A COMPUTER MODEL FOR THE REFRACTION OF NON-LINEAR WAVES

by

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ABSTRACT

A non-linear wave refraction model was developed which allows for the combined refraction and shoaling of Vocoidal waves over an arbitrary sea bed. The effects of bed friction and percolation are also catered for. The method is based on the cirular arc technique which is widely used for linear wave refraction. The method was extensively tested against Vocoidal wave refraction results obtained previously for a plane beach. A comparison of Vocoidal and linear wave refraction over an arbitrary sea bed indicated that Vocoidal waves refract less than linear theory, thereby yielding higher wave heights and angles of incidence at the breaker line. This result is in line with results of non-linear refraction over parallel bed contours quoted for other non-linear wave theories in literature. Further work is required before caustics can be adequately treated. Future research should include wave spectrum transfer and the re-evaluation of empirical relationships in use in the shallow water region and which will use this new higher-order refraction technique.

1. INTRODUCTION

The transformation of waves in shoaling water involves a change in wave height, length and velocity with depth. Wave refraction theory has been developed from solutions for light wave optics and its application is usually confined to the use of linear wave theory.

Wave ray methods are well suited to modelling of short waves. They solve the initial value problem so that there are few difficulties that can be encountered with boundary conditions as occurs with other methods. However, a number

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of deficiencies have been identified. These are associated with the representation of topography, methods of computation and interprestation of wave refraction coefficients and the validity of using linear wave theory right up to the breaker line.

A number of theoretical investigations into wave refraction results with non-linear wave theories have been carried out. These have mostly been confined to the simplest cases of parallel contoured slopes, the only exception being a model developed in France for the refraction of third order Stokes waves over arbitrary contours (Gaillard, personal communication, 1982). Nevertheless, they have shown that the results obtained from non-linear theories can be significantly different to the linear counterparts.

One of the reasons in the past that non-linear wave refraction has not been rigorously persued for the case of arbitrary topography would seem to be the lack of non-linear wave theory that would allow wave speeds to be calculated with reasonable ease and economy. The Vocoidal wave theory combined with a circular arc solution provides an excellent basis for a fast and accurate numerical model.

2. MOTIVATION FOR USING VOCOIDAL THEORY

Depth refraction using first order cnoidal wave theory has been investigated for the simple case of straight parallel bottom contours (Skovgaard and Petersen, 1977). It was shown that the cnoidal wave would theoretically break at a greater water depth than linear waves due to greater amplification. Also the cnoidal wave orthogonal was found theoretically to refract less than the linear wave.

In fact it was concluded in all previous shoaling and refraction studies that linear wave theory progressively underpredicts both wave height and angle of incidence to a greater extent as the wave shoals towards the breaker line (see for example Sakai and Battjes (1980); Chaplin (1980); Skovgaard and Petersen (1977); Skovgaard et al. (1976); Dean (1973)). This is of fundamental importance to coastal engineers involved in nearshore modelling particularly in the surf zone where theory dictates a strong sensitivity to breaking wave conditions, particularly the angle of attack.

Figure 1 gives an example comparison between refraction for Vocoidal and linear wave theories over a plane beach with parallel contours. The same tendency as for the other well-known higher-order theories, such as Dean's stream function, cnoidal and Cokelet's theories is observed, with this exception that the wave height does not increase as sharply close to the breaker line as predicted by, for example, Cokelet.

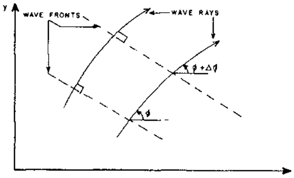
Although the non-linear refraction results referred to above differ from those for linear wave refraction, it still remains to ascertain whether this difference is significant or not. Swart and Loubser (1979) made a comprehensive comparison of measured wave properties with predictions from 13 different theories. The results indicate that although linear theory provides an adequate description of wave profile, wave celerity and orbital motions in deep water, the comparison deteriorates as the relative water depth becomes less (see for example Figure 2). This is also apparent in the mean error in the prediction of wave length:

F_c range	Mean error in λ/d	
	Airy theory	Vocoidal theory
0 - 50	0,012	0,021
50 - 100	0,030	0,041
100 - 200	0,006	0,028
200 - 500	-0,041	0,010
500 - 1 000	-0,069	-0,018
1 000 - 60 000	-0,093	-0,017

Since refraction depends heavily on the wave celerity (or wave length) this result is significant for the application of linear theory in wave refraction where results are required near or at the breaker line.

Swart and Loubser (1979) finally concluded that Vocoidal theory is the only readily-applicable analytical wave theory with a good correspondence to measured data and a good adherence to the original wave boundary value problem in the whole range of non-breaking waves. This theory therefore not only provides a good correspondence to data but also a sound framework for the derivation of expressions for the prediction of secondary wave-induced phenomena, and as such also of refraction.

