

WAVE REFLECTION FROM UNDULATING SEABED TOPOGRAPHY

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ABSTRACT

The results of experiments are described which show that surface waves may experience a resonant interaction with undulations on the seabed. This interaction manifests itself in a strong reflection of incident wave energy when the wavelength of the bottom undulation is about half that of the surface wave. It is shown that such a mechanism might enable a region of undulating seabed topography (eg sand bars or sand-waves) to extend in an up-wave direction, into a region of otherwise plane bed.

INTRODUCTION

The interaction of surface water waves with undulating seabed topography is a problem of fundamental importance to coastal engineers. While it has been shown, that, in the nearshore zone, quite complex patterns of wave motion (eg edge waves) may lead to beach cusps, shore parallel bars and even crescentic shore welded sand bars (Holman and Bowen, 1982), the problem that is considered in this study is how waves are likely to interact with a pre-existing pattern of regular undulations on the seabed. Such a pattern may consist of shore parallel bars formed by plane reflections of low amplitude swell waves from a beach, leading to standing waves of the type observed by Suhayda (1974). Alternatively, standing waves may occur seaward of the surf zone as a result of the time varying breakpoint forcing mechanism described by Symonds et al (1982). In this case a forced wave having incident wave group periodicity is radiated seaward from the breaker zone. Such a wave, interacting with incoming infragravity waves having periods in the range 30-300s, might lead to standing waves and consequently to bar formation. This latter mechanism seems to provide the most likely means of generating the multiple shore parallel bars which have been observed by Short (1975) and which would require wave periods of the order 100s.

A pre-existing pattern of bottom undulations might also be formed by tidally generated features such as sand waves (eg Langhorne, 1982) or sand ridges lying transverse to the general direction of wave propa-

gation and, as such, may occur well offshore away from the coastline.

In general, surface wave/seabed interactions may occur in any depth of water where the waves are able to "feel the bottom". It follows that such interactions may occur for a wide range of surface water wavelengths and bedform length scales.

Recent theoretical work Davies, 1980, 1982, has shown that large amounts of wave energy may be reflected as a result of resonant interactions between surface water waves and bottom undulations, the wavelengths of which lie in the ratio 2:1 approximately.

Davies (1980, 1982) has used linear perturbation theory to show that, to a first approximation, wave reflection from a finite number of submerged sinusoidal bars, having small amplitude and on an otherwise plane bed, is given by the wave reflection coefficient

$$K_r = \frac{a_r}{a_i} = \frac{2bk}{\{2kh + \sinh(2kh)\}} \cdot \left[\left(\frac{2k}{\ell} \right) \left| \frac{\sin\left(\frac{2k}{\ell} m \pi\right)}{\left(\frac{2k}{\ell}\right)^2 - 1} \right| \right] \quad (1)$$

where a_r and a_i are the reflected and incident wave amplitudes respectively, well away from the region of bedforms, b is the bar amplitude, h is the water depth, m is the number of bars and k and ℓ are the free surface and bar wavenumbers. Here $k = 2\pi/L$ and $\ell = \frac{2\pi}{L_b}$, where L and L_b are surface and bar wavelengths respectively. It should be noted that, strictly speaking, this is a two-dimensional formulation of the problem requiring long crested surface waves collinear with the bottom undulations.

Equation (1) illustrates that for a given number of bars (m), the wave reflection coefficient is oscillatory in $2k/\ell$, that is the quotient of twice the surface wavenumber and the bed wavenumber. The reflection coefficient is also resonant in the region $2k/\ell \approx 1$ and, at $2k/\ell = 1$ itself, is proportional to m which suggests that peak reflection coefficients are linearly dependent on the number of bars.

These results were without any detailed experimental proof and this paper describes experiments carried out in a wave tank to examine the nature of resonant interactions between surface waves and simple sinusoidally varying topography. Preliminary aspects of this study have already been described by Heathershaw (1982).

EXPERIMENTAL PROCEDURE

To test Davies' (1980, 1982) theoretical predictions, and in particular Equation (1), detailed measurements of wave reflection from submerged bars were carried out using the 45.72 x .91 x .91 m wave tank facility at the US Army Coastal Engineering Research Center, Fort Belvoir, Virginia, USA. Test sections of 10, 4, 2 and 1 x 1 m wavelength, .05 m amplitude sinusoidal bars were constructed in the tank and set in a false bottom. The barred test sections were

situated approximately mid-way between a hydraulically driven piston type wave generator, at one end of the tank, and a 1:10 slope wave absorbing beach at the other. Water surface elevations were measured using standard parallel-wire resistance type wave gauges and wave reflection coefficients determined using the method of Goda and Suzuki (1976).

