is introduced in the following form :

$$-\vec{V}.\vec{n} + C\,(h + z) = (1 - \vec{u}.\vec{n})\,C\,Wi$$

where $\vec{V}$ is the volume flux
$\vec{n}$ is the outgoing unit-vector, perpendicular to the boundary side.
$\vec{u}$ is the unit - vector of propagation direction
Wi is the sea surface elevation of the incident wave
C is the wave celerity

The same sort of condition has been imposed on inside boundaries to model partial reflections :

$$-\vec{V}.\vec{n} + C\,(1 - r)\,(h + z) = 0$$

Where r is a coefficient of reflection (if r = 1 : total reflection, if r = 0 : no reflection).

3. AGITATION IN THE PORT OF FECAMP

The calculation is fitted to computations of non-linear agitation (periodic or not) in harbours of any bathymetry, including partial reflections. It has been used in the case of the port of Fecamp on the English Channel coast.

In 1970, a scale model was done in order to study the agitation in this port. The comparison between computed and scale model results has been performed to check the numerical model.

The figure 1 presents the port shape. Break-waters are situated along the sides of the access channel and the other boundaries are vertical quays. The wave characteristics at the entrance were very severe (height = 6m, period = 7s).

The available results of the physical model (fig. 2) display a great agitation in the center of the outer-harbour.

The characteristics of the calculation used are :

mesh dimension : DX = 3,5 m
time step : DT = 0,35 s

The free surface and the agitation contours are shown in figure 3 for a coefficient of reflection equal to 0,90 at the place of the break-waters. Considering the way of measuring agitation which smooths the agitation pattern, the computed results and the scale model results compare well since the computed agitation pattern displays the areas of great agitation in front of the "Grand Quai" and in the centre of the outer-harbour which appears in scale model.