3. LINEAR WAVE REFRACTION



WAVE REFRACTION DIAGRAM

Wave refraction as a water-wave phenomenon has been realised for a long time. Early solutions used graphical techniques assisted with a template (Wiegel, 1963) and this still provides a useful means of obtaining a rapid overview for cases with simple topography. With the advent of the computer finite difference solutions were developed which used predictor-corrector techniques. A number of different schemes for representation of the bottom topography were proposed and included linear and quadratic surface fitting schemes. Topographic smoothing may also be recommended largely to reduce the number of caustics that may arise from small, but severe bottom irregularities. An interesting study was carried out by Poole (1975) who investigated a number of different interpolation functions for representation of sea bed topography. He found that for the cases tested there were broad similarities in the refraction patterns, but significant local variations occurred.

Abernethy and Gilbert (1975) investigated the anomalies than can arise when constructing conventional refraction diagrams, that is, a selected number of rays refracted from the offshore boundary towards the shoreline. They presented examples for which the method can result in large variations in refraction coefficients, depending on the density of rays used in the calculation. They concluded that there can be an inherent bias towards high wave heights, an excessive sensitivity to frequency, offshore direction and position which is greatest in region where refraction coefficients are large.

The curvature of a wave ray in still water expressed in cartesian coordinates is

$$\frac{d\theta}{ds} = -\frac{1}{c}\left(\frac{\partial c}{\partial x}\sin\theta - \frac{\partial c}{\partial y}\cos\theta\right) \qquad \ldots \quad (1)$$

where x and y are the coordinates, s the distance along the orthogonal and θ the angle between the orthogonal and the x-axis. This type of relationship or its equivalent can be solved most conveniently and accurately by using the 'circular arc' technique developed at the Hydraulics Research Station, Wallingford. Briefly the mehtod is based on the fact that, given linear variation of wave speed across a small triangular element, the ray path is described by a circular arc whose centre lies on the line of zero celerity and is tangential to corresponding arcs in adjacent elements. This provides a fast numerical solution which is not restricted to the use of linear wave theory.

The wave refraction coefficient reflects the change in wave height of a wave train purely due to convergence or divergence of the wave orthogonals. Over complex topography the ray separation factor may be evaluated from the well-known expressions derived by Munk and Arthur (1952) whence:

$$\frac{d^2\beta}{ds^2} + p(s)\frac{d\beta}{ds} + q(s)\beta = 0 \qquad \ldots \quad (2)$$

where
$$p(s) = -\frac{1}{c}\left(\frac{\partial c}{\partial x}\cos\theta + \frac{\partial c}{\partial y}\sin\theta\right) \qquad \ldots \quad (3)$$

$$q(s) = \frac{1}{c}\left(\frac{\partial^2 c}{\partial x^2}\sin^2\theta - \frac{\partial^2 c}{\partial x\,\partial y}\sin 2\theta + \frac{\partial^2 c}{\partial y^2}\cos^2\theta\right) \qquad (4)$$

In the above β is the ray separation factor which is related to the wave refraction coefficient K_r by the square root of its inverse, that is,

$$K_r = \beta^{-0,5}.$$

Some of the deficiencies in forward tracking methods discussed above may be overcome by back tracking very closely spaced rays from a point of interest for a wide range of frequencies. This allows the computation of energy transfer from two-dimensional energy direction spectra and provides results that are usually consistent and stable. The two main disadvantages are that results can only be obtained for individual points of interest after quite a large computational effort and it is not possible to introduce wave decay due to bottom friction as the method relies on the independence of wave height in order to provide a complete family of results. The first problem may be overcome by applying statistical treatment to forward tracking results over discrete areas (Bouws and Battjes, 1982). The second requires the introduction of wave height into the calculation and is only really sensible for forward tracking methods (Skovgaard et al., 1976) until such time that bivariate distributions of wave height and frequency can be reasonably well-defined.

The following sections will show how the circular arc technique has been extensively extended to deal with wave refraction of non-linear waves using Vocoidal wave theory. It is shown that there can indeed be significant differences in comparative results, particularly when the sea bed topography is complex.

4. VOCOIDAL WAVE REFRACTION

The principle adopted was to use an existing refraction model using the circular arc technique and to adapt it in such a way as to cater for non-linear vocoidal waves. The specific model used is based on the Wallingford refraction program (Abernethy and Gilbert, 1975) and has the capability of containing more than one grid up to a maximum of ten with a grid spacing that need not be the same for all grids. Each grid can contain up to 5 000 grid points. A more comprehensive review of the techniques used will be included in Swart and Crowley (1983b).

