ON THE RELATION BETWEEN CHANGES IN INTEGRAL QUANTITIES OF SHOALING WAVES AND BREAKING INCEPTION

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ABSTRACT

This paper describes a mechanism of breaking waves over sloping bottoms in terms of changes in integral quantities of the waves. Systematic computations are made of wave profiles of shoaling waves up to the numerical unstable points by using the K-dV equation with variable coefficients and internal properties such as horizontal and vertical water particle velocities by a stream function method satisfying the conservation laws of mass and energy. Applicability of the numerical results is examined and a relation between numerical unstable points and actual breaker points is found. Characteristics of the integral quantities of shoaling waves are investigated in relation to the existence of the extremum of the energy of the shoaling waves and their breaking inception.

INTRODUCTION

A sound knowledge on breaking waves is very important for coastal engineering, as stated by Longuet-Higgins(1980) at the Sydney Conference. However, theoretical elucidation on the mechanism of breaking waves on sloping bottoms is not enough, although many prominent theoretical investigations have been carried out on this problem. A series of studies by Longuet-Higgins et al.(1974 & 1975) obtained the very interesting conclusion that the integral quantities, such as the phase speed, momentum and energy increase with wave-height initially, become maxima and then decrease; that is, these quantities reach their extrema at a wave-height preceding the highest wave. So, a great interest is now being taken in the relation between the existence of the extrema of the integral quantities and breaking inception and in the derivation of the breaking inception from the behaviour of the integral quantities which reflect the properties of the whole wave field, by regarding the wave breaking as an instability of the field.

Because symmetrical wave formes were assumed in their calculation, the conclusion obtained by Longuet-Higgins et al. indicates only that the integral quantity of the highest wave is not usually maximum and does not directly show that extrema arise in the integrals of shoaling waves on sloping bottoms. Therefore, it is necessary to calculate the changes in the integral quantities of shoaling waves in order to investigate the mechanism of wave breaking on a sloping bottom which is important in coastal engineering.

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Here, based on the results of the authors' studies (1979 & 1982) on the transformation of shoaling waves over sloping bottoms, are made computations on the wave profiles of shoaling waves up to the numerically unstable points by using the K-dV equation with variable coefficients. At the same time, the internal properties such as water particle velocities are computed for the given wave profiles at each water depth by the stream function method, satisfying the conservation laws of mass and energy. The relation between numerically unstable points and the actual breaker points defined by Goda (1980) are also examined, and investigations are made of the characteristics of the integral quantities of shoaling waves over sloping bottoms. It is then shown that whether the energy of shoaling waves has extrema or not is a function of the surfsimilarity parameter (1974). Finally, examinations are made of the relation between the existence of the extrema of the energy of shoaling waves and breaking inception or breaker type.

CALCULATION OF INTERNAL PROPERTIES OF SHOALING WAVES ON SLOPING BOTTOMS

1. Reduction of Basic Equation

By considering irrotational wave motion over a uniformly sloping bottom and taking a Cartesian co-ordinate system as Shown in Fig.1, the governing equations are given as



where, the subscripts denote partial differentiation, ϕ is the velocity potential, z' the water surface displacement from the mean water level, B the height of the sea bottom above the horizontal co-ordinate, h, the depth of mean water at the origin and g the acceleration of gravity. The above equations can be directly solved by a numerical method, such as the MAC method. However, there arise the problems of the great effort necessary for the computation and the insufficient accuracy of numerical solutions in view of the labour expended, so that the following approach is adopted here.

For our calculations the following four assumptions are made. i) Bottom slopes are gentle and the water depth changes dependently on $X = \varepsilon x$ in which ε is a small parameter, and therefore,

ii) the wave reflection from the sloping bottom is negligible and mass

transport by waves is not bound by a sloping bottom. In addition, according to the conclusion by Stiass nie et al. (1975) that the effect of wave set-down is less than about one percent of the water depth, the third assumption is

iii) the effect of wave set-down is negligible. And lastly,

iv) the effects of nonlinearity and frequency dispersion of waves and bottom slope are of the same order.

