A NEW APPROACH TO TRANSIENT WAVE GENERATION

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ABSTRACT

An analytical model of an "episodic" wave as a function of time is given. The linear dispersion theory is applied to the transient to hindcast the wave train at a wave board which is a specified distance up-wave from a test site. First order wave board theory and linear system theory is used to convert the wave train at the wave board into a control signal. Conversion of the time discrete control signal into a smooth analog voltage signal by means of a real time digital program is described. Examples of numerical simulations and physical measurements are given.

INTRODUCTION

Testing of coastal structures in hydraulic laboratories is generally performed by wave attack with either regular or irregular waves. Occasionally it is desirable to study fixed or floating structures by exposing them either to extremely large waves or to particular successions of large waves as may be encountered in wave groups.

Whereas many researchers believe that the use of long sequences of random waves for testing purposes offers a high probability that all extreme test conditions will be encountered sooner or later, experience at NRC has indicated that this is not generally the case. During a capsize test of a communication buoy, irregular waves for a fully developed sea failed to capsize the buoy whereas a particular transient wave, designed to break at the buoy location, achieved a capsize every time (Photographs 1 and 2). Other examples for the application of transient wave conditions are overtopping tests and the measurement of pressure and forces due to extreme waves. Purely from the point of view of being able to exercise more precise control of wave profiles at the test site as well as the greater testing efficiency and resulting time saving, the deterministic approach to wave generation has an appeal to the experimental scientist.

Episodic waves have been generated successfully by various laboratories using a "sweep frequency" technique. The span and the

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PHOTO 1

PHOTO 2

SEA KEEPING TEST OF A COMMUNICATION BUOY sweep rate were determined experimentally to achieve a breaker at a certain distance from the wave board. A variation of this method was described by Funke and Mansard (1979) which determines the sweep rate by calculating the periods of successive wave singlets in a wave train which is being assembled to achieve breaking at a predefined distance from the wave board. This technique is described in Figure 1 which illustrates the principle that the singlet of period $1/F_a$ requires T_a seconds to travel a required distance D at group velocity. Therefore, if a second singlet is to be generated, which can catch up and overtake the first singlet within distance D, then it must do so in $T_a - 1/F_a$ seconds. Because there is now less time available, the second singlet must, of necessity, be of a longer period than the first because longer period waves travel faster.

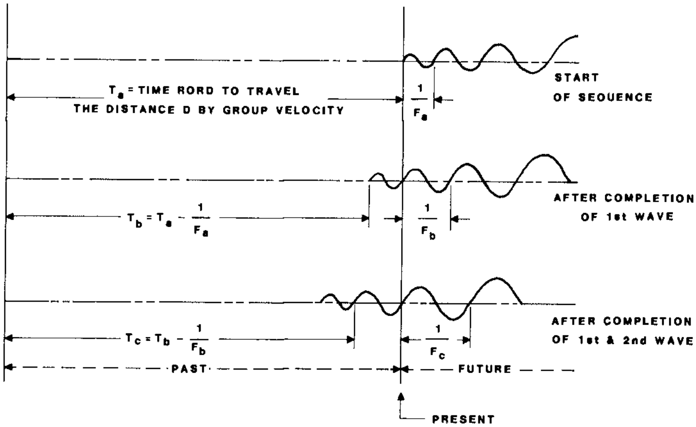

FIG.1 TRANSIENT WAVE GENERATION BY SWEEP FREQUENCY METHOD

Although the "sweep frequency" methods have proven that control over the wave breaker location is possible, they do not permit any control over the profile of the wave transient. The method by Funke and Mansard (1979) assumed that every singlet should have an amplitude of 1/10 of the associated wave length. As a result, the actual breaking occurs when the wave becomes critically steep due to the superposition of waves and not necessarily when all wave singlets have achieved phase coincidence.

The new method described here is based on a description of the desired wave transient at the test site and on the principle that, through the application of linear dispersion theory, the corresponding wave sequence at the location of the wave board can be hindcasted. It should therefore be possible to generate wave groups with an arbitrary number of waves per group and with arbitrary steepness. Limitations are, of course, imposed by physical realizability.

