

THE EFFECT OF BED SLOPE ON WAVE CHARACTERISTICS

by

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1. INTRODUCTION

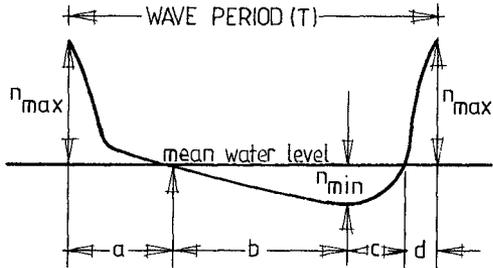
In all wave theories used in engineering applications it is assumed that the profile of the bed is horizontal which results in a symmetrical wave shape and velocity field, so that, strictly speaking, they can be applied only to this condition. Nearshore bottom profiles are, however, seldom horizontal, and a wave moving over a shoaling slope has an asymmetrical profile which is associated with an asymmetrical velocity field within the wave which, in turn, directly influences the movement of bed sediment. As a matter of necessity, but within reason, engineers have ignored the influence of bed slope on the wave theory used. This course of action was justifiable on many counts, not the least of which was that inaccuracies associated with the theories used were far greater than any inaccuracies introduced by ignoring bed slope parameters. However, wave theories developed in recent years have become increasingly accurate and reliable so that it may now be necessary to take account of bed slope parameters in applying these wave theories before further improvements can be made in the techniques used to predict wave-induced sediment transport.

2. AIMS AND OBJECTIVES

This paper describes an experimental programme which assessed and quantified the influence of bed slope on the characteristics of non-breaking waves. The objective was to determine, for individual wave characteristics, the maximum slope up to which insignificant effects are induced and, for steeper slopes, to describe quantitatively the divergence from those results for a horizontal bed. The specific wave characteristics measured and compared for various slopes were:

- (i) wave profile (n versus t);
- (ii) wave celerity (and hence, by definition, wavelength);
- (iii) potential energy;
- (iv) ratio of crest time to wave period;
- (v) ratio of trough time to wave period;
- (vi) ratio of crest height to wave height;
- (vii) ratio of trough depth to wave height;
- (viii) crest skewness;
- (ix) trough skewness; and
- (x) ratio of asymmetrical wave slope to bed slope.

The definitions of these characteristics are given in Figure 1.



WAVE PROFILE (n versus t)

WAVE CELERITY (and hence, by definition, wave length)

POTENTIAL ENERGY

RATIO OF CREST TIME TO WAVE PERIOD $(a + d)/T$

RATIO OF TROUGH TIME TO WAVE PERIOD $(b + c)/T$

RATIO OF CREST HEIGHT TO WAVE HEIGHT $n_{max}/(n_{max} + n_{min})$

RATIO OF TROUGH DEPTH TO WAVE HEIGHT $n_{min}/(n_{max} + n_{min})$

CREST SKEWNESS $d/(a + d)$

TROUGH SKEWNESS $b/(b + c)$

RATIO OF WAVE SLOPE TO BED SLOPE $|n_{min}|/b \tan \theta$

Figure 1 : Definition Sketch

3. PREVIOUS WORK

Adeyemo (1968) investigated experimentally, for six bed slopes ranging from 1:4 to 1:18, the effects of bed slope on the ratio of crest height to wave height and various measures of crest skewness. The work covered a wide range of wave non-linearity using values of d/λ between 0.08 to 0.26. Adeyemo demonstrated that crest skewness was greater on steeper bed slopes and that it increased as the wave moved into shallower water, being a maximum at the breaker point. He also found that for small values of d/λ the ratio of crest height to wave height became progressively less on steeper slopes. Unfortunately, Adeyemo did not give any observed values of H/d at the points of measurement which limited severely the comparisons that can be made with other experimental studies and prevented the incorporation of the results into such studies. The work described in this report demonstrates, for example, that the ratio of crest height to wave height is also a function of the ratio of wave height to water depth.