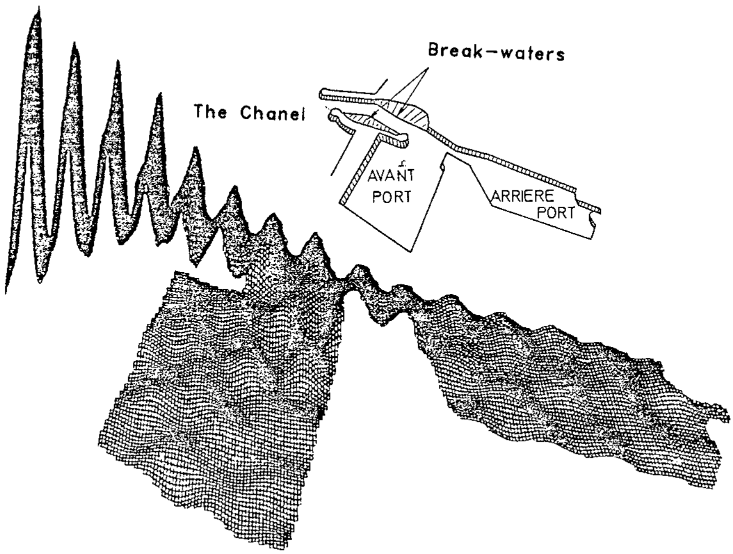

Fig.1 Computed Agitation Pattern

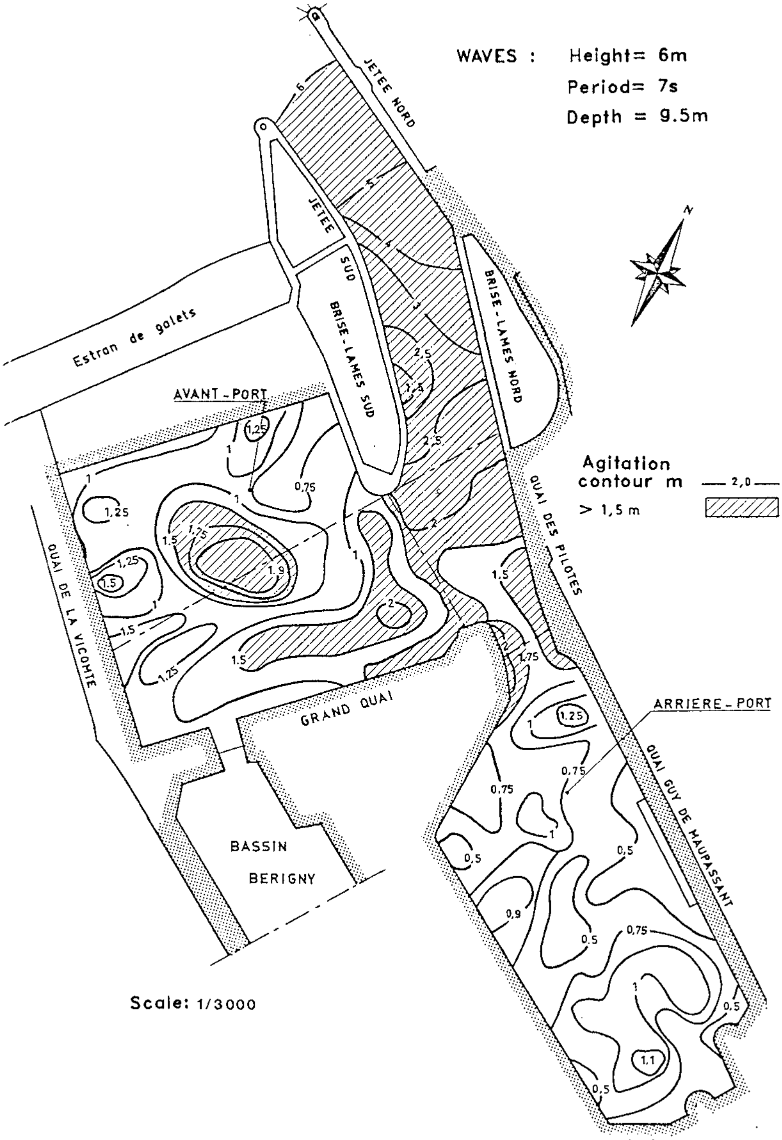

Fig.2 Agitation Contours of the Scale Model

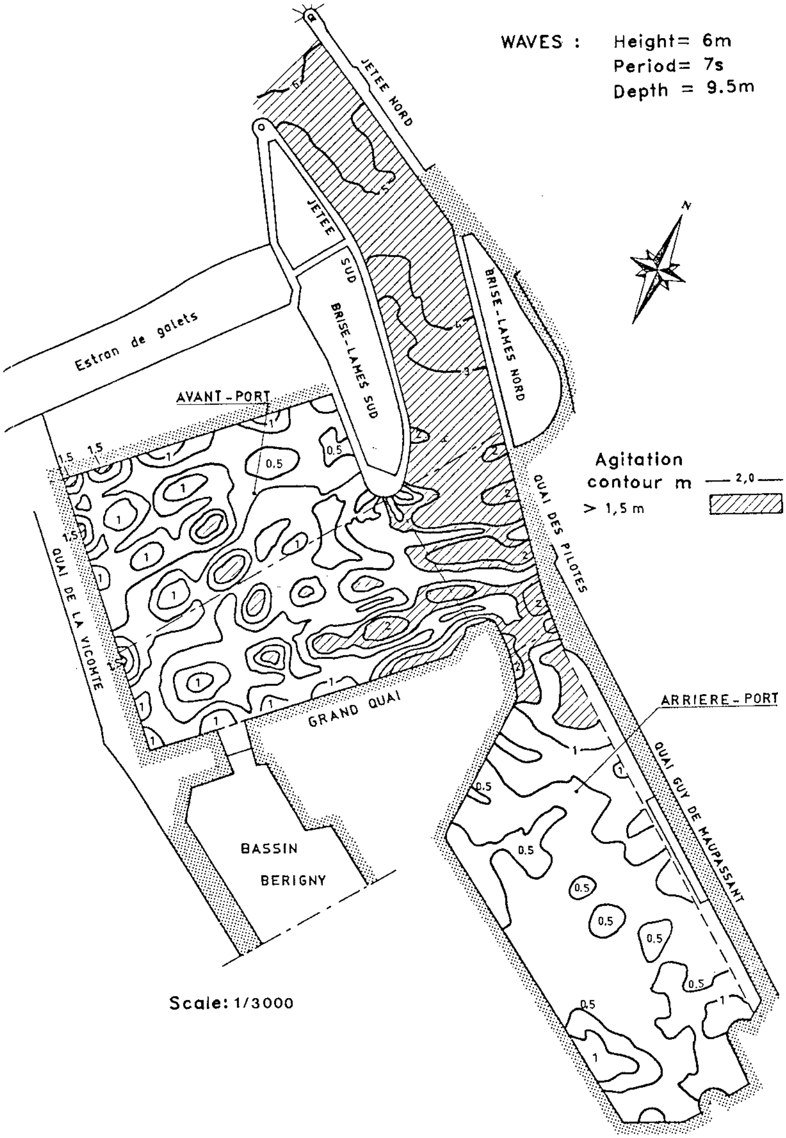

Fig.3 Agitation Contours of the Numerical Model

4. SUBMARINE LANDSLIDE NEAR NICE AIRPORT

As the model takes into account vertical accelerations resulting from the bottom shape, it allows the modelling of tsunamis. In october 1979, a mini-tsunami induced by a submarine landslide near the works of the new airport of Nice, damaged a part of the coast of "La Baie des Anges" in Mediterranean sea.

The introduction of sea-bottom changes, from the bathymetry before the tsunani to the bathymetry after it, has induced the rising of the wave (figure 4).

The model has been able to reproduce the first waves which occurred at the beginning of the landslide but secondary waves of great height which appeared 3 minutes after the first ones were not reproduced in the numerical model. This results shows that the secondary waves did not come from reflection effects but that they could be induced by turbidity currents or a second landslide.

5. SIMULATION OF BREAKING WAVES

Description

A simulation of breaking waves has been added in the numerical model. Taking into account the fluctuation of velocity (u', v') over the depth, the following terms appear in the governing equations :