DEFINING THE WAVE TRANSIENT

To illustrate the method, the generation of a wave group with a finite number of waves is being described. In the absence of an analytical description of such a wave or wave train as a function of time, a mathematical function has been composed which gives the water surface elevation $\eta(t)$ as may be observed at a single point at the test site in a wave flume or basin. This function is given as:

$$\eta(i\cdot\Delta t) = \sum_{n=1}^{n_{max}} \frac{H\cdot T\cdot e^{-(\gamma\cdot t)^2}\cdot \sin(n\pi\rho/T}{\pi^2\cdot n^2\cdot\rho\cdot(1-\rho/T)}\cdot \sin(2\pi n\cdot i\Delta t/T) \qquad (1)$$

where n_{max} = is the maximum number of participating harmonics

H = is the wave height of an unmodulated saw tooth wave form

T = is the period of the saw tooth

γ = $\sqrt{3}/(\beta\cdot T)$

β = is the half amplitude width of the Gaussian envelope function

ρ = is a steepness parameter.

Equation (1) may best be understood with reference to Figure 2. See also Moskowitz and Racker (1951). The term

$$\frac{H\cdot T\cdot \sin(n\pi\rho/T)}{\pi^2\cdot n^2\cdot\rho\cdot(1-\rho/T)}$$

gives the nth Fourier coefficient for a saw tooth wave form with steepness parameter ρ. If ρ=0.5, the saw tooth is symmetrical with front and back steepness being equal. If ρ=1.0, then the wave front is infinitely steep. In practice, ρ=0.75 appears to be satisfactory. The

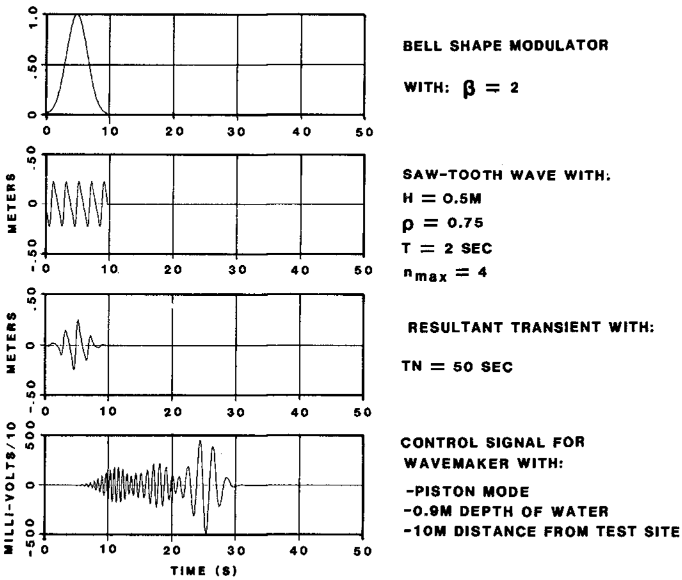

FIG.2 SYNTHESIS OF AN EPISODIC WAVE

second graph in Figure 2 illustrates five cycles of a saw tooth wave for H=0.5 m, $\rho=0.75$, T=2 seconds and $n_{max}=4$. This trace results from evaluating $\sin(2\pi \cdot n \cdot i\Delta t/T)$ for $0<i\Delta t<10$ seconds.

n_{max}, in the summation of equation 1, may be limited in order to avoid very sharp, non-realizable crests and troughs in the saw tooth. Alternatively, n_{max} may be used to limit the maximum frequency in the wave machine control signal.

The term $e^{-(\gamma \cdot t)^2}$ in equation (1) gives an amplitude modulator which is applied to the saw tooth. $\gamma=\sqrt{3}/(\beta \cdot T)$ may be used to control the width of this bell shaped function. In the example, $\beta=2$, which implies that the half amplitude width is $\beta \cdot T=4$ seconds, the bell function is centred at $t=1.21 \cdot \beta \cdot T$.

The result of modulating the saw tooth is given as the third trace in Figure 2. This is the theoretical wave transient as a function of time and represents the wave which is to be generated at a test site some distance from the wave board.