Adeyemo (1970) investigated experimentally the relationship between wave shape asymmetry and wave-induced velocities near the breaker zone. From the results of experiments using a fixed wave period of 0.8 seconds, bed slopes of 1:9 and 1:18 and values of d/λ between 0.0800 and 0.1245, he concluded that there are qualitative and quantitative relationships between the asymmetry of wave shape, caused by bed slope, and the asymmetry of the resulting velocity field. Indeed, his observed velocity profiles shown in his Figures 3 and 4 for the two slopes tested demonstrate all the characteristics and trends shown on the observed wave shape profiles of Figure 4 in this paper.

4. EXPERIMENTAL EQUIPMENT AND TECHNIQUE

The basic apparatus used for the experimental programme was a glass-walled tilting flume with a test section 30m long, 0.99m deep and 0.75m wide. Tilting was effected by means of electrical drives about a central pivot point. The flume had a tilting slope range of between horizontal and 1:67. Other slope ranges could be obtained by building in fixed slopes with respect to the flume, and tilting these. The flume was fitted with a pneumatic, uniform wave generator. A mobile instrument carriage located on rails along the top of the flume was fitted with two parallel-wire, resistance wave transducers located centrally in the flume and 0.22m apart along the flume axis.

The carriage also carried the associated electrical equipment together with a water level follower and two analogue chart recorders. The carriage instrumentation was in turn connected to a Hewlett Packard System 1000, model 31 computer with a 21MXE central processing unit, an RTE II operating system and a 2240A measuring and control processor.

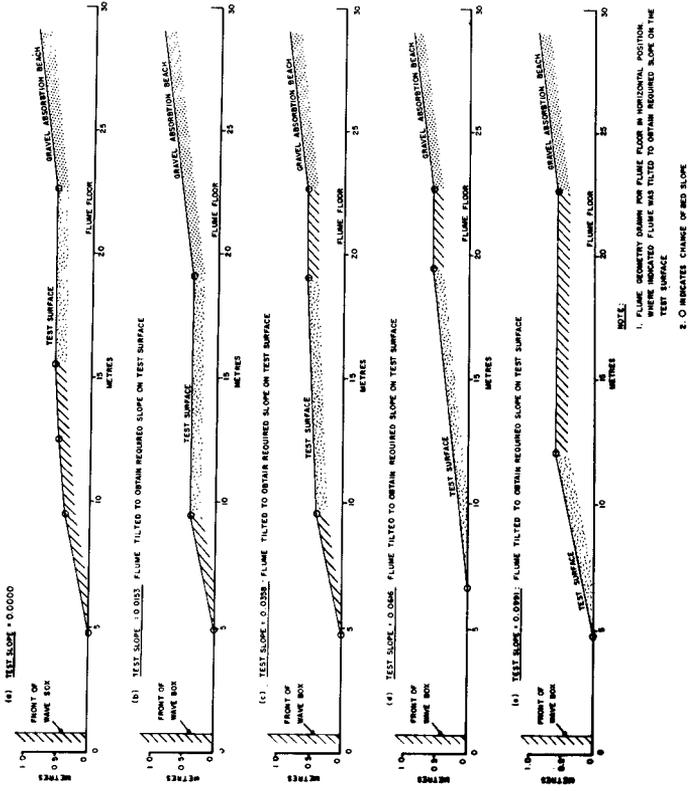


FIGURE 2 : Flume Profiles Used in Test Programme

NOTES:

1. (3) DENOTES REGION No.
2. --- DENOTES LINES OF EQUAL P (VOCCOIDAL PROFILE PARAMETER)
3. HEAVILY FRAMED REGIONS INDICATE THOSE COVERED BY USEFUL EXPERIMENTAL DATA