As stated previously, the application of the circular arc technique is not restricted to the linear wave theory, provided that wave celerities are available at the corner points of the grid element under consideration. This poses the first problem in the application of the circular arc method to predict refraction for higher-order waves, namely, **the celerity of higher-order waves is a function of the wave height, which is not known until after the wave refraction has been done.**

Furthermore, since the celerity is a function of wave height, which is determined in turn by refraction, shoaling, friction and percolation, it is not possible to separate the effects of these four processes as is customary for linear waves. In fact, the wave height change due to each of these processes in turn depends on the resultant of the others. A technique will therefore have to be found to solve for the wave height along the ray in an analogous way as was done for linear theory by using the ray separation equation (2) and which also includes the effect of percolation and bed friction.

This dependence of wave propagation on the wave height will make it difficult to deal with wave rays near caustics. Ways and means of overcoming this problem for the case of very irregular topographies should be investigated.

These three aspects will be discussed below. For the present only forward tracking is considered.

Wave celerity at grid points

Swart (1981) tabulated values of various wave properties in terms of H/d and $T\sqrt{g/d}$. From this tabulation one can observe the following typical variation in $c^2/(gd)$:

$T\sqrt{g/d}$	Values of $c^2/(gd)$					
	H/d=0,01	0,1	0,45	0,5	0,55	1,0
5	0,553	0,554	0,590	0,593	0,596	-
10	0,872	0,878	1,015	1,040	1,065	-
15	0,942	0,950	1,103	1,133	1,168	1,677
20	0,968	0,976	1,131	1,162	1,211	1,789
30	0,986	0,995	1,148	1,179	1,245	1,884
40	0,992	1,002	1,151	1,190	1,261	1,923

It is apparent that the value of c^2/gd is indeed strongly dependent on wave height.

An indication of the error introduced in $c/\sqrt{gd}$ as a result of using a wrong value of H/d, can be obtained by looking at the tabulated values for H/d = 0,45; 0,5 and 0,55.

$T\sqrt{g/d}$	5	10	15	20	30	40
1) $E_{10\%}$ in %	0,25	1,20	1,43	1,71	2,04	2,30

$E_{10\%}$ is the error in $c/\sqrt{gd}$ for a 10 per cent deviation from the assumed mean value for H/d of 0,5.

It can thus be seen that the error in $c/\sqrt{gd}$ increases to about 20 per cent of the error in H/d for very high values of $T\sqrt{g/d}$. Since there is some correlation between the wave period and the wave height in the incident wave spectrum, one normally finds that the value of the parameter $T\sqrt{g/d}$ at wave breaking rarely exceeds 30. It can therefore be safely assumed that the error in $c/\sqrt{gd}$ will be restricted to 20 per cent of that in H/d.

On the other hand, if the celerities at H/d = 0,5 and 1,0 are compared with those at H/d = 0,01, it is found that the errors are appreciably more substantial.

$T\sqrt{g/d}$	5	10	15	20	30	40
1) $E_{0,5}$ in %	3,43	8,43	8,88	8,73	8,55	8,70
2) $H_{1,0}$ in %	-	-	25,05	26,44	27,66	28,18

1) $E_{0,5}$ is the relative difference in $c/\sqrt{gd}$ for H/d = 0,01 and 0,5, if the $c/\sqrt{gd}$ at H/d = 0,5 is assumed to be the correct value.

2) $E_{1,0}$ is the same as $E_{0,5}$ but with H/d = 1,0.

It is therefore quite clear that linear wave celerities, which are very close to those in Vocoidal theory for H/d = 0,01, should not be used for refraction of finite waves in shallow water.

The behaviour of Vocoidal wave celerity outlined above will form the basis of the discussion of a method for obtaining wave celerities at the corner points of grid elements.

Swart (1981) tabulated values of wave height H and angles of incidence θ for combined shoaling and refraction of Vocoidal waves on a plane beach with parallel depth contours, for deep water angles of incidence of 0°, 5°, 10°, 20°, 30°, 40°, 50° and 60°. The values of H/d and θ were tabulated for values of the ratio deepwater wave height H_o to water depth d between 0,01 and 1,30 and for values of the period parameter T_c ($= T\sqrt{g/d}$) between 1 and 60.

Results were obtained by simultaneously solving the energy flux equation, Snell's law and the Vocoidal wave length relationship.

The technique adopted for the computation of celerities at the corner points of the grid elements was based on these tabulated values of combined refraction and shoaling and consists of the following steps:

(i) It is assumed that the grid element is plane, that is, the contours across the element are parallel.