$$\bar{T} = \frac{1}{h} \begin{bmatrix} \int_0^h u'^2 \, dz & \int_0^h u' \, v' \, dz \\ \int_0^h u' \, v' \, dz & \int_0^h v'^2 \, dz \end{bmatrix}$$

Assuming an analogy with Fick's law for turbulent momentum fluxes, the tensor T is written :

$$\bar{T} = - R \text{ grad } \vec{V}$$

Where $\vec{V}$ is the average of horizontal velocity over the depth.

Before breaking, the dispersion coefficient R is equal to zero, whereas it is high in the breaking zone so that the large variations of velocity over the depth in the breaking zone are simulated.

Wave breaking is assumed to occur at the location where the wave height H exceeds the Weggel's criterion :

$$H = \left(b(m) - a(m) \frac{H}{T^2}\right) d$$

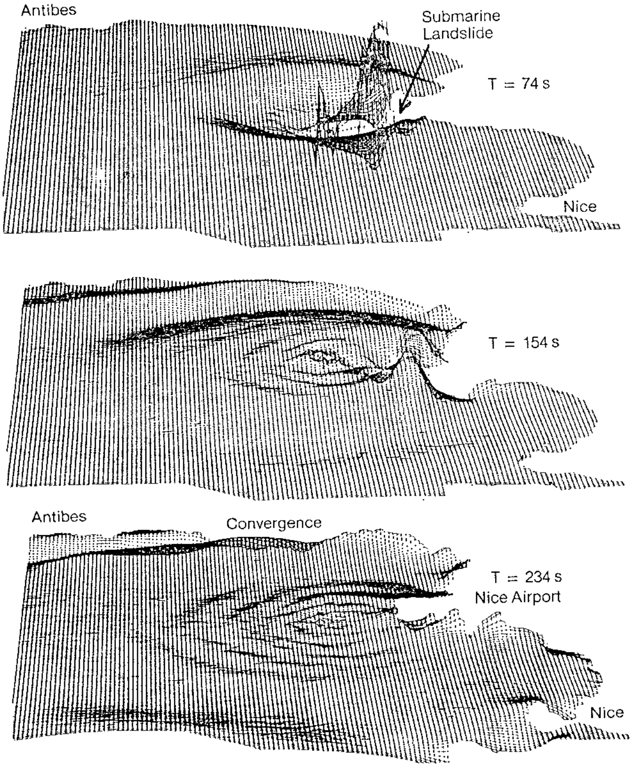

Fig.4 Simulation of a Submarine Landslide

Near Nice Airport

Where m = bottom slope
T = period
d = still water depth (wave set-up is neglected).
$a(m) = 4,46\ (1 - e^{-19m})$
$b(m) = 1\ /\ 0,64 \cdot (1 + e^{-19,5m})$.

The simulation of breaking waves has been tested over two cases :

- in a flume with a constant sloped bottom, using a one dimensional model
- in a rectilinear beach, limited by two groynes, under the action of waves with an offshore incidence of 30°.

One - dimensional model

The propagation and the breaking of waves over a constant sloped bottom has been tested in a flume geometry with various wave conditions and slope values.