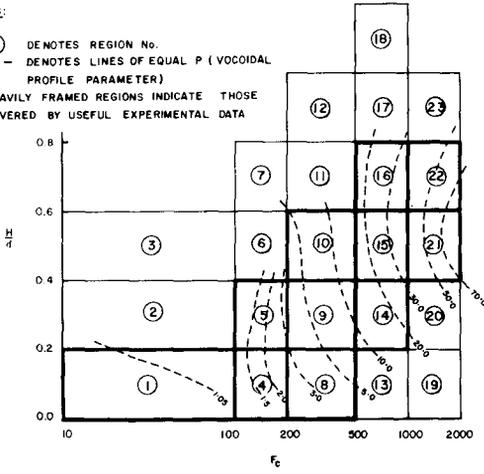


FIGURE 3 : Adopted Regions on $H/d, F_c$ Field

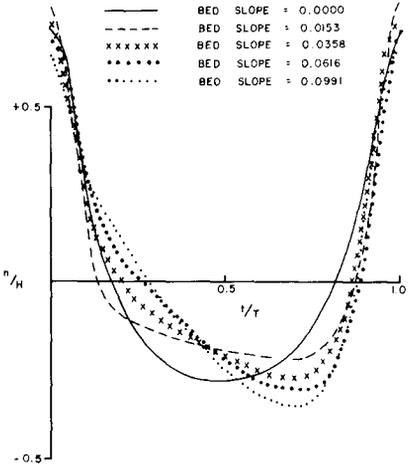


FIGURE 4 : Typical Observed Wave Profiles

All instrument calibration and data acquisition were performed on-line to the computer, the data being analysed immediately and then stored on disc file as well as printed out as a hard copy. Water level readings from each probe were logged every 30 milliseconds. A sample comprised of a number of consecutive, individual readings acquired simultaneously at each probe during a time period of at least two wave periods. A test run comprised of a specified number of samples acquired simultaneously at each probe, the wave height, water depth and wave period being the same for each sample.

It was possible to acquire any number of samples for any given test run but three was determined to be the optimum. This number was based on the results of pilot experiments in which no definite improvement in the standard deviation of the measured characteristics could be discerned for greater numbers of samples.

The occurrence of any water level set-down or set-up at the wave transducers was automatically accounted for in the data acquisition system.

Five slopes were used during the experimental programme, namely, 0.0000, 0.0153, 0.0358, 0.0616 and 0.0991. The flume profiles are shown in Figure 2. The slopes were not tested in this order. The testing order was 0.0153, 0.0358, 0.0000, 0.0991 and 0.0616. The order was determined as the programme proceeded but was governed mainly by two factors:

- (i) the ease with which the flume could be adapted to another slope, and
- (ii) the desire to test extremes so that the intermediate slopes that should be tested could be determined.

It was found that the experimental technique used precluded experimentation on slopes steeper than 0.100 because:

- (i) wave reflection created significant secondary effects,
- (ii) random ripples caused by wave-breaking and downrush created significant secondary effects, and
- (iii) differences in depth between probes became significant.

5. EXPERIMENTAL DESIGN

Any dimensionless numbers used in the basic classification or grouping of data had to be free of theory-dependent parameters and preferably contain only those which could be measured directly. The only parameters capable of direct measurement at a point over a sloping bed, with a high degree of precision, are wave period (T), water depth (d) and wave height (H). Hence, use was made of the non-linearity parameter, F_c , after Swart (1978) and Swart and Loubser (1979). This is defined as;

$$F_c = (H/d)^{0.5} (T_c)^{2.5}$$

$$T_c = T (g/d)^{0.5}$$

One helpful property of F_c is that waves of equal F_c have approximately the same relative wave shape. It was decided, therefore, to establish a system of comparison based on the measured experimental values of H/d and F_c .

Another problem to be solved was that a combination of test values T , d and H used on one slope would be physically impossible to reproduce exactly on each of the other slopes tested. The solution lay in the division of the H/d versus F_c plane into regions so that groups of results from identical regions could be compared. For this to be done two criteria would have to be satisfied. One of these was that the range of relative profile shape covered by each region should not be too large. The second, and more important criterion, was that the standard deviation of results obtained over any one region should not be too large, otherwise no significance could be attributed to differences between mean values obtained from various regions on different slopes. To determine the boundaries and size of these regions a pilot experiment was undertaken and the results investigated statistically. Regions were adopted which required between 4 and 6 test runs per region. The regions adopted are shown in Figure 3 together with the relationship between H/d , F_c and P the vocoidal wave profile parameter, which is a measure of relative wave shape after Swart (1978) and Swart and Loubser (1979).