Two pairs of gauges and a single gauge were used to make two types of measurement; first, incident and transmitted wave conditions were measured with one gauge pair 5 m on the up-wave side of the bars and the second gauge pair 5 m on the down-wave side. The remaining gauge was positioned midway along the test section. The up-wave gauge pair thus gave information on wave reflection from the bars while the second gauge pair provided data on the transmitted wave heights and the amount of wave energy reflected from the beach. In the second type of measurement two pairs of gauges were moved along the tank in such a way as to give surface elevation data every 0.25 m and to determine how wave reflection varied throughout the tank, first from the barred test section and finally from the beach. The remaining gauge was positioned at the end of the tank at the foot of the beach. These experimental arrangements are illustrated in Fig 1, with further details given in Davies and Heathershaw (1983).

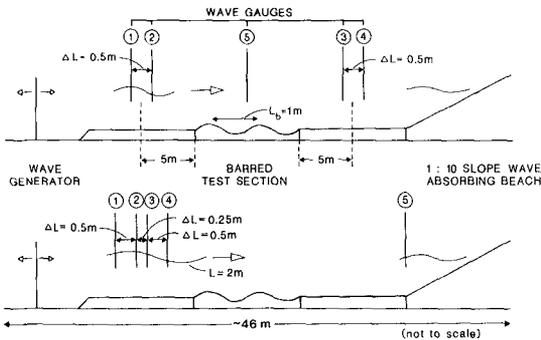


Figure 1 Position of gauges, in relation to barred test section (2 bars only), and wave absorbing beach, for two main types of measurements. Typical values of the wave gauge spacing ΔL are also shown.

With the bar wavelength, L_b , fixed at 1 m, incident surface wavelengths were varied over a range giving $.5 < 2k/\ell < 2.5$, by varying the wave period in steps of .01 s. Thus, good resolution in non-dimensional wavenumber space $2k/\ell$, of the order of .01, enabled detailed investigations to be made of the oscillatory nature of the wave reflection coefficient and of the main resonant interaction peak. These tests were carried out using small amplitude monochromatic waves only. For the results shown here, water depths were varied to give bar amplitude - water depth ratios, b/h , in the range $.08 < b/h < .40$.

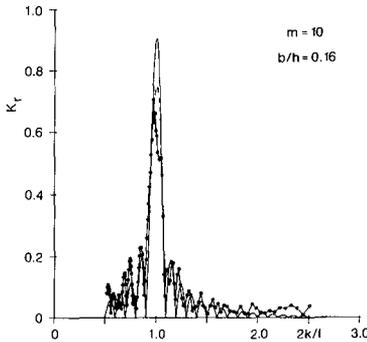


Figure 2a The variation of wave reflection coefficient K_r with the ratio $2k/\ell$ for $m = 10$ bars and for $b/h = .16$ ($h = 31.3$ cm). The broken and solid curves represent corrected and uncorrected theoretical predictions respectively; corrected theoretical predictions assume a linear attenuation of incident wave amplitude across the bar patch and uncorrected predictions assume no attenuation.

RESULTS

For surface water wavelengths approximately twice the bar wavelength, strong resonant interactions were observed leading to large reflection coefficients (in some cases as large as $K_r = .8$) and to dramatic partially standing wave patterns on the up-wave side of the bars. On the down-wave side of the bars the standing wave pattern gives way to progressive waves leaving the test section and travelling towards the wave absorbing beach.

Figure 2 shows the variation of wave reflection coefficient, K_r , with the wavenumber ratio $2k/\ell$ for 10, 4 and 2 bars and bar amplitude/water depth quotients of $b/h = .16$ and $.32$, corresponding to depths of $h = 31.3$ cm and $h = 15.6$ cm respectively. Also shown are the first order predictions from Davies (1980, 1982) both corrected and uncorrected for the effects of wave attenuation as the incident waves propagate over and are reflected by the bars. A striking feature of

these results is the large resonant interaction peak at $2k/\ell = 1$, and the oscillatory nature of K_r in respect of $2k/\ell$. Figure 2a, for $m = 10$ bars, shows that the measured reflection coefficients follow the

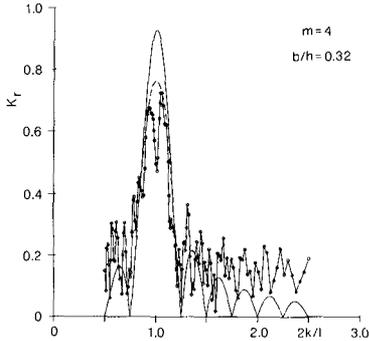


Figure 2b The variation of wave reflection coefficient K_r with the ratio $2k/\ell$ for $m = 4$ bars and for $b/h = .32$ ($h = 15.6$ cm). The broken and solid curves represent corrected and uncorrected theoretical predictions respectively (see Fig 2a).