(ii) The entry conditions, that is, wave height, angle of incidence relative to grid element contours and water depth, at the point where the wave ray first enters the grid element, are used to refract the ray back to deepwater assuming contours parallel to those in the grid element. For this purpose the above-mentioned shoaling/refraction tables (Swart, 1981) were used to obtain curve-fitted equations for fictitious deepwater wave conditions, H_o and θ_o, which would have existed if the offshore bed topography had indeed consisted of a plane bed with contours parallel to those in the grid element under consideration.

(iii) These deepwater conditions are then used in the same curve-fitted equations to in turn obtain wave heights at the corner points of the grid element under consideration, again by assuming a plane bed seawards of the element. It should be emphasized that the wave heights computed in this manner do not form part of the output of the refraction computation, but are only used to obtain celerities at the corner points of the grid element which are in turn used as basis for the application of the circular arc method. A systematic comparison was made between values of wave height computed in this way and those obtained from the tables (Swart, 1981). It was concluded that the wave height is predicted to within acceptable limits. The extent of the difference between the tabulated (true) values of wave height and those computed in the manner described above depends on the variation in water depth over the grid element. Tests were done for water depth variations of up to 100 per cent between the minimum and maximum depths in the element. Nevertheless, the difference in wave height was less than 10 per cent with a corresponding difference in predicted wave celerity which was always less than 2 per cent and mostly much less than 1 per cent. It will be shown later that this accuracy is sufficient to ensure an accuracy in the predicted wave heights and angles of incidence along the wave ray which are of the order of 0,1 per cent. The results also showed that the accuracy of predicted wave height can be significantly improved by reducing the grid size in the shallow water region, which can be easily done in the refraction program used.

(iv) The wave heights and celerities at the corner points of the grid element, computed in this manner, are then used to apply the circular arc method in exactly the same way as for linear wave theory.

(v) the accuracy of this procedure can be tested by comparing the angle of wave incidence along the wave ray for refraction over a plane beach against the tabulated results (Swart, 1981). An extensive comparison was made for a wide range of initial conditions and it was found that the true vocoidal angle of incidence at any relative water depth from deep water to the breaker line never differed from the computed angle by more than 0,01 degree. The method described is therefore considered to yield the correct result, even though the wave heights and celerities at the corner points of the grid elements did contain some inaccuracies.

Wave height along wave ray

The output of the circular arc method consists of the ray trajectory and thus also the ray orientation at the intersection points of the wave ray with the grid elements. The computations are continued until wave breaking occurs. The specific choice of the wave breaking criterion is left to the user of the program. Apart from the ray trajectory, however, wave heights are also required along the ray. In the linear wave theory the wave height along the ray is found by calculating the wave shoaling and refraction as well as the wave height reductions due to bed friction and percolation separately at each intersection point. The wave height H at that point is then simply the product of these four effects.

$$H = K_s K_r K_f K_p H_o \qquad \text{(for linear theory)} \qquad \ldots (5)$$

where K_s, K_r, K_f and K_p are coefficients of shoaling, refraction, friction and percolation relative to the starting point and H_o is the wave height at the starting point of the calculation.

This method does not hold for any higher-order theory, for the reasons given in the introduction to this section. Equation (2) which defines the ray separation factor β in linear wave theory, can however be used as the basis for the calculation of wave height by using the principle of energy flux conservation along the wave ray, namely,

$$\beta \, Enc = (Enc)_o \qquad \ldots (6)$$

where β is again the ray separation factor, E is the total wave energy per unit surface areas, c is the wave celerity and n is the ratio group velocity/wave celerity. Subscript "o" refers to the starting point of the wave ray. By putting $(Enc)_o = K_o$, where K_o is a parameter defined by the starting conditions, equation (6) can be rewritten as

$$\beta = K_o \zeta \qquad \ldots (7)$$

where $\zeta = (Enc)^{-1}$ ... (8)

Combination of equations (2) and (7) yields

$$K_o \frac{d^2\zeta}{ds^2} + p(s) \, K_o \frac{d\zeta}{ds} + q(s) \, K_o \, \zeta = 0$$

or

$$\frac{d^2\zeta}{ds^2} + p(s) \frac{d\zeta}{ds} + q(s) \, \zeta = 0 \qquad \ldots (9)$$

This equation can be solved numerically along the wave ray using Vocoidal wave properties in the same way as equation (2) was solved for linear wave theory. Values of wave height H along the ray are then found from equation (8) by using the Newton-Raphson iteration technique. This approach is necessary since E, n and c are all functions of wave height H (see Swart, 1978).