By developing Kakutani's approach(1971) to a higher order approximation on the basis of these assumptions, Eq.(1) yields the following tractable wave equation using a new expression of wave profile, n.

$$\begin{split} n_{\tau} + & 3n_{\xi}/2c_{0}^{2} + c_{0}n_{\xi\xi\xi} - B_{\tau}^{*}n/4c_{0}^{2} = \epsilon(-c_{0}^{3}n_{\xi\xi\xi\xi\xi}/15 \\ & -3n_{\xi}n_{\xi\xi}/2c_{0} - 2n_{\xi\xi\xi}/3c_{0} - 2c_{0}n_{\xi\xi\tau}/3 + B_{\tau}^{*}n_{\xi\xi}/2 - 3n^{2}n_{\xi} \\ & /2c_{0}^{6} - n_{\xi}\Omega_{\tau}/c_{0}^{2} - 3n_{\xi}/c_{0}^{2} - \Omega_{\tau\tau}/2c_{0} - 5B_{\tau}^{*}n^{2}/8c_{0}^{4} \\ & + B_{\tau}^{*}\Omega_{\tau}/2c_{0}) + o(\epsilon^{2}) \end{split}$$

$$\end{split}$$

$$\end{split}$$

where

$$c_{0} = \sqrt{1-B^{*}} = \sqrt{h^{*}}, \ \xi = e^{1/2} \left(\int dx^{*}/c_{0} - t^{*} \right), \ \tau = e^{3/2} x^{*},$$

$$B^{*} = B/h_{1}, \ x^{*} = x/h_{1}, \ z^{*} = z/h_{1}, \ t^{*} = t\sqrt{g/h_{1}}, \ h^{*} = h/h_{1},$$

$$\varepsilon = (h_{1}/L_{1})^{2} <<1, \ \zeta = z^{*} - B^{*}, \ \varepsilon \eta = z^{1}/h_{1}$$
(3)

 $L_{\rm I}$ is the wave-length at the origin and a new expression of the velocity potential, $\Omega,$ has the following relations

$$\Omega_{\xi} - \eta = \epsilon (c_0 \eta_{\xi\xi} / 2 + \eta^2 / 2 c_0^2) + o(\epsilon^2)$$
(4)

$$\frac{\Phi}{h_1 \sqrt{gh_1}} = \epsilon^{V_2} \left(\Omega - \frac{\epsilon}{2} z^{*2} \Omega_{\xi\xi} + \frac{1}{4} \epsilon^2 z^{*4} \Omega_{\xi\xi\xi\xi} - \cdots \right)$$
(5)

The applicability of the numerical results of Eq.(2) has already been examined in detail by comparing them with the results of the experiment without the restriction on mass transport by waves due to a sloping end wall of the wave flume and the influence of wave reflection from the end wall. As a result, it was concluded by authors(1979) that the first order solution, which is the numerical solution of the lowest order equation of Eq.(2) given by

$$_{\tau} + \frac{3}{2} c_{0}^{3} \eta \eta_{\xi} + \frac{1}{3} c_{0} \eta_{\xi\xi\xi} - \frac{1}{4} c_{0}^{2} B_{\tau}^{*} \eta = 0$$
(6)

is stable and applicable to shoaling waves over sloping bottoms. The second order solution, which is the numerical solution of the second or-

der equation of Eq.(2), however, because of its round-off errors and secular terms, is not as appli-In Fig.2 is shown an excable. ample of the comparisons between the numerical and experimental results of wave profiles of shoaling waves. Here, the thin, broken and solid lines indicate the results of the first and second order solutions, respectively, and the heavy lines indicate the experimental results. Therefore, based on the assumptions i), ii), iii) and iv), and as far as the wave profiles of shoaling waves as concerned, it could be said that the numerical solution of Eq.(6) can be used instead of the direct solution of Eq.(1). Hence, the present problem is reduced to finding the velocity potential ¢ satisfying Eq.



