SENSITIVITY OF PARAMETERS AND APPROXIMATIONS IN MODELS OF TIDAL PROPAGATION AND CIRCULATION

by
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Introduction

In the last ten years a system has been designed for the two-dimensional simulation of the hydrodynamics and water quality in well-mixed estuaries, coastal seas, harbors, and inland waters. The system called SIMSYS2D or WAQUA, can simulate the hydrodynamics in complicated geographical areas, and the model can determine the land/water boundary during simulation [1]. The system accounts for the sources of discharges, tidal flats, islands or dams, and time-varying or time-invarying flow restrictions such as generated by openings in dams, sluices, or storm-surge barriers. In the SIMSYS2D system numerous finite difference approximations of the vertically integrated hydrodynamic equations and their boundary conditions are available to the investigator.

A large number of models and model experiments with two-dimensional models have been described in the literature. In many instances, authors have emphasized the advantages of the particular approximations they were using in their model and sometimes made comparisons with other approximations by use of hypothetical geographical areas. Generally, no comparisons are given of different computation methods applied to actual estuaries.

Currently, no comprehensive overview of the comparative importance of the approximations of the terms of the hydrodynamic equations based on experiments of typical estuaries, is available. Nor has a practical assessment been made on the comparative importance of timestep size, grid size, depth accuracy, roughness estimates, and approximation of the closure term by viscosity or other expressions.

To obtain an insight into the relative importance of computational and systems parameters, a large number of experiments were made with models of the Eastern Scheldt using the SIMSYS2D system in many of its modes. Some of the more important results of this extensive analysis will be presented in this paper.

Method of Analysis

The Eastern Scheldt model, used for most of the experiments, had a grid size of 800 m. For a few experiments a model with a grid size of 400 m was used (Fig. 1).

A periodic tide was taken at the boundary of the model. The amplitude and phase relation of the boundary points to a fixed observation station near the location of the boundary was determined by means of cross-spectral analysis from a 5½ day simulation of a model with a grid size of 800 m, which included the coastal region outside the estuary and the estuary itself [2].

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After determination of a yearly averaged tide from observations at the fixed station, amplitudes and phases of every boundary point could be calculated from the amplitude and phase relations for the M_2, M_4, and M_8 tidal component.

Simulations, as long as 15 tidal cycles, indicated that a quasi-steady state condition was never obtained. The tidal residual currents computed as low-passed filtered Eulerian transports did slowly fluctuate. Mean water levels computed by the same low-pass filter (tidal eliminator) also showed some fluctuation. In addition, the computation appears to contain noise in a few areas due to the large grid size and the discrete changes in area's size when tidal flats and sandbars become dry or flood.

Consequently, comparisons of tidal wave propagation and amplification by means of a Fourier analysis of the computed records at stations along the periphery of the estuary were difficult to make. A much better approach appeared to be optimal estimation of amplitude and phase relationships between pairs of stations by cross-spectral analysis. In addition to the amplitude and phase relations, data about the confidence of the linear estimate could be computed.

Simulations of 5½ days were made and as at least one day is required for dissipation of the starting transient, 100 hours of the simulation was available for analysis. The 100 hours is a short series for cross-spectral analysis as only eight cycles of the primary tide are available, nevertheless, a very high coherency was always obtained for this frequency. This was not the case for the quarter- and sixth-diurnal tides. These appeared to be influenced by leakage from the very strong semi-diurnal tidal component.

To alleviate that problem, band pass filters, as described by Godin [3], were applied to the computed time series which eliminated the semi-diurnal tide but retained the quarter-diurnal or the sixth-diurnal tide. Subsequently, cross-spectral analyses were made on pairs of the resulting time series. The analyses were made of the records of adjacent stations on a line along the estuary as shown in Fig. 1.

To investigate the effect of the method of approximation of the finite difference equations or the sensitivity of certain parameters on residual circulation, (Eulerian residual transport) the mass transport rates through certain cross-sections were filtered with a low pass filter. Also, the mass transport rate at each section with a dimension of the grid size of Δx was computed and filtered with a boxcar filter with the length of the tidal period. Alternately, the filtering was done with a low pass filter which completely eliminated components in the tidal frequencies. Subsequently, the vector plots of the residual transport fields could be prepared.