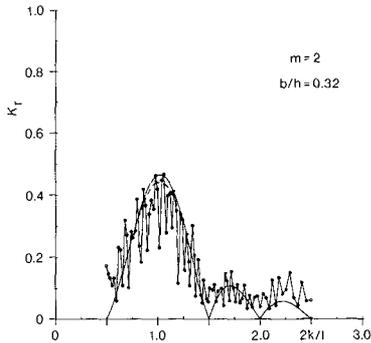


Figure 2c Variation of the wave reflection coefficient K_r with the ratio $2k/\ell$ for $m = 2$ bars and $b/h = .32$ ($h = 15.6$ cm). The broken and solid curves represent corrected and uncorrected theoretical predictions respectively (see Fig 2a).

trend of the theoretical curves closely throughout the resonant peak and on either side of it. Similarly in Figures 2b and 2c for $m = 4$ and 2 bars the central resonant peaks are well reproduced. However, in these cases the level of the measured reflection coefficient is in general above that predicted by the theory. Davies and Heathershaw (1983) have shown that this is probably due to a small, though non-negligible, amount of wave energy (less than ~4%) being reflected from the wave absorbing beach at the far end of the tank.

The solid curves in Figure 2 represent theoretical values calculated assuming no attenuation of the incident waves across the bar patch and, consequently, no proper energy balance. The broken curves represent theoretical values calculated assuming a linear attenuation of the incident wave across the bars and giving the required energy balance. Details of the energy balance procedure may be found in Davies (1980).

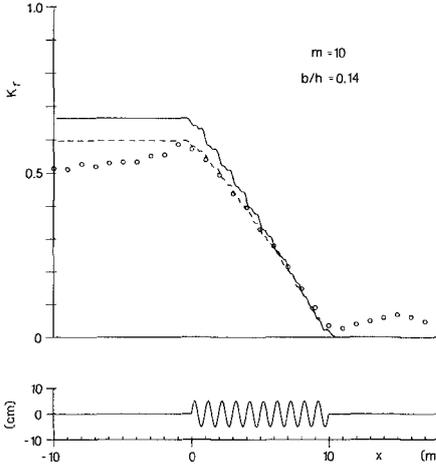


Figure 3a The variation of measured reflection coefficient K_r over the barred test section and on either side of it for $m = 10$ bars and for $b/h = .14$ ($h = 35.7$ cm). Broken and solid curves represent corrected and uncorrected theoretical predictions respectively. The corrected theory assumes a linear decrease in incident wave amplitude across the bar patch.

Figure 3 shows the results from 10 and 4 bars, of measurements of the reflection coefficient, K_r , at resonance, at different positions, x , along the tank and throughout the barred test section. In particular it should be noted that K_r is a value calculated by the method of Goda and Suzuki (1976) and that this may only be expected to agree with

the K_r given by (1) well away from the bars on the up-wave side of the bar patch. Measurements are shown for bar amplitude-water depth quotients $b/h = .14$ and $.32$, corresponding to water depths of 35.7 cm and 15.6 cm respectively. The resonant wave period settings for these measurements were 1.28 s and 1.73 s.

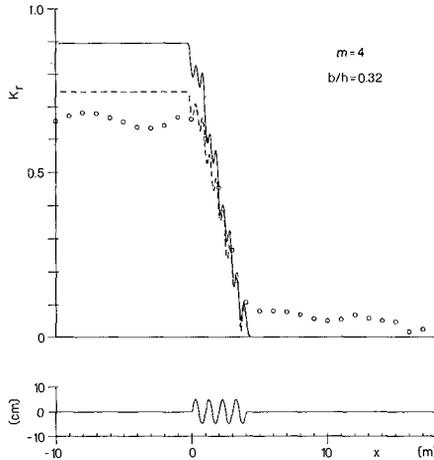


Figure 3b The variation of measured reflection coefficient K_r over the barred test section, and on either side of it for $m = 4$ bars and for $b/h = .32$ ($h = 15.6$ cm). Broken and solid curves represent corrected and uncorrected theoretical predictions respectively. The corrected theory assumes a linear decrease in incident wave amplitude across the bar patch.