An extensive comparison was made between wave heights obtained in this way for refraction of a plane underwater topography and those tabulated by Swart (1981). In this comparison it was assumed that the effect of bed friction and percolation can be neglected. It was found that the difference between these two quantities was never more than 0,1 per cent of the "true" wave height, as interpolated from the tables. However, at least part of this already negligible difference can be attributed to rounding off errors in interpolating from the tables. It is therefore considered that the method described herein for the computation of a total wave height along the wave ray yields reliable results of comparable accuracy to those obtained for linear wave refraction by using the ray separation technique.

Effect of dissipative forces

The effect of bed friction and percolation can be included in the technique described above by realising that the effect on wave height of these two phenomena can be assumed to be

$$H_{fp}/H_o = e^{-(\alpha_f + \alpha_p)t} \quad \ldots (10)$$

(Swart and Crowley, 1983a)

where H_{fp}/H_o is the additional wave height reduction due to bed friction and percolation, t is the time travelled by the wave and the parameters α_f and α_p are coefficients of friction and percolation respectively. These latter two parameters can be expressed in terms of vocoidal wave parameters. The effect of these two dissipative processes on the refraction is included by reducing the wave heights at the corner points of each grid element and the wave height along the ray trajectory in accordance with equation (10) before equation (8) is used with Newton-Raphson to solve for the wave height H. Since no test runs have as yet been done to test this part of the program, it will not be discussed further at the present.

5. DISCUSSION

The Vocoidal refraction program VOCREF developed along the lines outlined in Section 4 is at present being tested extensively for a number of prototype applications in parallel with a version of the program which used linear wave theory (LINREF). After completion of this sensitivity analysis and production testing it is intended to use this method (VOCREF) for routine analysis instead of LINREF. A number of interesting results have been obtained to date.

Since wave propagation is so strongly dependent on wave height in the case of non-linear waves and because of the fact that non-linear waves are not really defined for wave heights in excess of the cirtical breaker height, problems are encountered at or near caustics. Two ways of dealing with this problem have been investigated. By smoothing the bed in a linear manner in both directions along grid lines a sea bed topography is obtained which although smoother still exhibits the same overall tendencies as before smoothing. This process of two-way linear smoothing can be repeatedly applied. With each application the contours are smoothed even more. Obviously there should be some optimal number of smooths beyond which the effect of the irregular topography becomes mushed. Kluger (CSIR, 1979) concluded that about eight smooths would be optimal in the case of very irregular topography for linear wave refraction although this would seem to be very high. See for example Figure 3 for the effect of three degrees of smoothing as compared to the unsmoothed case. Smoothing tends to reduce the number of caustics and allows for wave propagation across areas of possible caustic action. Figure 4 shows the results of refraction with various degrees of smoothing. The refraction diagrams are very similar except for the fact that a caustic at the right of the plot for the unsmoothed case is eliminated for three and eight smooths of the bed. The reason for the wave ray terminating at a caustic is that for higher-order theory it is not possible to calculate wave heights at the corner points of the grid elements in areas where the wave height would be higher than the critical breaker height.

At present the effect is being investigated of artificially putting the wave height equal to the critical breaker height in the vicinity of a caustic for as long as the wave height as obtained from the method given in Section 4 is higher than the critical height. In this manner the non-linear wave rays can be allowed to propagate through the region of the caustic.

It has proven useful to include in the standard output of the program a series of four plots containing the breaker line values of breaker depth, breaking wave height, angle of incidence at the breaker line relative to the local bed contours and the longshore energy flux factor ($0{,}5 e_t H^2 nc \sin 2\theta$, where e_t is the total energy coefficient) (see Figure 5). Such plots clearly outline shore areas where caustic activity play a role. Overall tendencies in breaker line characteristics are also visible at a glance. At present the possibility is investigated of using the Monte Carlo technique as described by Bouws and Battjes to render the breaker line data even more useful. The effect of more smooths on the breaker line characteristics is to smooth out the variability and to enhance the clarity of the tendencies in these characteristics.

It appears that about three smooths of the contours would be optimal in defining all parameters sufficiently.

At a first glance the refraction diagrams produced by LINREF and VOCREF are very similar (see Figure 6). However, a comparison of breaker line characteristics shows up the differences remarkably (compare Figures 5 and 7; note that the scales in these figures are not all the same). In general the breaking wave height and the angle of incidence relative to the local contours at the breaker line are higher for VOCREF than for LINREF. Correspondingly the longshore energy flux factor is also appreciably higher for the non-linear case. This has important consequences for sediment transport prediction and in fact for all design work in and near the breaker zone. The area affected by caustics is shifted laterally (alongshore) by going from linear to non-linear refraction, which is again of importance in interpreting refraction diagrams for design purposes.