Fig.2 Comparison of wave profiles deter determined from numerical solutions and from experimental data

(1) against the water surface displacement z' derived from Eq.(6). The mathematical formulation of ϕ has already by the authors(1980) as:

$$\frac{\Phi_x}{\sqrt{\rho h}} = \varepsilon \eta + \varepsilon^2 \left\{ \frac{1}{2} \eta^2 + (z^{*2} - 1) \eta_{\xi\xi} + B_{\xi}^* \eta \right\} + o(\varepsilon^3)$$
(7)

The characteristics of the horizontal water particle velocities were numerically examined through Eq.(7), and it was found that the accuracy of the high-order differential quotients computed numerically became lower with shoaling water. So, by paying attention to the physical significance of Eq.(7), i.e., that the velocity potential of shoaling waves is mainly subject to the water surface displacement and suffers the direct effect of bottom slope only in the second order, the following approach is here adopted, where the effect of water surface displacement given by Eq.(6) is evaluated as accurately as possible instead of ignoring the direct effect of bottom slope as assessed in Eq.(7). Therefore, in order to use the WKB method, a fifth assumption must be made:

v) Both water surface displacement and velocity potential depend on two variables, v and X, expressing the phase change and the gentle change in water depth, respectively:

$$z'(x,t) = z'(v,X), \quad \phi(x,z,t) = \phi(v,X,z)$$
 (8)

where

$$\upsilon = \frac{1}{\varepsilon} \int_{0}^{K} K(X) dx - \omega t$$
(9)

 $K\left(X\right)$ is the wave-number dependent on the change in water depth and $\omega,$ the angular frequency, independent from the change according to the conservation law of wave-number.

This assumption can be sufficiently justified because the expression,

$$z'(\xi,\tau) = z'(\xi - \int c d\tau), \qquad c = \frac{3z' c_0 z_{\xi\xi\xi}}{2\epsilon c_0^3 - 3z'_{\xi}} \frac{B^* \tau}{4c_0^2}$$
 (10)

can be derived by considering that the water surface displacement, z'= $(h_1/\epsilon)\eta,$ is a solution of Eq.(6), and then

$$\xi - \int c d\tau = \frac{\varepsilon^{1/2}}{\omega} \left\{ \int K(X) dx - \omega t \right\}$$
(11)

can be obtained.

Application of the assumptions of i) and v) to Eq.(1) yields the following equation, where the effect of the water depth change rate is explicitly expressed by the small parameter, ε .

$$K^{2} \phi_{\upsilon \upsilon} + \phi_{zz} + \epsilon (K_{X} \phi_{\upsilon} + 2K \phi_{\upsilon X}) + \epsilon^{2} \phi_{XX} = 0$$

$$-\omega \phi_{\upsilon} + \frac{1}{2} (K^{2} \phi_{\upsilon}^{2} + \phi_{z}^{2}) + gz^{2} + \epsilon K \phi_{\upsilon} \phi_{X} + \frac{\epsilon^{2}}{2} \phi_{X}^{2} = 0 \quad \text{at} \quad z = h_{1} + z^{2}$$

$$-\omega z_{\upsilon}^{2} + K^{2} \phi_{\upsilon} z_{\upsilon}^{2} - \phi_{z} + \epsilon (K \phi_{X} z_{\upsilon}^{2} + K \phi_{\upsilon} z_{X}^{2}) + \epsilon^{2} \phi_{X} z_{X}^{2} = 0 \quad \text{at} \quad z = h_{1} + z^{2}$$

$$-\phi_{z} + \epsilon K B_{X} \phi_{\upsilon} + \epsilon^{2} B_{X} \phi_{X} = 0 \quad \text{at} \quad z = B$$

$$(12)$$

Here, a final assumption is made concerning the velocity potential $\phi:$ vi) The velocity potential ϕ can be expanded as a power series with respect to the small parameter ϵ as:

$$\phi(\upsilon, X, z) = \phi_0(\upsilon, X, z) + \varepsilon \phi_1(\upsilon, X, z) + \cdots$$
(13)

Moreover, by exchanging υ for a new variable θ and defining the stream function $\psi,$ respectively, as:

$$\frac{\partial}{\partial \theta} = K \frac{\partial}{\partial \upsilon} , \quad \frac{\partial^2}{\partial \theta^2} = K^2 \frac{\partial^2}{\partial \upsilon^2} , \quad \theta = x - c(X)t , \quad (14)$$

$$\psi = \phi - c , \quad \psi_{\pm} = -\phi . \quad (15)$$

$$\psi_{z} = \phi_{\theta} - c \quad , \quad \psi_{\theta} = -\phi_{z} \quad , \tag{15}$$