When the wave height is smaller than Weggel's criterion, the dispersion coefficient is equal to zero.

When this criterion is reached, the dispersion coefficient is high from the breaking line to the coast. This value is chosen so that the ratio of wave height over still water depth is constant in this domain.

The bottom friction is assumed to be proportional to the squared velocity :

$$\vec{\pi} = -\frac{\rho g}{C^2}\ \vec{V}\,||\vec{V}||$$

Considering Naher's studies, the coefficient of Chezy is equal to 25 $m^{1/2}/s$.

The obtained propagation in the flume is presented in fig. 5, with the following characteristics.

Off-shore wave conditions: period 9 s, wave height 2 m

Bottom slope 1,3 °/₀

Dispersion coefficient in the surf zone : 5 m^2/s
out the surf zone : 0 m^2/s

At first the wave is refracted and its height increases. When the height exceeds Weggel's criterion, the wave breaks, the height decreases from the breaking line to the coast. The wave surge coming from the wave energy loss has been obtained in the surf zone. In this case, the set-up is of 0,47 m.

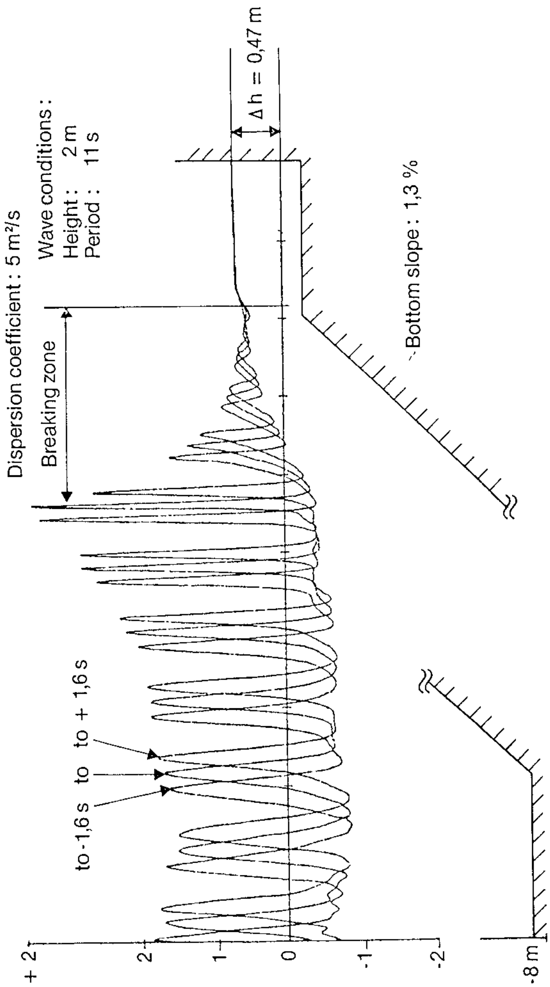

Fig.5 Wave Breaking Test with a One-dimensional Model

The dispersion coefficient has been estimated for various values of the slope and of the ratio of the wave-height over the wave length (fig. 6)

The one-dimensional tests have shown that the numerical model is able to simulate wave breaking and to predict wave surge.

A two-dimensional case has been computed to simulate longshore currents.

<u>Two - dimensional model</u>

The current induced by breaking waves has been studied in a 700 meters length rectilinear shore, limited by two groynes under the action of waves generated at the seaward boundary (fig. 7)

The off-shore wave conditions are : height 3 m
period 12 s
angle of incidence 30°

The mean level set-up at the coast is equal to 0,21 m (fig. 8)