Although comparisons between results of linear and vocoidal refraction over arbitrary contours is still in the early stages, it is already abundantly clear that it will be essential to preferentially use the non-linear refraction program VOCREF. However, it is quite clear that before the use of non-linear refraction programs such as the one described here can become standard practice a substantial re-evaluation of presently used empirical techniques will have to be carried out. While this is being done the present model should be extended to also include wave spectrum transfer. To allow this the problem of defining three-dimensional spectra or some relationship between the wave height distribution and frequency should be tackled.

6. SUMMARY AND CONCLUSIONS

The main conclusions of this study can be summarised as follows:

(i) A computer program was developed which allows for the computation of the combined shoaling and refraction of non-linear Vocoidal waves over an arbitrary sea-bed topography and which includes the dissipative effect of bed friction and percolation. The basis of the derivation is the circular arc method which is extensively used in linear wave refraction.

(ii) The computer program was verified by comparing its output for the case of combined refraction and shoaling over a plane bed with parallel bed contours with known results for this case.

(iii) Comparison of results obtained from the Vocoidal refraction model and its linear theory counterpart for the case of irregular topography indicates, in line with previous findings quoted in literature for non-linear wave refraction on a plane slope, that Vocoidal waves refract less than linear waves. Consequently the breaker-zone wave characteristics, that is, wave height and angle of incidence, are higher for Vocoidal than for linear refraction.

(iv) The influence area of caustics is shifted alongshore when Vocoidal refraction is done instead of linear refraction.

(v) Future reseach should be aimed first, at finding a workable solution for the problem of caustics (such as, for example, the Mote Carlo approach of Bouws and Battjes, 1982) and second, at the extension of the technique to include wave spectrum transfer.

(vi) In parallel to the research indicated under (v) all presently used empirical techniques for processes within the shallow-water area, such as longshore current prediction, should be re-evaluated to assess the effect of non-linearity of the waves on the values of the emprirical parameters.

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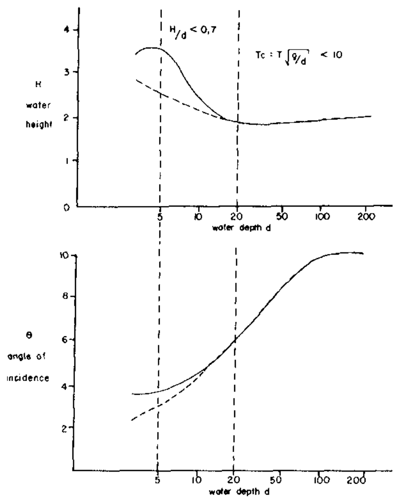

FIG. I REFRACTION OF VOCOIDAL AND LINEAR WAVES FOR PARALLEL CONTOURS $H_0 = 2m$; $\theta_0 = 10°$; T = 14,35

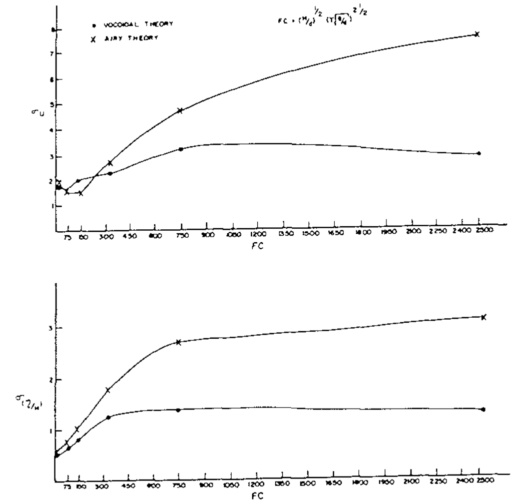

FIG. 2 STANDARD DEVIATION OF ERROR IN PREDICTION OF HORIZONTAL ORBITAL VELOCITY UNDER WAVE CREST (σ_u) AND WAVE PROFILE ($\sigma_{Z/H}$)