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the following equation, expressed with the stream function, is derived in the lowest order O(1) from Eq.(12).

$$\frac{1}{2}(\psi_{0}^{2}+\psi_{z}^{2})+gz'=\frac{1}{2}c^{2} \quad \text{at} \quad z=h_{1}+z',$$

$$\psi=\psi_{0} \quad \text{at} \quad z=h_{1}+z',$$

$$\psi=0 \quad \text{at} \quad z=B,$$
(16)

where ψ_O is a constant denoting the total volume rate of flow underneath the steady wave per unit length in a direction normal to the x plane.

As mentioned above, the water depth change rate is of the order ε and from the assumption i) the value of ε could be considered small enough that the velocity potential ϕ can be sufficiently evaluated by the lowest order term ϕ_O , that is, the stream function defined in Eq.(14). Thus, under assumptions i)-v), although ψ is an explicit function of θ and z alone because all terms dependent on the change of water depth, such as bottom slope, are neglected, and although ψ is indirectly affected by water depth change through the conservation laws, the mathematical formulation satisfying both Eq.(15) and the conservation laws against the water surface displacement z' given numerically from Eq.(6) is synonymous with Eq.(1).

2. Examination on Conservation Laws

The effect of wave set-down is ignored by the assumption iii), so that the conservation laws to be satisfied become those of mass and energy alone. They can be expressed in a two-dimensional wave field of steady state, shown in Fig.1, as:

$$\frac{d}{dx}\left\{\int_{B}^{h_{1}+z}\rho u dz\right\}=0$$
(17)

$$\frac{d}{dx}\left[\int_{B}^{h_{1}+z} \rho u\left\{\frac{1}{2}(u^{2}+w^{2})+\frac{p}{\rho}+q(z-h_{1})\}dz\right\}+\rho g \eta_{S} \frac{d}{dx}\left\{\int_{B}^{h_{1}+z} u dz\right\}=0$$
(18)

where the bar — indicates averaging over one period, and $\eta_{\rm s}$ is wave set-

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 $\nabla^2 h = 0$

down. It is found from Eq.(18) that the uniformity of energy flux by waves is satisfied independently from the evaluation of wave set-down when the conservation law of mass is satisfied. Substitution of Eq.(15) into Eq.(17) yields

$$\varepsilon \frac{d}{dX} \left\{ \int_{B}^{h_{1}+z'} K \Phi_{00} dz \right\} + o(\varepsilon^{2}) = 0$$
(19)

The conservation law of mass is satisfied unconditionally in the same order O(1) as in Eq.(16), so that it can be considered in the order of ϵ . Hence, the law is expressed as

$$I = \varepsilon \int_{B}^{h_{1} + z} (\psi_{z} + c) dz = \varepsilon \{\psi_{0} + c(h_{1} - B)\} = const.$$
 (20)

where I is the mass flux by waves.

By similar examination on the conservation law of energy, the law can be written in the order of ϵ independently from wave set-down as:

$$W = \varepsilon \int_{B}^{h_{1}+z} \hat{\rho} \left(\psi_{z} + c \right) \left\{ \frac{1}{2} \left(\psi_{z} + c \right)^{2} + \frac{1}{2} \psi_{\theta}^{2} + \frac{p}{\rho} + g \left(z - h_{1} \right) \right\} dz$$
$$= \frac{\varepsilon}{2} \rho c \left\{ \left(3c + \overline{u} \right) 1 - c \overline{u} \left(h_{1} - B \right) - 2g \overline{z^{2}} \right\} = const.$$
(21)

where W is the energy flux by waves.