Effects of the Main System Parameters

In this study an alternating direction implicit scheme is used to solve the set of partial differential equations which represent the two-dimensional equations representing the two-dimensional flow [4]. For the analyses of the effects of modifications of the main system parameters a higher order finite difference approximation in space was used as the base case. This representation was first used by Arakawa and

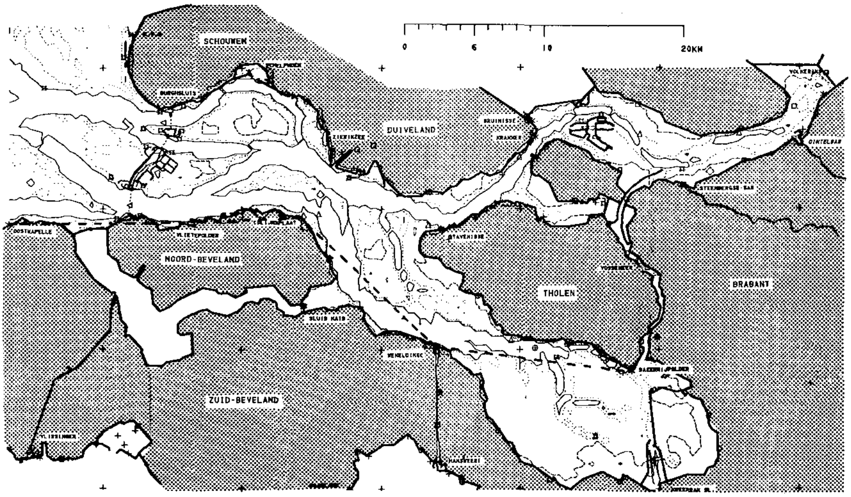

Fig. 1--Model area used for the experiments (dashed line indicates line of stations used for determination of amplification and phase lags)

has a second order accuracy in time [4]. The bottom stress terms in the semi-momentum equations were computed in a conventional manner using a Chezy value. The Chezy values were computed every half-hour from the transport depth from the bottom roughness expressed with a Manning's n coefficient.

In an earlier study it was indicated that the finite difference methods cause dispersion and amplification of a propagating tidal wave compared to solutions of the differential equations. These effects can be expressed with the complex amplification factor [5]. The dispersion and amplification is a function of the timestep, the grid size, and of the period of the wave. The base case was a simulation with the model which was already adjusted.

For the study of the effects of the main system parameters, five simulations were made, each one was a variation of the base case. We investigated a 10% decrease in Manning's n, a 5% increase in depth, a 50% increase in the multiplier (α) of the advection term, a doubling of the momentum diffusion and a reduction of 50% of the timestep (Fig. 2).

In the base case, we used for the momentum diffusion a value of 15 m^2/sec, $\alpha = 1$ and a timestep of 2.5 min. To show the effect of the parameter modifications the time lag between a boundary station (Oostkapelle) and an inland station (Razernijpolder) was computed for several components. The confidence intervals shown in this figure are for the analysis of the last two stations on the line. It will be noted that the confidence interval for the sixth-diurnal tide is much larger than for the semidiurnal tide. In the figure the results of an analysis of

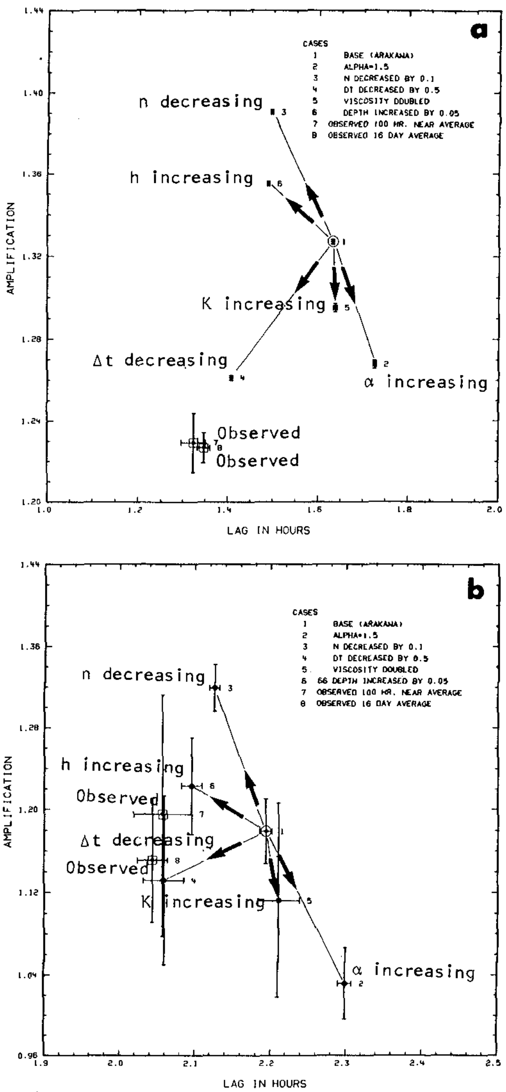

Fig. 2--Amplification versus phase lag for the relation between a station near the boundary (Oostkapelle) and an inland station (Razernijpolder): (a) semidiurnal tide (b) sixth-diurnal tide

field data is also shown. The spectral densities of the observed records in the main tidal frequency bands was nearly the same as for the boundary conditions used in the experiments.