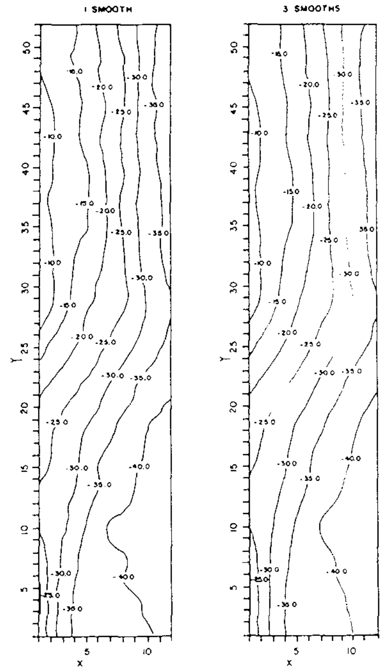

FIG. 3 RESULT OF VARIOUS DEGREES OF SMOOTHING ON SEABED CONTOURS

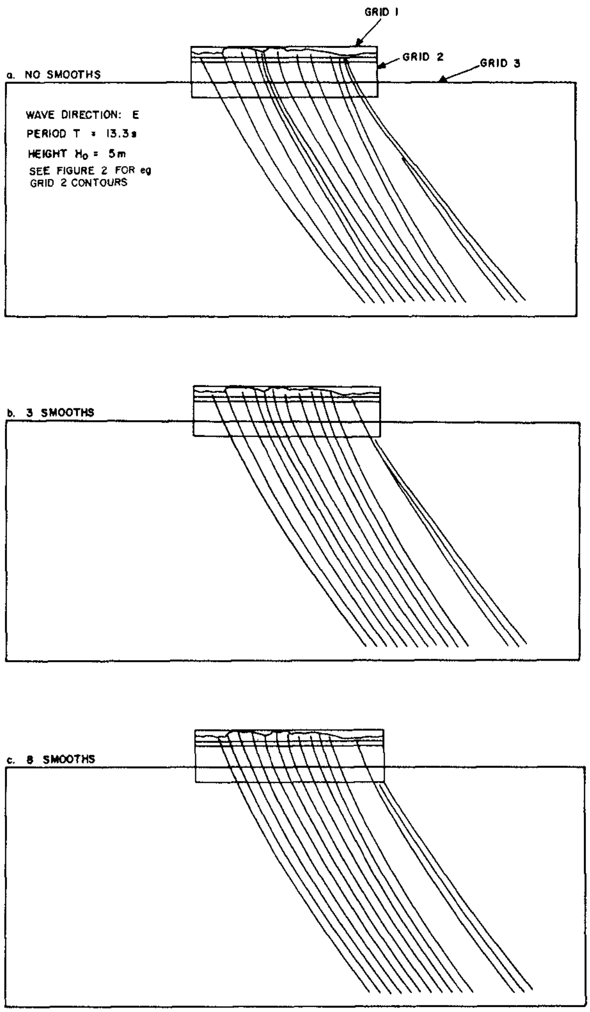

FIG. 4 RESULT OF VARIOUS DEGREES OF SMOOTHING ON REFRACTION PATTERN

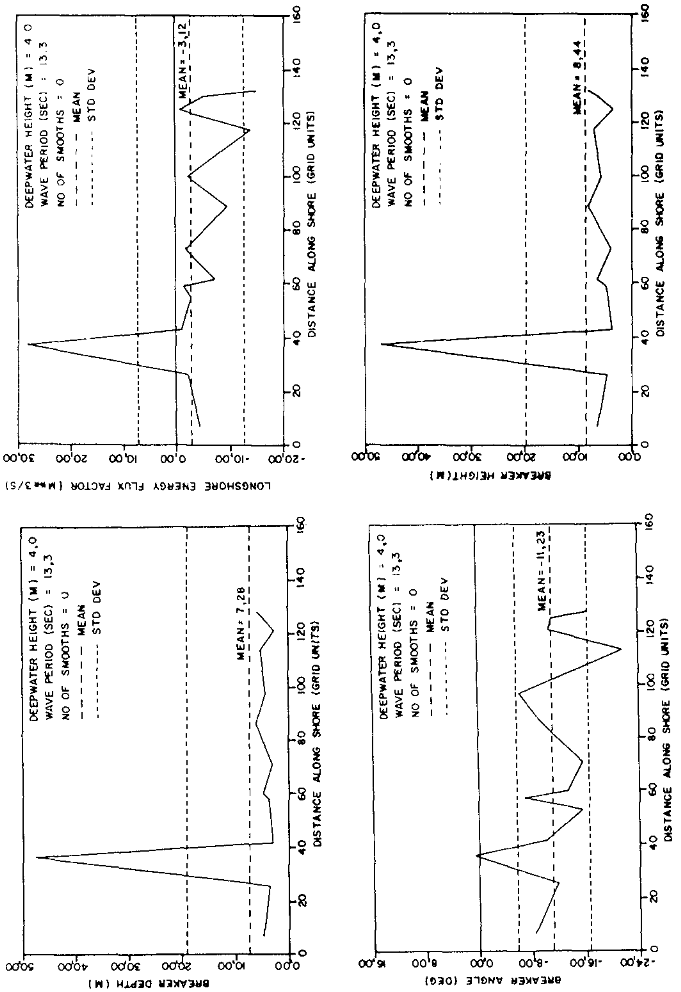