3. Calculation of the Stream Function of Shoaling Waves

By considering the theoretical result(1980) that mass transport velocity \bar{u} is given in Eulerian co-ordinate due to a nonlinear effect when the phase of z' is assumed to agree with that of ϕ , a generalized mathematical formulation of the stream function is assumed as

$$\frac{\Psi}{h\sqrt{gh}} = -\left\{\frac{1}{X_{(1)}X_{(2)}} - \frac{\bar{u}}{\sqrt{gh}}\right\} \quad \zeta \qquad + \frac{\sum_{n=4,6}^{N-1} \frac{\sinh(n-2)\pi X_{(2)}\zeta}{\cosh(n-2)\pi X_{(2)}}}{\left[X_{(n)}\cos((n-2)\pi \chi) + X_{(n+1)}\sin\{(n-2)\pi \chi\}\right]}$$
(22)

where $\zeta = (z-B)/h$, $\chi = (x-ct)/L$, $X_{(n)}$ is the n-th coefficient, especially $X_{(1)} = T\sqrt{g/h}$, $X_{(2)} = h/L$, $X_{(3)} = \Psi_0/h$ gh and T wave-period. The dynamic boundary condition at free surface shown in Eq.(16) can be rewritten with respect \bar{u} as:

$$\frac{1}{2}\bar{u}^{2} + (u_{w} - c)\bar{u} + \frac{1}{2}(u_{w}^{2} + w^{2}) - cu_{w} + gz' = 0 \quad \text{at} \quad z = h_{1} + z'$$
(23)

where u_w and w are the periodic component of the horizontal and vertical water particle velocities respectively. They are in the following relation with the stream function.

$$\Psi_{x} = -\Psi \qquad \Psi_{y} = \overline{u} + u_{w} - c \qquad (24)$$

Hence, u_w and w are expressed by Eq. (22) and (24), respectively, as: $u_w = \sum_{n=4,6}^{N-1} \frac{(n-2) \chi_{(2)} \cosh\{(n-2) \pi \chi_{(2)} \zeta\}}{\cosh\{(n-2) \pi \chi_{(2)} \}} \{\chi_{(n)} \cos((n-2) \pi \chi_{(n+1)} \sin((n-2) \pi \chi_{(2)})} + \chi_{(n+1)} \sin((n-2) \pi \chi_{(2)})} \{\chi_{(n+1)} \sin((n-2) \pi \chi_{(2)})} \{\chi_{(n+1)} \cos((n-2) \pi \chi_{(2)})} \}$ (25) $w = \sum_{n=4,6}^{N-1} \frac{(n-2) \pi \sinh((n-2) \pi \chi_{(2)})}{\cosh((n-2) \pi \chi_{(2)})} \{\chi_{(n+1)} \cos((n-2) \pi \chi_{(2)})} - \chi_{(n)} \sin((n-2) \pi \chi_{(2)})} \}$

Moreover, the mathematical formulation of water surface displacement using the stream function is derived through the kinematic boundary condition at free surface:

$$Y_{i} = \frac{\frac{-X_{(1)}X_{(2)}}{1 - (\bar{u}/\sqrt{gh})X_{(1)}X_{(2)}} [X_{(3)} - \frac{\sum_{n=4,6}^{N-1} \sinh\{(n-2)\pi X_{(2)}(1+Y_{i})\}}{\cosh\{(n-2)\pi X_{(2)}\}}$$

$$\{X_{(n)} \cos(n-2)\pi \chi_{i} + X_{(n+1)} \sin(n-2)\pi \chi_{i}\}] - 1$$
(26)

Hence, as long as the above expression is used, the conditions to be satisfied by Eq.(22) become a binding condition, i.e., the wave profile expressed by Eq.(26) must agree with both the numerical solution given by Eq.(6) and the conservation laws mentioned above. Although these conditions determine the coefficients $X_{(n)}$ uniquely, some of them are subject to nonlinear equations with respect to $X_{(n)}$. So, the determination of the stream function satisfying the above conditions is carried out by a method similar to that used by Rienecker et al.(1981). The unknown coefficients to be determined are $X_{(2)}, X_{(3)} \cdots X_{(n)}$, their total number being N-1 quantities.