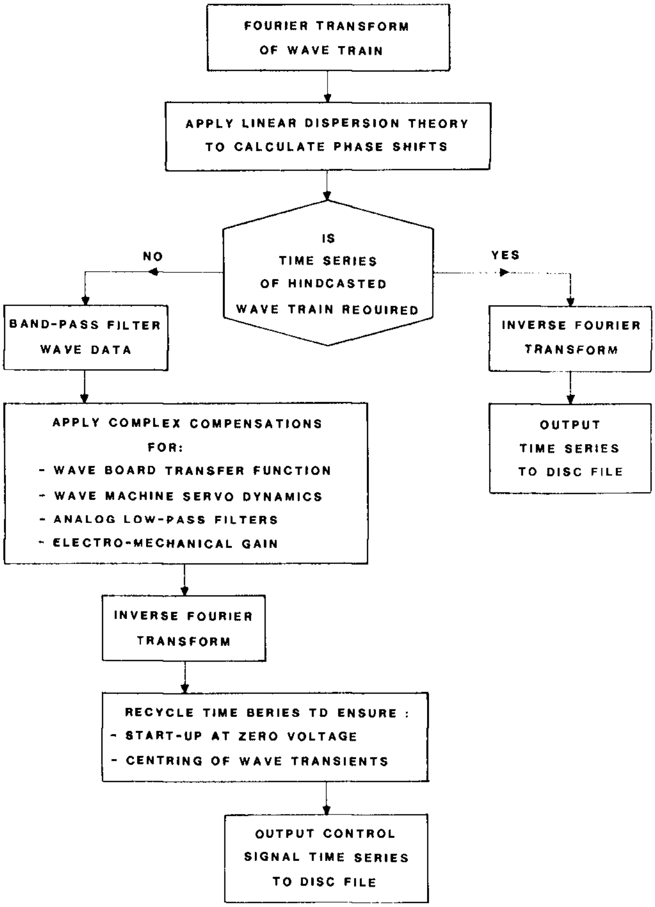

FIG.3 SUMMARY OF OPERATIONS REQUIRED FOR CONVERTING A WAVE TRAIN TO A WAVE MACHINE CONTROL SIGNAL

Discrete Fourier transform considerations require that a recycling period is specified. Also, for purposes of conducting a test, it is preferable to introduce a quiescent period between successive wave transients in order to permit disturbances in the flume to settle down after each transient has traversed the length of the flume. For this reason a recycling period T_n is introduced which was set to 50 seconds for the example of Figure 2.

COMPUTATION OF THE WAVE GENERATOR CONTROL SIGNAL

The conversion of the wave train, defined in Equation (1) and Figure 2, to a wave generator control signal is accomplished by first submitting the time series to a program known as RWREP which is described by Funke and Mansard (1983). This computer program performs a variety of operations which are summarized in Figures 3 and 4.

All these operations are being implemented in the frequency domain and therefore the desired wave train is first Fourier transformed.

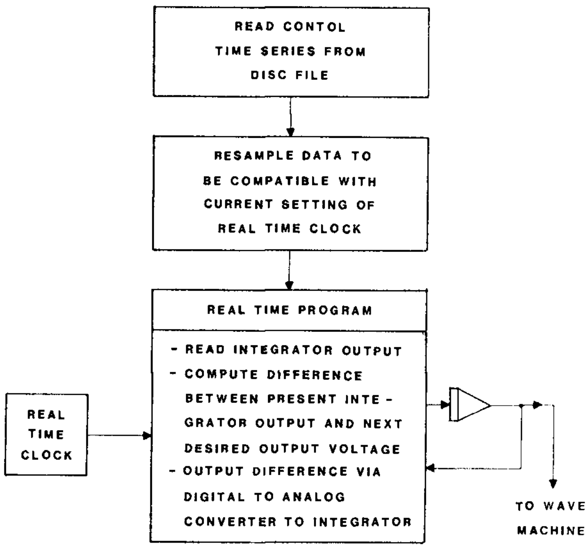

FIG.4 CONVERTING A CONTROL TIME SERIES TO A VOLTAGE SIGNAL

Linear dispersion theory states that all individual frequency components, which are constituent parts of a progressive wave train, propagate at their own phase velocity. This is, however, a function of both period and water depth. Under shallow water depth conditions, the phase velocities are all the same. But, under intermediate and deep water conditions, the low frequency waves propagate faster than the high frequency waves.