Even though the use of a multiplier for the advection is commonly used in one-dimensional computations to account for the non-linear velocity distributions over the vertical, this multiplier is not used much for two-dimensional models.

From the results shown here it can be inferred that use of this coefficient does not seem effective as increasing n will give similar effects as increasing α.

Adjustment of parameters other than the timestep appears to be ineffective in obtaining better results. This conclusion can also be reached by a careful analysis of the behavior of the complex propagation factor. Only when the timestep is reduced does the amplification and phase lag approach the observed amplification in lag. A good agreement of all tidal components including the overtides could be obtained by reducing the grid size from 800 m to 400 m. Figure 3 shows the comparison between the observed and the computed tide at the inland station for 4 September 1975.

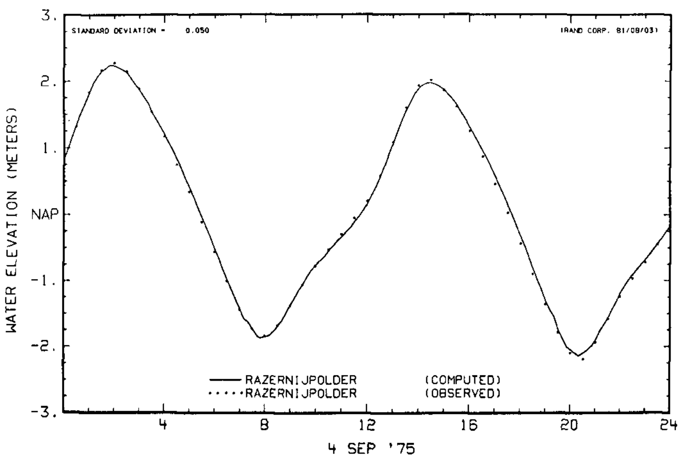

Fig. 3--Comparison of computed and observed water levels at an inland station (400 m grid model)

Choice of Advection Approximation

The approximation used for the advection terms in the momentum equations influences the tidal amplification and the transfer of energy of the primary tides to overtides.

Simulations were made with different advection terms. It appeared that the most simple advection term was the most effective in transferring energy from the semidiurnal tides to other tidal frequencies as can be seen from the amplification and phase lag plot (Fig. 4). The amplification and phase of other tidal components was not influenced much by the choice of the approximation. The choice of the approximation influences the residual circulations to a certain extent, but it did not change the general pattern. Omission of the advection terms reduces the computed residual circulations and one can conclude that advection is the primary mechanism generating residual circulations. A comparison of the computed residual circulation with and without the advection term is presented in Fig. 5.

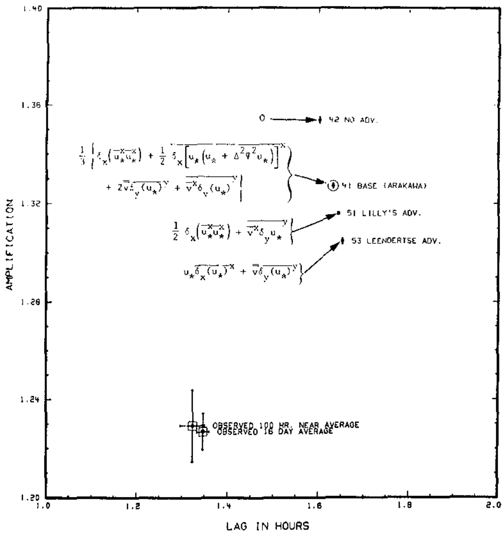

Fig. 4--Amplification versus phase lag of the semidiurnal tide for the relation between a station near the boundary (Oostkapelle) and an inland station (Razernijpolder) for computations with different advection term approximations