The measurements show good agreement with theoretical predictions throughout the barred test section although in general they underestimate the theory on the up-wave side of the bars and overestimate it on the down-wave side. The results in Figure 3 show that on the up-wave side of the bars the measured reflection coefficient is more or less constant and rises to a peak value within a few water depths of the bars before falling, linearly throughout the test section, to a value of the order of .05 or less, which is the reflection from the beach alone. The increase in K_r towards the patch is believed to be due to viscous dissipation in the tank (see Davies and Heathershaw, 1983).

Figures 4a and 4b represent corresponding measurements of the amplitude of surface elevation throughout the barred test section and on either side of it for the conditions described above. These wave envelope observations confirm the presence of a standing or partial standing wave between the bars and the wave generator and show how

this gives way to progressive waves leaving the test section and propagating towards the beach. The results in Figures 4a and 4b show good agreement with the corrected theoretical curve (b), supporting the use of linear attenuation of incident wave amplitude across the bar patch. Details of the theoretical predictions of surface elevation amplitudes, in the vicinity of the bar patch, are given in Davies and Heathershaw (1983).

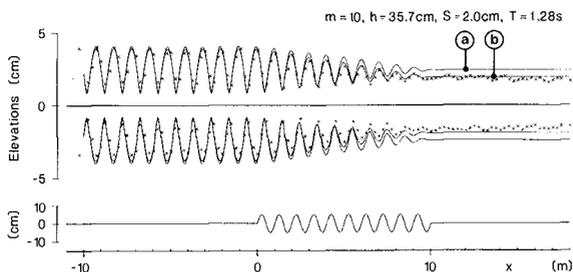


Figure 4a Surface elevation amplitudes measured throughout the barred test section and on either side of it for $m = 10$ bars and $b/h = .14$ ($h = 35.7$ cm). Curves (a) and (b) represent uncorrected and corrected theoretical predictions respectively. The corrected theory assumes a linear decrease in incident wave amplitude across the bar patch.

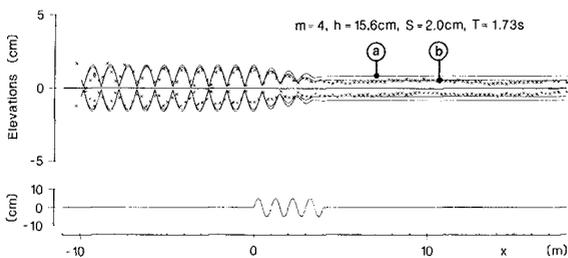


Figure 4b Surface elevation amplitudes measured throughout the barred test section and on either side of it for $m = 4$ bars and $b/h = .32$ ($h = 15.6$ cm). Curves (a) and (b) represent uncorrected and corrected theoretical predictions respectively. The corrected theory assumes a linear decrease in incident wave amplitude across the bar patch.

Results illustrating the variation of the maximum possible value of K_r with the number of bars (m) and the bar amplitude-water depth quotient (b/h) are shown in Figure 5. For each case, measurements of K_r were made over a range of $2k/\lambda$ values at or near resonance and the reflection coefficient values averaged. Further details of these measurements and the averaging procedure are given in Davies and Heathershaw (1983). It should be noted that the number of measurements of K_r at or near a resonant peak may not have been representative of the true variation in K_r and for this reason values of K_r in Figure 5 are shown as means with standard deviation error bars.

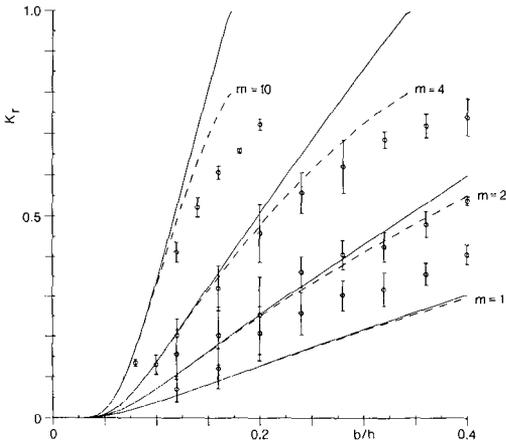


Figure 5 Measured peak reflection coefficients for $m = 1, 2, 4$ and 10 bars and for different values of b/h .