Experiments have indicated that the linear dispersion theory is a reasonable assumption under non-breaking wave conditions. Figure 5 illustrates the results of one experiment carried out in a wave flume with 0.92 m depth of water. A "random" wave train with a Pierson-Moskowitz spectrum with $f_p=1$ Hz was generated. Ten wave probes were placed along the wave flume at various intervals from 2 to 19 meters from the wave board. The signals monitored by each of the probes were Fourier analysed and the phase lags, relative to the first wave probe, were computed for 5 frequencies from 0.9 to 1.3 hz. Figure 5 gives the results and compares these with the phase shift based on linear dispersion theory. From this it was concluded that linear dispersion relations represent a reasonable approximation in calculating phase shifts for frequency components in complex wave trains and this is being applied by the program RWREP to hindcast the wave train in terms of its Fourier transform at the wave board.

The next step in Figure 3 gives the option to inverse Fourier transform the hindcasted data in order to provide a time series describing the wave train which will be generated at the wave board. The fourth trace on Figure 2 provides an interesting example of the wave train which must be generated at the wave board in order to produce the wave transient shown in the third trace. Although the sweep of frequencies from high to low frequencies are reminiscent of previously used techniques, the groupy modulation of amplitudes are a surprising result.

Following the computation of the hindcasted wave train, the data is optionally band pass filtered and then compensated for a variety of dynamic transfer functions which are in the path of a computer generated control voltage. The wave board transfer function described by Gilbert, Thompson and Brewer (1970) was expressed as a complex function to allow for the $\pi/2$ phase advance between the wave board position and the water surface elevations. To convert the time series for water surface elevation at the wave board to a time series of wave board position, the Fourier transform of the former is divided by this wave machine transfer function.

The electro-hydraulic servo transfer function of the wave machine can be obtained from measurements. Depending on the depth of water and the particular settings of the control loop parameters, this transfer function may vary somewhat. For most machines currently in use at the National Research Council this transfer function was found to be:

$$G(j\omega) = \frac{1}{(j\omega+1)^3}$$

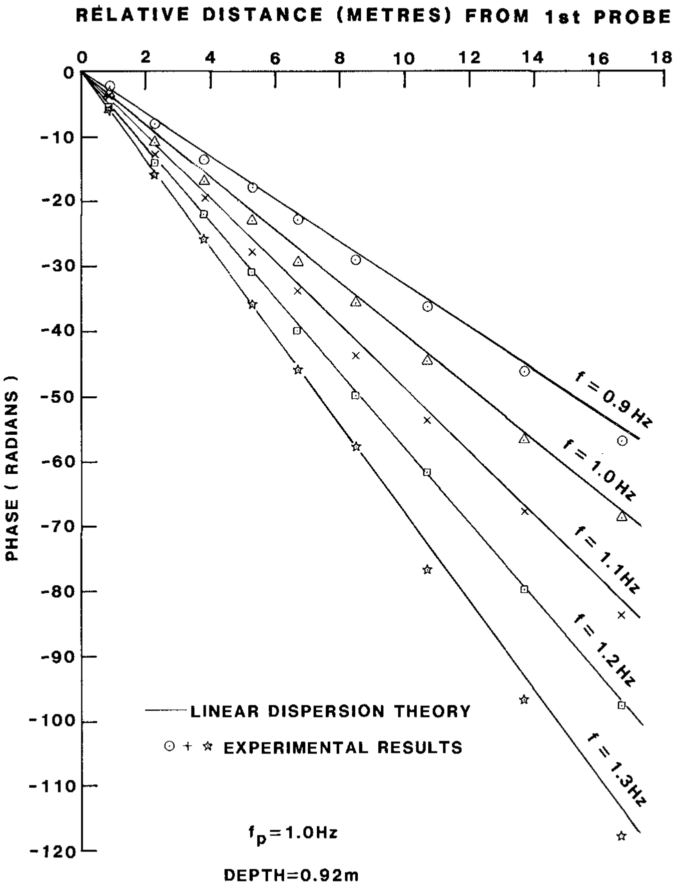

FIG.5 PHASE PROPAGATION OF INDIVIDUAL FREQUENCY COMPONENTS IN A PIERSON MOSKOWITZ SPECTRUM

Consequently, the Fourier transform of the wave board position signal is divided by $G(j\omega)$.