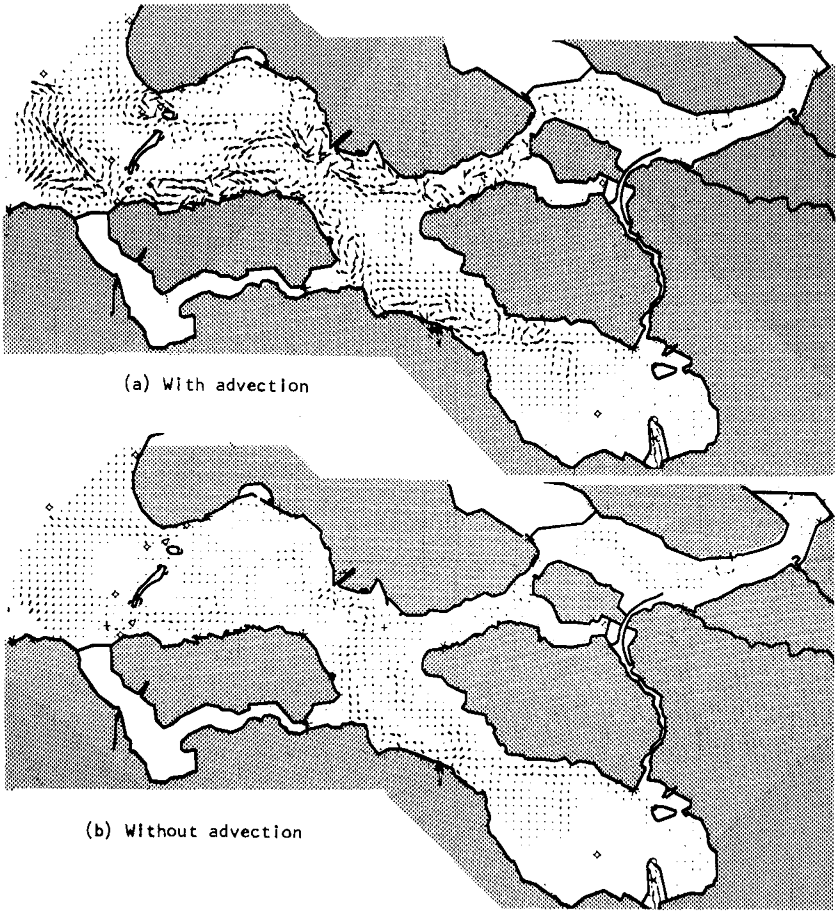

Fig. 5--Computed Eulerian residual transports for a computation with and without the advection terms

Bottom Stress Approximation

In tidal computations the bottom stress is generally taken as a function of the squared velocity and of the inverse of the depth and the squared Chezy coefficient.

In the design of a finite difference model the modeler has the option to compute the bottom stress term on the lower time level or make an expression which is central in time. Comparison of simulations with these options indicated that for practical purposes the same results are obtained. Nevertheless, the expression which is central in time is to be preferred when tidal flats and wind effects are simulated. Otherwise, a strong wind will accelerate water with a very limited depth which has just started to participate in the computational field due to flooding, to unrealistic high velocities because the bottom friction is zero.

The bottom stress can also be expressed as a function of the root of the local subgrid scale energy, the local velocity and a length scale. The local subgrid scale energy is computed as a constituent in the mass transport model. The energy is generated by the bottom stress and this generation is taken exactly the energy loss in the momentum equations. Its decay is taken as a function of the 3/2 power of the local energy intensity. The mixing length was assumed to be a fraction of the depth. By making these computations one has a two-dimensional model with a simple turbulence closure. Comparison of a simulation made with this closure model and a simulation with the bottom stress computed directly from the velocities reveal only minor differences as shown in Fig. 6.

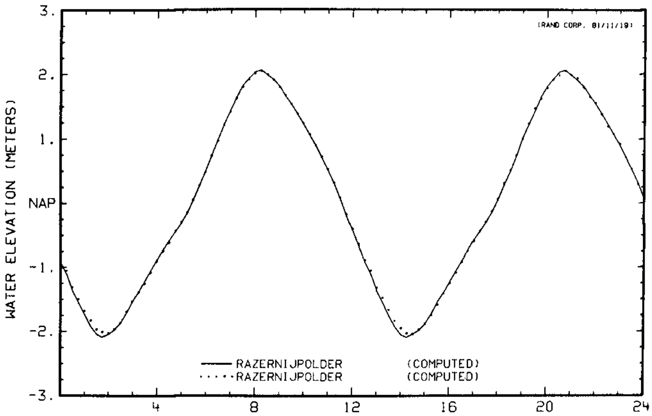

Fig. 6--Comparison of water levels at an inland station computed with a simple turbulence closure model (····) and computed by use of a stress term which is a function of the squared velocity (——)

Conclusions

From the experiments with an estuary model and from analytical considerations it can be concluded that a high resolution in time and space is required to obtain a good agreement between model and prototype. Adjustments of depth, viscosity or bottom roughness cannot compensate for insufficient resolution.

The advection terms cannot be omitted from the model as otherwise the tidal residual circulations are not properly computed. A simple expression for the advection terms appears to be the most effective in transferring tidal energy from the main tidal frequency to the overtides.

The use of a simple turbulence closure model appears to give virtually the same results as a computation with the bottom stress computed in a conventional manner.

References

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