The conditions to be satisfied by them can be expressed as follows: Firstly, as to the wave profile conditions, the numerical solutions of Eq.(6) and z'_c must agree with the expression shown in Eq.(26), so the conditions written as

$$f_i = z_{ci}/h - Y_i = 0, \quad i = 1, 2, \cdots, N-3$$
 (27)

must be satisfied. Here, i indicates the i-th phase during one period divided impartially by N-3. Secondly, the conservation laws of mass and energy expressed in the order of ε are, respectively,

$$f_{N-2} = \varepsilon(I-I_0) , \quad f_{N-1} = \varepsilon(W-W_0)$$
(28)

where the subscript o denotes the quantity in deep water and I is written by the expression shown in Eq.(22) as:

$$I = \overline{u}h + h \sqrt{gh} \sum_{n=4,6}^{N-1} \tanh\{(n-2)\pi X_{(2)}\} \{X_{(n)}\cos(n-2)\pi X_{i} + X_{(n+1)}\sin(n-2)\pi X_{i}\}$$
(29)

Here, the subscript i denotes the pahse giving $Y_i = 0$. The N-l numbers of equations shown above are solved by Newton-Raphson's method so that the internal properties of shoaling waves can be calculated under the six assumptions mentioned above.

4. Examination of the applicability of Numerical Solutions

An examination by Iwagaki et al.(1974) has already been made of the applicability of Dean's stream function method(1965) to the waves on sloping bottoms. Water surface displacement was given by experimental results at each water depth with normal wave flumes, and its applicability was determined for shoaling waves with deformed wave profiles. Their conclusion was that the stream function method can be applied to waves with asymmetrical wave profiles on sloping bottoms as long as the wave profiles can be accurately calculated, although the method should be primarily developed for uniform waves with symmetrical profiles. Hence, it is thought to be possible to apply the present approach, as well as the calculation by Iwagaki et al.(1974). However, the present approach is different in that the satisfying of the conservation laws, is not required in the usual methods. So, an examination is here made of the applicability of the horizontal water particle velocity computed by the present approach.



Fig. 3 Comparison between numerical solution and experiment by Iwagaki et al. of vertical distributions of horizontal water particl particle velocities

Fig.3 shows the comparisons between the numerical results obtained by the present approach where the measured wave profile at $h/L_0 = 0.069$ was given as initial value and experimental results by Iwagaki et al. of vertical distribution of horizontal water particle velocities of shoaling waves. And Fig.4 shows comparisons of their corresponding wave profiles up to the breaker point noted by $h/L_0 = 0.028$ in the figure. Here, the solid line shows the numerical results determined by



Fig. 2 Comparison of wave profiles determined from numerical solutions and from experimental data

the present approach and both of circles and broken lines show the experimental results. It might be said from the figures that the present approach is applicable for the calculation of the internal properties of shealing waves up to a breaker point, although the examination was made only of the water particle velocities and wave profiles and although their numerical results seem to give larger values at the breaker point than the experimental ones which probably suffered from the effect of back currents from the end wall of wave flume.

CALCULATION OF INTEGRAL QUANTITIES OF SHOALING WAVES

1. Definition of Integral Quantities

The method used by Longuet-Higgins(1975) can be applied to the present approach, and the expression of the various integrals can be derived as in the following, if the integrals are evaluated in the same order, O(1), as the terms in the equations mentioned above. The potential energy $E_{\rm p}$ at arbitrary water depth is defined as:

$$E_{\rm p} = \frac{1}{2} \rho g \bar{z}^{1/2}$$
(30)

and the kinetic energy Ek is defined as:

$$E_{k} = \int_{B}^{h_{1}+2} \frac{1}{2} \rho \{ (\psi_{z} - c)^{2} + \psi_{\theta}^{2} \} dx$$
(31)

By applying the expression defined by Longuet-Higgins, Eq.(32) can be rewritten as:

$$E_{k} = \frac{1}{2} \rho \{ c^{2}h - (\bar{u} + c)I \}$$
(32)

In the same way, the radiation stress S can be expressed as:

$$S = \int_{B}^{h_{1}+z'} (p + \rho u^{2}) dz - \frac{1}{2} \rho g h^{2}$$

= 2cI - 3E_p (33)

Then, the energy flux W shown in Eq.(29) can be expressed as:

$$W = \frac{1}{2} \rho c \left\{ (3c + \tilde{u})I - \tilde{u}ch - 4E_{p} \right\}$$
(34)

Therefore, the integral quantities of shoaling waves can easily be calculated within the limits of O(1), as long as the values of the wave celerity, the potential energy and the mass transport velocity can be given at an arbitrary water depth.