A low pass filter is frequently introduced at the output of a digital to analog converter in order to remove the sharp corners of the converter's output or to protect the machine from dynamic shock. A correction for such a filter should also be included.

At this stage all dynamic effects have been allowed for. However, in order to convert a wave board position signal to a voltage signal, a known wave machine calibration factor in terms of meters of board displacement per voltage input to the servo must be applied as a scale factor. It is also important to ensure that a positive value in a digital control signal leads to forward displacement of the wave board.

Application of the inverse Fourier transform supplies then the time series for the wave generator control signal. Some adjustments are then made to avoid severe start-up discontinuities or, as is the case for the generation of wave transients with long quiescent periods following the transients, the control signal is cycled around the vector length to ensure that the signal starts during the quiescent stage and not in the middle of the transient.

SIGNAL GENERATION

Whereas the above computations are all carried out in a digital computer resulting in a time series of the desired control signal, the ultimate step is the conversion of the time series into a smooth analog voltage signal. This is performed by a real time computer program operating in the "foreground" under control of the computer's real time clock as shown in Figure 4.

The real time clock is usually preset to be compatible with the highest frequencies which may have to be generated. Experience at the National Research Council over a period of ten years has indicated that a real time clock rate of 10 steps per second is more than adequate. However, during the digital wave synthesis process, such as described by equation (1), the intersample spacing, Δt, may not necessarily be 0.1 seconds. Therefore, an operation of re-sampling may be required to preserve the temporal integrity of the control signal. Under the GEDAP operating system described by Funke, Crookshank and Wingham (1980) this operation is transparent.

The actual operation of generating a smooth analog signal is performed by a digital to analog converter in conjunction with an integrator. The real time program monitors the output of the integrator by means of an analog to digital converter and computes the difference between the present integrator output and the desired integrator output one time step later and transmits this value to the digital to analog converter. This results in a straight line interpolation of analog voltages between successive discrete digital values as described in Funke, Crookshank and Wiegert (1981).

EXPERIMENTAL RESULTS

An example similar to Figure 2 has been tested in a flume and the results are shown in Figure 6 for various distances from 1.8 m to 22.8 m. This test was carried out at a depth of water of 0.60 meters. The transient was synthesized for a focal distance of 20 meters. It may be noticed from Figure 6 how the wave transient converges as it propagates towards its cataclysmic destiny. It should be noted that the actual breaking took place at about 23.4 meters. Photograph 3 shows the resultant appearance of the plunger.

Further experimental results are illustrated in Figure 7. The first three traces 'a' to 'c' show the degree of success with which a variety of wave groups can be generated. There are notable differences between the desired and the measured transients which may be explained by the fact that the formulation of a wave group in terms of equation (1) is not physically realizable for the water depth to wave length conditions of the test. The desired vertical asymmetry in Figures 7b and 7c did not reproduce and, instead, a severe horizontal asymmetry resulted.