It will be noted later that in Tables 1, 2 and 3 experimental values of celerity have not been used on their own, but rather the ratio of observed celerity to that predicted by vocoidal theory. This is called the celerity ratio and is simply a device to reduce the standard deviation of celerity measurement over any one region. The experimental values of celerity on their own showed too great a variation. This device in no way makes the final objective (the influence of slope on wave celerity) dependent on any wave theory. Any wave theory could have been used providing its celerity estimates varied with respect to T , d and H in a manner similar to that in real situations on horizontal beds. Obviously the greater this similarity, the less would be the standard deviation. It is emphasized that it is this similarity of variation with T , d and H that is important here and not the accuracy of the absolute value of celerity predicted by the theory. However, if there was similarity in this respect also, the ratio should be close to 1.0 in each region for a horizontal bed. This was in fact so when vocoidal theory was used.

TABLE 1

PERCENT VARIATION OF REGIONAL MEANS FROM HORIZONTAL BED RESULT WHERE
A STATISTICALLY SIGNIFICANT VARIATION OCCURRED

(a) Celerity Ratio

Slope	0,0153	0,0358	0,0616	0,0991
Region 1				-3
Region 4				
Region 5	-3	-4		
Region 9			+4	
Region 10				
Region 15				

(b) Potential Energy Coefficient

Slope	0,0153	0,0358	0,0616	0,0991
Region 1	-2			
Region 4				
Region 5				
Region 9		-5	-5	
Region 10				
Region 15				

(c) Ratio of Crest Time to Wave Period

Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4				
Region 5				
Region 9				+13
Region 10			+12	+23
Region 15			+13	+21

(d) Ratio of Trough Time to Wave Period

Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4				
Region 5				
Region 9				-8
Region 10			-6	-11
Region 15			-7	-11

TABLE 1 (Cont'd)

PERCENT VARIATION OF REGIONAL MEANS FROM HORIZONTAL BED RESULT WHERE
A STATISTICALLY SIGNIFICANT VARIATION OCCURRED

(e) <u>Ratio of Crest Height to Wave Height</u>				
Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4				
Region 5				-6
Region 9			-5	-11
Region 10			-7	-13
Region 15			-7	-12
(f) <u>Ratio of Trough Depth to Wave Height</u>				
Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4				
Region 5				+11
Region 9			+9	+21
Region 10			+19	+34
Region 15			+21	+33
(g) <u>Crest Skewness</u>				
Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4			-10	-10
Region 5				-7
Region 9	-7	-14	-18	-25
Region 10	-10	-18	-27	-32
Region 15	-21	-37	-47	-49
(h) <u>Trough Skewness</u>				
Slope	0,0153	0,0358	0,0616	0,0991
Region 1				
Region 4				
Region 5				
Region 9		+33	+32	+24
Region 10		+46	+43	+41
Region 15	+63	+82	+80	+77

TABLE 2

RANKING OF WAVE CHARACTERISTICS IN RISING ORDER OF SLOPE INFLUENCE

Rank	Wave Characteristic	Fc range affected	Slope range affected	Greatest observed regional variation
(i)	Potential energy coefficient and celerity ratio	No consistent trend	No consistent trend	5%
(ii)	Ratios of crest time to wave period and trough time to wave period	>200	$\geq 0,062$	23%
(iii)	Ratios of crest height to wave height and trough depth to wave height	>100	$\geq 0,062$	34%
(iv)	Crest and trough skewness	>100	$\geq 0,015$	82%

TABLE 3

RECOMMENDED FUNCTIONAL RELATIONSHIPS

- Note: * denotes applicable to all $\tan \theta < 0,100$
- + denotes that values may be in error by an amount approaching 10% for $\tan \theta > 0,100$ when $F_c > 500$. With this proviso the value shown is applicable to all $\tan \theta \leq 0,100$
- # denotes that functional relationships should not be used for values of $H/d < 0,05$ when $F_c > 200$. In general all values applicable to all $\tan \theta \leq 0,100$