2. Definition of Breaker Point

In order to carry out the computation of the integral quantities of shoaling waves up to the breaker point, the breaker points must be reasonably and accurately defined and the computation of the stream function with sufficient accuracy must be possible. The former must be investigated here because the definition of breaker points has not yet been established theoretically, although the latter is satisfied by comparison with experiments including the actual breaker points. So, some examinations must be made of the breaker points to be used here.

Although the theoretical breaker point is thought to be defined by the singular point of Eq.(6) in the lowest order, the equation has not been yet analytically solved, so the condition for the singularity cannot be obtained theoretically. Therefore, the breaker point is conjectured from the behaviour of the results computed numerically under the conditions of the wave steepness H_O/L_O and bottom slope B_X described in

Table 1, where the values of the well-known surf-similarity parameter μ = $B_{\rm X}/~/{\rm H_O}/{\rm L_O}$ are also shown. The computations start from the point of h/L_O = 0.08 and the values of L/L_O and H/H_O at the point are computed by the energy flux method (1977).

Fig.5 depicts the changes in waveheights of shoaling waves computed from Eq. (6) as a function of the surf-similarity parameter. Here, the solid circles indicate the breaker points defined by Goda and expressed as

$$\frac{H_{b}}{L_{0}} = 0.17 \left[1 - \exp\left\{-1.5\pi\left(\frac{h}{L_{0}}\right) \right]$$

$$\left(1 + 15 B_{x}^{*/3}\right) \right]$$
(35)

It can be found from the figure that the extrema always exist in the changes in wave-height. The occurence of the extrema is conjectured to be caused by a kind of instability in the numerical solutions, which can be attributed to the excessive value of the curvature of the wave profile at crest. So, the relation between the existence of the extremum noted in the figure and the upper limit of a curvature of water surface displacement at crest of the numerical solution of Eq.(6) were examined to discover the reason for such an extremum. In Fig.6 white circles shows the values of the curvatures at the point of maximum wave-height of the numerical solution, and the points defined by Goda's breaking inception are indicated by solid circles for every value of the surf-similarity parameter. It can be noticed that the upper limit of the curvature exists in the vicinity of the values of 10⁴ for every value of the parameter, and the maximum value

Table 1 Conditions of wave parameters computed

Ho/Lo	^B x	μ
0.004	1/10	1.58
0.008	1/10	1.12
0.01	1/10	1.00
0.004	1/20	0.79
0.008	1/20	0.55
0.01	1/20	0.50
0.01	1/50	0.20
0.02	1/50	0.14
0.04	1/100	0.05



Fig. 5 Changes in wave-height of shoaling waves as a function of the surf-similarity parameter

for wave-height computed by Eq.(6) depends on the upper limit of the curvature of the numerical solution. Therefore, it is assumed that the theoretical breaker point in the numerical simulation generated by Eq.(6) is given by the upper limit.

Fig.7 shows the relation between the theoretical breaker indices, i.e., the ratio of wave-height to water depth at the breaker points, $\rm H_{b}/$ $\rm h_{b}$, and the actual indices defined by Eq.(35). Here, the solid lines



Fig. 6 Curvatures of the heighest waves and the surf-similarity parameter

describe the breaker indices given by Eq.(35), and circles indicate the theoretical indices for all bottom slope conditions. It could be considered from the results shown in the figure that most of the theoretical breaker indices exceed the actual ones and that the numerical solutions are stable and continue to maintain sufficient accuracy in the shallower water beyond the actual breaker points. And, it is found that the dependency of the theoretical breaker indices on bottom slopes reasonably corresponds to that of the actual breaker indices, although there are a few difference with regard to the



Fig. 7 Comparison between the theoretical breaker indices and the actual ones defined by Goda's breaking inception



Fig. 8 Changes in the ratio $u_{\rm C}/c$ of shoaling waves with the surf- similar-ity parameter

absolute values between them. Thus, the definition of the breaker points as being dependent on the upper limit of the courvature is regarded as reasonable. So, an investigation is made on the relation between the theoretical breaker point and Rankine-Stokes' condition of greatest wave-height.