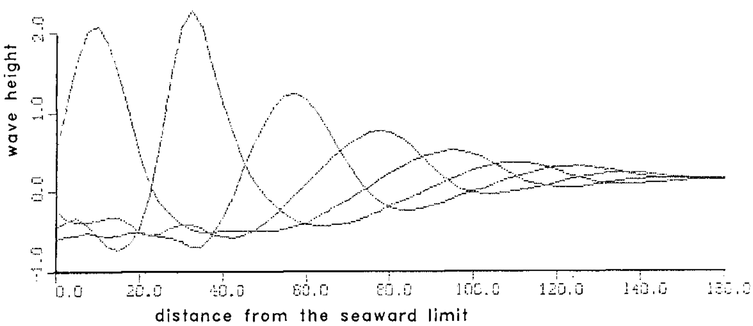

Fig.8 Wave Propagation along Section A-A

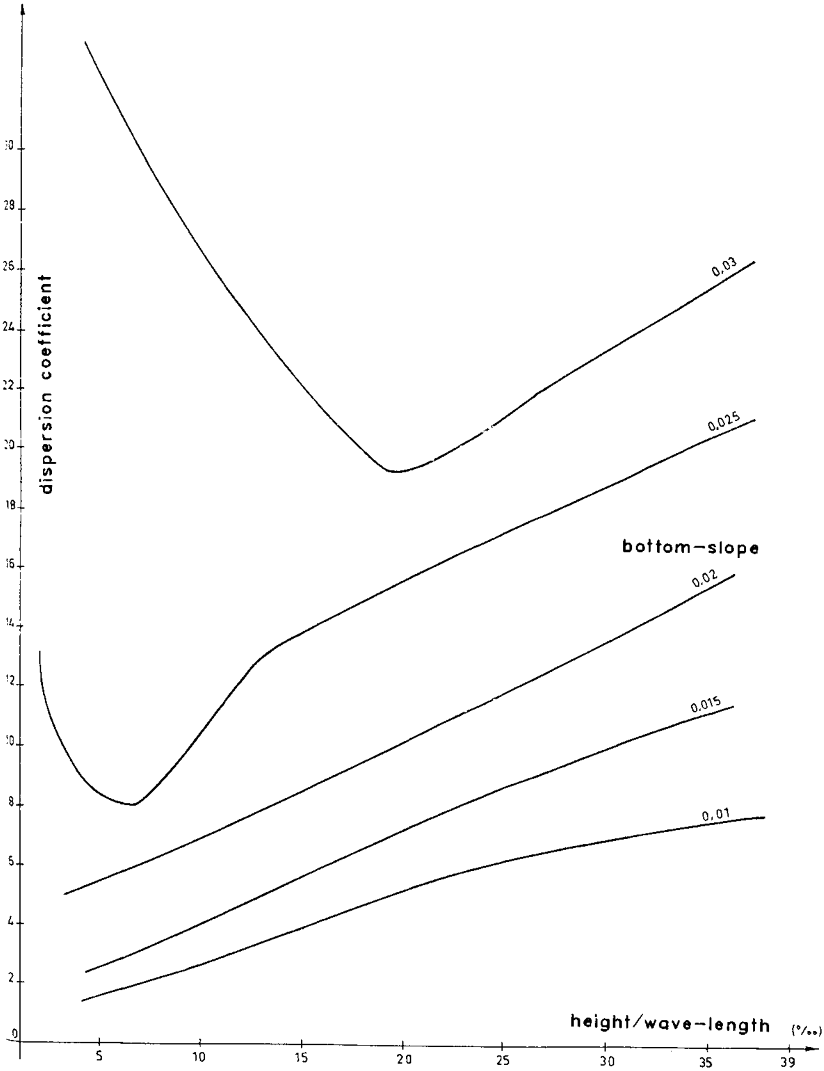

Fig.6 Dispersion coefficient R

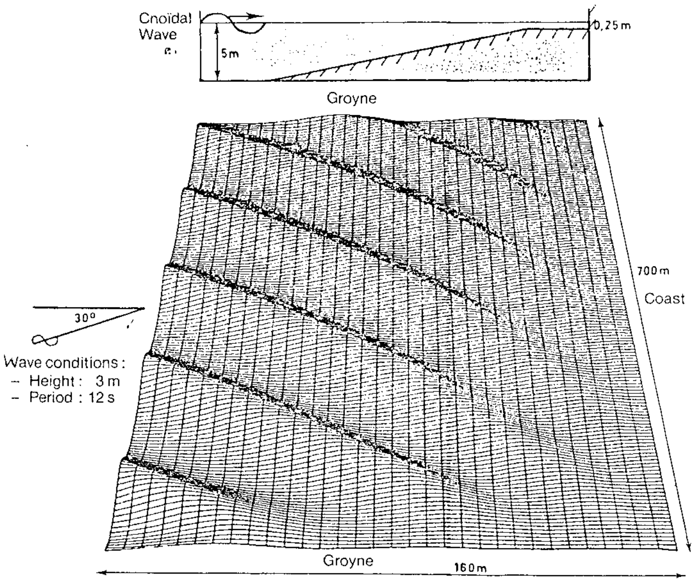

Fig.7 Simulation of Longshore Currents

The fig. 9 shows the instantaneous fluxes and the longshore current obtained by averaging fluxes over a period. A mean current parallel to the coast appears ; this is in good agreement with natural phenomenon. But a current takes place in the domain between the off-shore limit and the breaking line, in the opposite direction. Vortices appear between these two zones. This last current is due to the boundary condition which permits the waves to go out of the domain but prevents mean currents to go out.