Results are shown in Figure 5 for $m = 10, 4, 2$ and 1 bars. For 10 and 4 bars ($m = 10$ and 4) measured reflection coefficients in general underestimate the theory which is shown uncorrected (solid curve) and corrected (broken curve) for the effects of wave attenuation across the bars. For 2 and 1 bars ($m = 2$ and 1) the measurements tend to overestimate the theory and, as shown by Davies and Heathershaw (1983), since the measured bar reflection coefficient is equal to the actual bar reflection coefficient plus or minus the value for the beach (dependent upon phase) this result most probably indicates the increasing importance of beach reflections as the predicted value of the bar reflection coefficient decreases with a smaller number of bars. Despite these shortcomings the measurements are generally supportive of the main theoretical conclusion that the peak wave reflection coefficient increases linearly with the number of bars (m) and as the water depth is decreased (b/h increased). However, it should be noted

that strictly speaking, K_r is only expected to increase linearly with m at $2k/\lambda = 1$ and that in most cases the peak values shown in Figure 5 correspond to mean $2k/\lambda$ values which are not exactly equal to 1 but which are in the range .9647 - 1.0171 with standard deviations of .0077 - .0538. Further details of these results may be found in Davies and Heathershaw (1983).

Following a suggestion of Davies (1980, 1982) some observations of sediment movement were also carried out in the wave tank. Davies suggested that as a result of the partial standing wave which forms up-wave from a reflecting bar systems, the pattern of wave orbital motions near the bed may lead to areas of preferential erosion and deposition of sediment. Potentially, at least, this provides a mechanism for bars to grow in the up-wave direction.

To confirm this result, fine sand of about 235 μ m mean diameter was sprinkled in a thin uniform layer throughout the barred test section (with 2 bars only) and for about 2-3 m on either side of it. Small amplitude waves were started and the wave amplitude increased until sediment motion was initiated. Sediment movement was then observed for a resonant wave reflection condition ($K_r = .34$) and the evolution of ripple patches recorded. On the up-wave side, ripple patches with a 1 m spacing were observed while down-wave a more or less continuous sheet of ripples developed. Erosion, and ripple formation, was observed to occur beneath the nodes of surface elevation of the partially standing wave. With increasing time, ripple heights were observed to grow on the up-wave side of each patch in such a way as to bring about an accumulation of material approximately mid-way between node and antinode and roughly in the position where bar crest formation would be expected to occur. These results confirm that a potential bar-growth mechanism exists up-wave of the bars but not on the down-wave side.

An example of the observed sediment distribution is shown in Figure 6. It should be noted that sediment accumulation at an antinode would not be expected in this case since the horizontal component of the wave induced current at this location is minimal and usually below the threshold of movement of all but the finest sediment. As was observed in this study, sediment accumulation and the cessation of sediment transport, would be expected to occur at a point intermediate between the high velocities at the nodes and the low velocities at the antinodes. This result may be contrasted with that of Nielsen (1979) who found that for fine sand ($d_{50} = 80\mu$ m), sediment accumulation did occur beneath the antinode of a partially standing wave. In this case sediment movement was principally as suspended load whereas in this study material moved mainly as bedload or in a thin suspension layer due to vortex shedding from ripple crests.

CONCLUSION

In conclusion the results of these experiments have shown that significant and large amounts of wave energy may be reflected from submerged bar like structures and that these reflections are brought about by

resonant interactions between surface water waves and the bedforms. In particular, at resonance, incident surface water wavelengths are approximately twice the bedform wavelengths. The results have implications not only in terms of wave reflection from naturally occurring bedforms, say bars on beaches, but also for sediment transport processes in general.

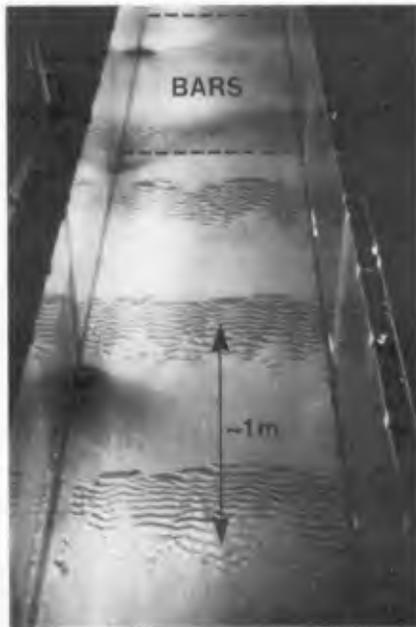


Figure 6 Ripple patches with a 1 m spacing formed beneath a partial standing wave on the up-wave side of 2 x 1 m wavelength bars. The bar amplitude is 5 cm and for these observations the water depth was $h = 15.6$ cm and the wave reflection coefficient was $K_r \approx .34$. Maximum ripple heights and wavelengths, in the ripple patches, were of the order 1.5 cm and 5.5 cm respectively.

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