Fc range	+ Celerity ratio
All Fc	1,00

Fc range	* $\tan \alpha_w$
All Fc	0,4 C $\tan \theta$

Fc range	* Potential energy coefficient
Fc < 10	0,062
10 < Fc < 100	0,061
100 < Fc < 200	0,059
200 < Fc < 500	$0,062 - 0,019 \left(\frac{H}{d}\right)^{0,56}$
500 < Fc < 1 000	$0,062 - 0,028 \left(\frac{H}{d}\right)^{0,56}$
Fc > 1 000	$0,062 - 0,036 \left(\frac{H}{d}\right)^{0,40}$

TABLE 3 (Cont'd)

RECOMMENDED FUNCTIONAL RELATIONSHIPS

Fc range	# $\frac{\text{Crest time}}{T}$
Fc < 10	0,50
10 < Fc < 100	0,46
100 < Fc < 200	$0,47 - 0,17\left(\frac{H}{d}\right)$
200 < Fc < 500	$0,42 - 0,20\left(\frac{H}{d}\right) + 0,70 \tan \theta$
500 < Fc < 1 000	$0,42 - 0,20\left(\frac{H}{d}\right) + 0,90 \tan \theta$
Fc > 1 000	$0,38 - 0,20\left(\frac{H}{d}\right) + 2,70 \tan \theta$

Fc range	# $\frac{\text{Trough time}}{T}$
Fc < 10	0,50
10 < Fc < 100	0,54
100 < Fc < 200	$0,53 + 0,17\left(\frac{H}{d}\right)$
200 < Fc < 500	$0,58 + 0,20\left(\frac{H}{d}\right) - 0,70 \tan \theta$
500 < Fc < 1 000	$0,58 + 0,20\left(\frac{H}{d}\right) - 0,90 \tan \theta$
Fc > 1 000	$0,62 + 0,20\left(\frac{H}{d}\right) - 2,70 \tan \theta$

Fc range	# $\frac{\text{Crest height}}{H}$
Fc < 10	0,50
10 < Fc < 100	0,54
100 < Fc < 200	$0,53 + 0,26\left(\frac{H}{d}\right)$
200 < Fc < 500	$0,60 + 0,27\left(\frac{H}{d}\right) - 0,90 \tan \theta$
500 < Fc < 1 000	$0,63 + 0,25\left(\frac{H}{d}\right) - \tan \theta$
Fc > 1 000	$0,67 + 0,20\left(\frac{H}{d}\right) - 1,3 \tan \theta$

TABLE 3 (Cont'd)

RECOMMENDED FUNCTIONAL RELATIONSHIPS

Fc range	# $\frac{\text{Trough depth}}{H}$
Fc < 10	0,50
10 < Fc < 100	0,46
100 < Fc < 200	$0,47 - 0,26 \left(\frac{H}{d}\right)$
200 < Fc < 500	$0,40 - 0,27 \left(\frac{H}{d}\right) + 0,90 \tan \theta$
500 < Fc < 1 000	$0,37 - 0,25 \left(\frac{H}{d}\right) + \tan \theta$
Fc > 1 000	$0,33 - 0,20 \left(\frac{H}{d}\right) + 1,3 \tan \theta$

Fc range	*Crest skewness
Fc < 10	0,50
10 < Fc < 100	0,50
100 < Fc < 200	$0,50 - 0,40 \tan \theta$
200 < Fc < 500	$0,50 - 0,71 (\tan \theta)^{0,85}$
500 < Fc < 1 000	$0,50 - 0,83 (\tan \theta)^{0,56}$
Fc > 1 000	$0,50 - 1,06 (\tan \theta)^{0,45}$

Fc range	*Trough skewness
Fc < 10	0,50
10 < Fc < 100	$0,50 + \frac{\tan \theta}{0,07 + 30 \tan \theta}$
100 < Fc < 200	$0,50 + \frac{\tan \theta}{0,06 + 14 \tan \theta}$
200 < Fc < 500	$0,50 + \frac{\tan \theta}{0,02 + 6,5 \tan \theta}$
500 < Fc < 1 000	$0,50 + \frac{\tan \theta}{0,015 + 4,3 \tan \theta}$
Fc > 1 000	$0,50 + \frac{\tan \theta}{0,01 + 4,0 \tan \theta}$

6. EXPERIMENTAL RESULTS

6.1 Coverage of F_c , H/d Field

It was not possible to acquire useful data in all the regions shown in Figure 3. The regions for which useful data were acquired, and their relation to other regions, are shown in Figure 3. In general, the extent of the coverage achieved was limited by the following factors.