The velocity distribution in a section perpendicular to the coast is shown on fig 10. The maximum value does not reach 0,3 m/s. This value seems rather small compared to measurements in scaled models or in nature. The reason of this difference could be imputed to the back current.

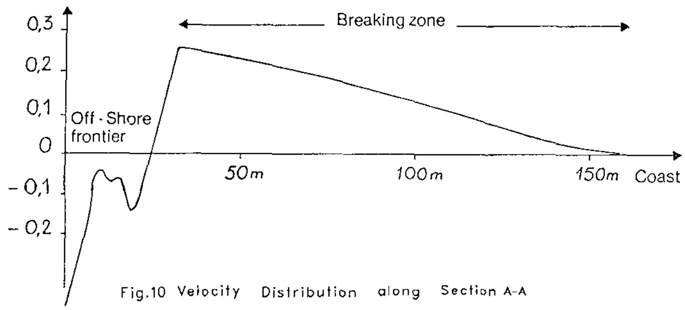

Fig.10 Velocity Distribution along Section A-A

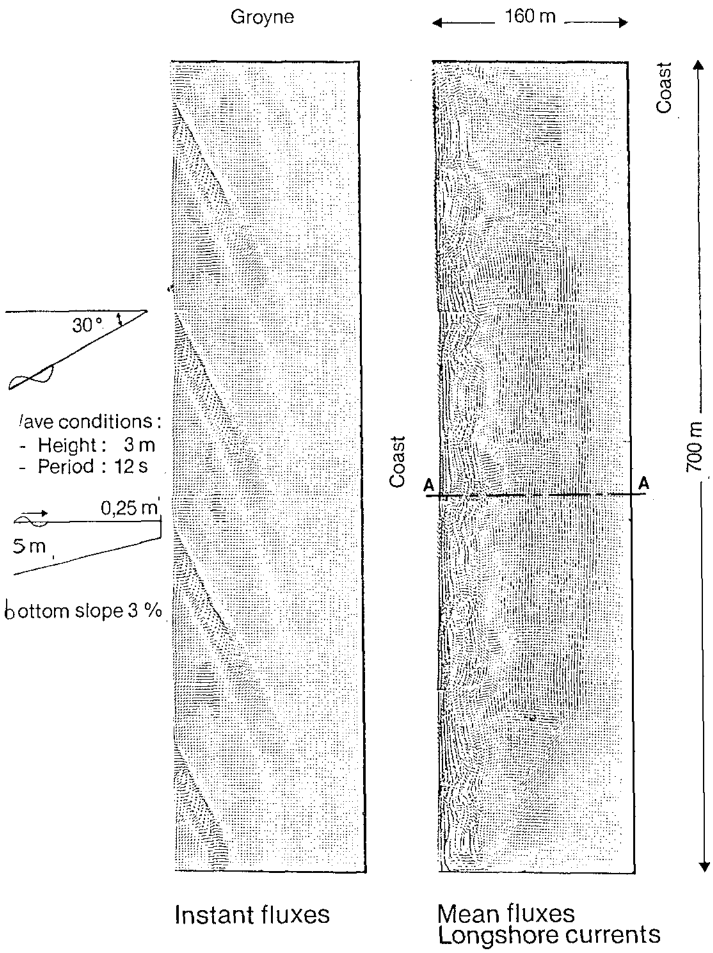

Fig.9 Fluxes Pattern in a Reactilinear Shore

6. CONCLUSION

The results of the numerical model applied to the agitation in the port of Fecamp and the submarine landslide near Nice airport show that the model is able to compute refraction, diffraction and reflection for non linear waves over any given bathymetry with a good accuracy.

The model appears to be very useful for breaking waves studies applied to longshore currents predictions. In the near future such results could give interesting informations for sedimentological studies concerning sea-bottom evolutions under waves action.

As the cost of numerical models is moderate in front of physical models, it is thought that the use of such numerical models in studies will increase substantially.

7. ACKNOWLEDGEMENT

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