PHOTO 3

PLUNGING WAVE ON A CONICAL STRUCTURE

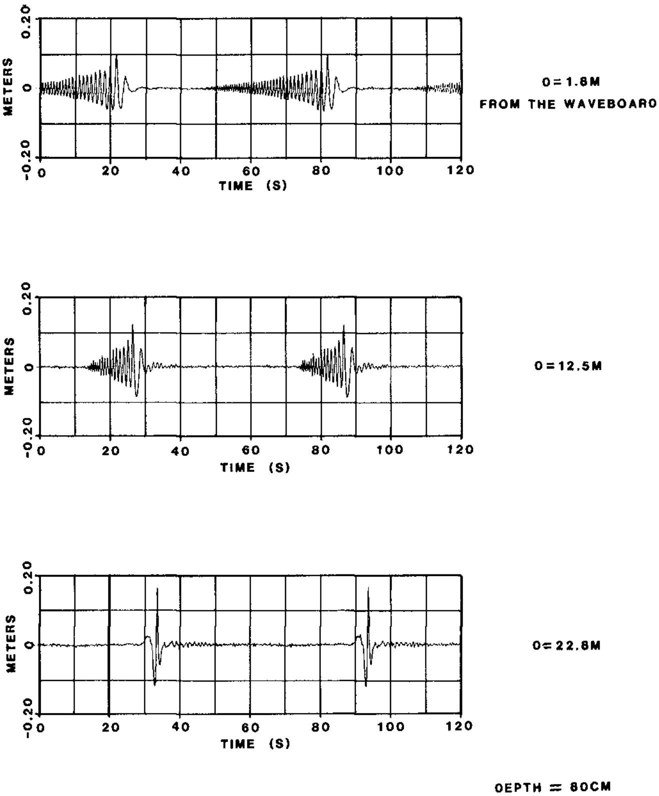

FIG.6 EPISODIC WAVE AS OBSERVED ALONG THE FLUME

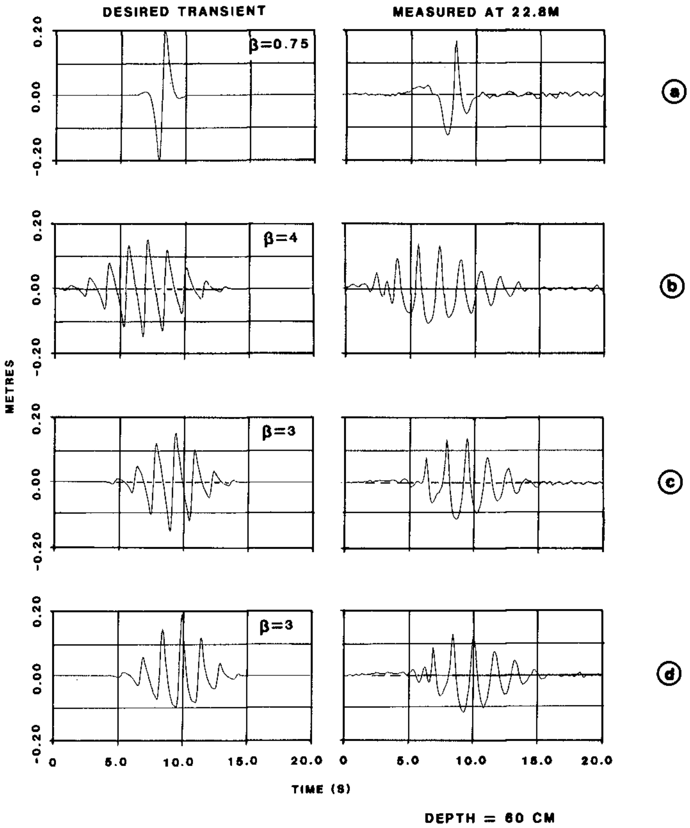

FIG.7 GENERATION OF WAVE TRANSIENTS IN THE FLUME

In order to improve on the formulation of equation (1), a new technique was introduced which was to lead to a more realistic definition of the wave transient. This technique is described by Funke and Mansard (1982). It involves a variety of non-linear distortions designed to increase crest heights while reducing the size of troughs and to shorten crest durations while lengthening trough durations without altering wave periods.

Figure 7d is an example of such a non-linear transformation applied to the group of Figure 7c. The resultant measured wave shows some improved reproduction which suggests that, through trial and error, further improvements may be realizable.

CONCLUSIONS

It has been demonstrated that transient wave forms can be generated in a flume or basin. Episodic waves could be generated with larger amplitudes than has previously been possible. The fidelity of reproduction is reasonable but can probably be improved by the application of non-linear dispersion theory and second order wave generator theory. The definition of the desired transient is, at this time, restricted to an amplitude modulated saw tooth with controlled front steepness. It is hoped that improved theoretical models of extreme wave transients as a function of time may also contribute in realizing more accurate generation of extreme wave conditions in the laboratory environment.

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