- (i) Secondary wave effects (solitons) became significant at values of F_c greater than 2 000
- (ii) The occurrence of solitons and/or random ripple effects became significant in regions 13, 19 and 20
- (iii) Test runs in regions 11, 12, 17, 18 and 23 could not be obtained because of wave breaking
- (iv) Test runs in regions 2, 3 6 and 7 were precluded because the necessary wave heights could not be achieved by the wave generator at smaller wave periods.

The coverage obtained on the two extreme test slopes (0.0000 and 0.0991) requires some additional discussion. Testing beyond $F_c = 1\ 000$ was not possible on the 0.0991 slope, because of secondary effects caused by wave reflection and random ripples created by wave breaking and down-rush. Testing beyond $F_c = 1\ 000$ on the horizontal slope was also precluded because of the presence of solitons in the wave form. This was partly because when tests were done at small depths on the test surface, water depths at generation were less than those for other test slopes for which the flume was tilted to maximise the depth at the generator. Earlier preliminary testing had shown that solitons were less evident for greater depths at generation (see also Hulsbergen (1972) and Van Wyk (1975)).

However, of greater importance was the fact that values of H/d greater than 0.55 could not be obtained over a horizontal bed due to wave instability and wave breaking. This was confirmed by using three different arrangements of the experimental equipment. Initially, testing on the horizontal slope was attempted using the test surface shown in Figure 2b with the flume in the horizontal position, but this failed to produce ratios of H/d greater than 0.50 because of wave instability and wave breaking. This was conceivably due to the sharp change in bed slope at the top of the shoaling slope. Further testing on the horizontal bed was postponed until the arrangement of Figure 2a could be constructed with a smooth transition between the initial shoaling slope and the horizontal test surface. This arrangement had the added advantage of increasing the depth at generation. However, the same limitation on values of H/d remained but experimentation was continued and as large a coverage as possible was achieved.

When testing had been completed on all five test slopes a third attempt was made to achieve values of H/d greater than 0.55 over a horizontal bed using a modification of the flume profile shown in Figure 2d. A horizontal, symmetrical contraction to half the flume width was constructed over the top 5m of the shoaling slope, so that a horizontal bed, half the flume width, was available prior to the absorption beach. This increased the number of available combinations of T , d and H but the same phenomena of wave instability and wave breakup were observed.

The value of H/d of 0.5 at which breaking of waves occurred over a horizontal bed was well below the values of 0.8 and greater obtained on beds with slopes as low as 0.0153. The experience of other researchers (see Nelson, 1980) tend to confirm this limiting value of H/d for horizontal beds, and leads to the conclusion that unbroken, regular waves with values of H/d exceeding 0.55 cannot exist on a horizontal bed.

6.2 Statistical Significance and Magnitude of Slope Influence

One objective of the project was to determine those slopes on which wave characteristics became significantly different from those of waves moving over horizontal beds. Unfortunately, for five of the eleven regions tested no data were obtainable for the horizontal bed. For three of these regions (8, 14 and 21) this was due to secondary wave effects. The limiting value of H/d of 0.55 accounted for the other two (16 and 22). The fact that values of H/d of 0.8 were possible for these two regions on slopes as small as 0.0153, in itself indicates a significant slope effect.

For the six remaining regions (1, 4, 5, 9, 10 and 15) a direct comparison of all four finite slope results with the horizontal bed results, was possible for all measured wave characteristics. A statistical test of significance at the 5 per cent level was made to determine which slopes produced results which varied significantly from those obtained on the horizontal bed. Table 1 shows the percentage variations of regional means from the horizontal-bed result where a statistically significant variation occurred. All tables show that with respect to the combined considerations of magnitude of variation, range of slopes responsible and F_c range affected, the ranking in Table 2, in rising order of slope influence, applies.