FIG.5 TYPICAL OUTPUT OF BREAKER ZONE CHARACTERISTICS FOR VOCOIDAL REFRACTION

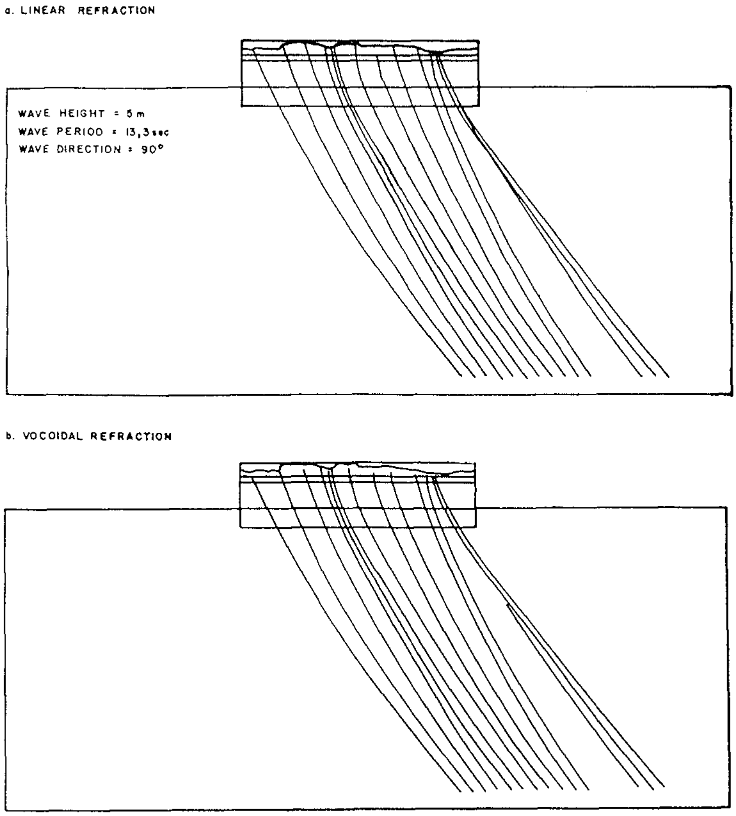

FIG. 6: COMPARISON BETWEEN REFRACTION DIAGRAMS WITH LINEAR AND VOCOIDAL THEORIES

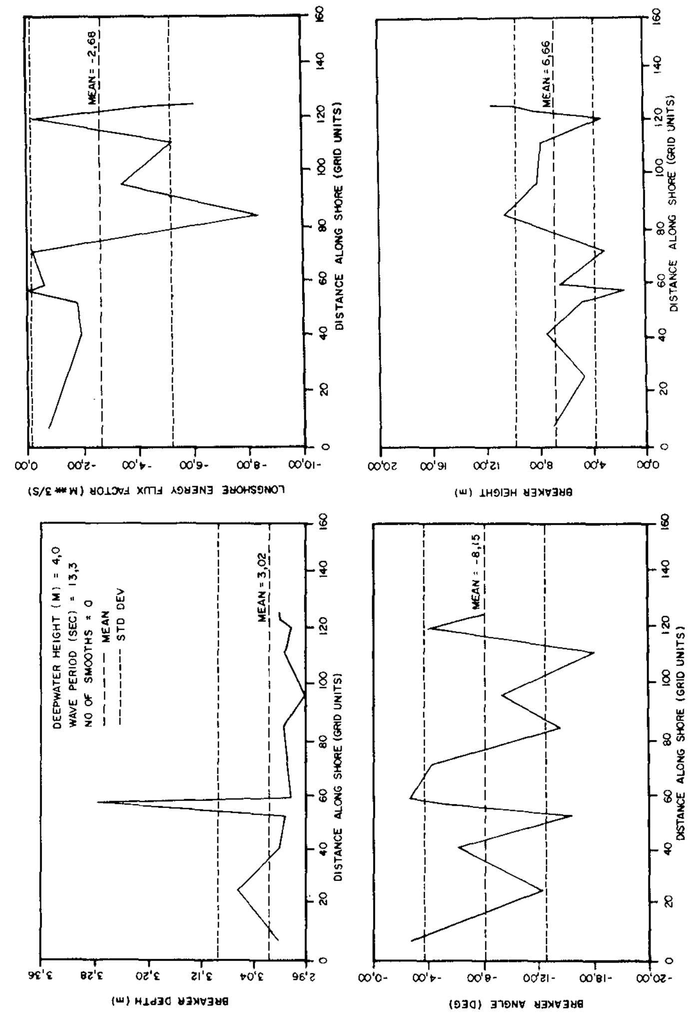

FIG. 7 TYPICAL OUTPUT OF BREAKER ZONE CHARACTERISTICS FOR LINEAR REFRACTION