The only parameter which was treated differently was the ratio of asymmetrical wave slope to bed slope. The reason for a different treatment is that it was difficult to read off (define) the asymmetrical wave slope when (i) the wave height to water depth ratio H/d was less than 0.2, and (ii) the non-linearity parameter F_c was less than 200. Sometimes the asymmetrical slope could be interpreted in more than one way. Such results were also neglected. Therefore the regional mean data were lumped together in two different ways rather than interpreting them separately, namely,

- (i) by grouping the wave profile data for all regions together for any slope; and
- (ii) by grouping all wave profile data for the same region (all slopes) together.

These results indicate that the overall mean of $(C \tan \theta / \tan \alpha \omega)$ has a value of about 2.50 with a standard deviation of 0.25. This implies that the 95 per cent confidence band would extend from about 2.0 to 3.0 with a best estimate of 2.50. No subgroup (population comprising the results of a given slope or region) had a mean result which differed significantly at the 95 per cent level from this overall mean result.

6.3 Recommended Values of Wave Characteristics

For the range of slopes tested, all the experimental data were used to derive empirical relationships for wave characteristics. These are shown in Table 3. Due cognizance must be given to the remarks shown in the table when using these relationships.

The tables apply to bed slopes of less than 0.100. They show that wave characteristics can be functions of F_c , H/d and $\tan \theta$. This is so for the ratios of crest time to wave period, trough time to wave period, crest height to wave height and trough depth to wave height. The observation that for shallower water, the ratio of crest height to wave height becomes progressively less on steeper slopes agrees with the findings of Adeyemo (1970).

Crest and trough skewness are shown to depend only on F_c and $\tan \theta$. The results indicate that there may be some dependence on H/d but that it is either very small, or the data are inconclusive and conflicting.

For all practical purposes the potential energy coefficient is a function of only F_c and H/d on slopes less than 0.100, and the celerity ratio is independent of all three parameters. Since the celerity ratio used was that of experimentally measured values to those predicted by the vocoidal wave theory, the results tend to enhance the practicability of applying the vocoidal theory to a wide range of slopes for the purpose of celerity and wave length determinations.

The asymmetrical wave slope depends on the bed slope and the wave celerity and can be used in conjunction with the skewness parameters to reconstruct the wave profile schematically.

6.4 Typical Observed Wave Profiles

To assist the reader to appreciate the progressive influence of bed slope on wave profile some typical observed profiles are shown for region 15 in Figure 4.

7. CONCLUSIONS AND RECOMMENDATIONS

Experiments on the influence of bed slope on some parameters associated with unbroken regular waves lead to the following findings:

- (i) It is doubtful whether unbroken, regular waves on a horizontal bed with wave height-to-water depth ratios exceeding 0.55, can ever exist. Bed slopes as low as 0.015 influence the wave mechanics sufficiently to increase this ratio to greater than 0.80. Therefore, the characteristics of waves near breaking on a gentle slope cannot be assumed to be the same as those on a horizontal bed.
- (ii) The effect of bed slopes of less than, or equal to, 0.100 on potential energy and wave celerity is minimal and, for all practical purposes, can be ignored.
- (iii) Bed slopes greater than or equal to 0.062 can significantly influence the ratios of crest height to wave height, trough depth to wave height, crest time to wave period and trough time to wave period. These modifications of profile shape will create a redistribution of potential bed particle transport because of consequent changes in the positive and negative wave-induced bed velocities.
- (iv) The greatest influence of bed slope is on crest and trough skewness and the asymmetrical wave slope and becomes significant on bed slopes as low as 0.015. These resulting asymmetries of wave profile will result in an asymmetry of the velocity field within the wave, which will have a direct influence on potential bed-particle transport.
- (v) Functional relationships relate bed slope and wave characteristics and these are as shown in Table 3.

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