Fig.8 shows the change in the ratio of the horizontal water particle velocity at crest, $u_{\rm C}$, to the wave-celerity, c, of the shoaling waves computed by the present approach up to the theoretical breaker points. It is noticed that all the values of the maximum ratio $u_{\rm C}/c$ do not exceed 1, although the ratios at the breaker points approach one as the value of the surf-similarity parameter increases. This result means that shoaling waves on sloping bottoms become unstable and begin to break down before Rankine-Stokes' condition is satisfied and that the breaking inception of shoaling waves is controlled by another condition besides Rankine-Stokes' condition.

Fig.9 shows the change in the small parameter σ with the ratio u_c/c definded by Longuet-Higgins(1975) as:

$$\sigma = \frac{1 - (u_c - c)^2 (u_t - c)^2}{c^2 c_0^2}$$
(36)

It is found that the parameter σ has a tendency similar to the ratio uc/c and that any given value of σ does not exceed 1 because of the similarity of the parameter σ to the ratio u_{c}/c , although the value σ of the highest wave becomes 1 in the calculations by Longuet-Higgins et al.. Therefore, it is to be expected that results considerably different from his will be obtained from the changes in the integral quantities of shoaling waves computed by the present approach.



Fig. 9 Changes in the ratio $u_{\rm C}/c$ of shoaling waves with the surf-simi-larity parameter

CHANGES IN INTEGRAL QUNATITIES WITH SHOALING WATER

Fig.10 shows the changes in the potential energy of shoaling waves with the surf-similarity parameter up to the theoretical breaker point computed by the present approach. It is found that the extrema of potential energy always exist before both the breaker points are attained and that the integral quantities of the highest wave are not always maxima. This agrees with the result shown by Longuet-Higgins et al.(1974), although the effect of the surf-similarity parameter is not taken into account in their calculation. It is thought that the existence of



Fig. 10 Changes in the potential energy of shoaling waves with the surf-similarity parameter

Fig. 11 Changes in the kinetic energy of shoaling waves with the surf-similarity parameter

such extrema is due to the sharpening of the wave crest caused by the tendency of the competition between the nonlinear effect and the dispersive effect to deform the wave profile which is strengthened with shoaling water.

Fig.ll shows the change in the kinetic energy of shoaling waves obtained by the same computation continued to the breaker points. It is found that the extrema of kinetic energy do not always exist before the breaker points are attained, in contrast to the case of potential energy and that the existence of the extrema depends on the value of the surf-similarity parameter. This is very different from the results of Longuet-Higgins(1975) which state that these extrema are always found in both the potential and the kinetic energies. This result indicates that there is a difference in the existence of the extrema of kinetic energy between shoaling waves with asymmetrical wave profiles on a sloping bottom and uniform waves with symmetrical wave profiles on a flat bottom. Therefore, it could be said that there is a difference in the internal properties, such as water particle velocities, between these wave types and that the conclusion obtained for one cannot be directly applied to the explanation of the wave breaking for the other.

Fig.12 shows the change in the total energy E_{t} of the same shoaling waves. Several matters are clear from this figure. The existence of the extremum of total energy depends on the value of the surf-similarity parameter just as that of the kinetic energy does. The extrema arise with the value of the parameter μ under 1.12 before the theoretical breaker points are attained and under 0.79 before the actual breaker points are attained. The critical value of the parameter controlling the existence of the extremum seems to be between 1. 12 and 1.58 where the theoretical breaker points are applied and between 0.79 and 1 where the actual breaker points are appli-It is well-known from the ed. experiments by Galvin(1969) that the breaker type depends on the value of the surf-similarity parameter and that a spilling breaker occurs when the value of the parameter is less than 0.5. Hence, it could be judged that the existence of the extremum closely relates to the breaker type and that a spilling breaker occurs due to the instability of the transfer of wave energy when the value of the parameter μ is less than the critical value mentioned above. The extremum exists because the conservation law of energy is satisfied in the present approach, the decreasing of the total energy with shoaling water requiring the increasing of the energy transport velocity of shoaling waves.

Fig.13 describes the change in the ratio of the energy transport velocity to the wave-celerity, c_g/c , of shoaling waves continued to the theoretical breaker point. It can be noticed that there is a tendency for the ratios c_g/c at both breaker



Fig. 12 Changes in the total energy of shoaling waves with the surf-similarity parameter



Fig. 13 Changes in the ratio c_g/c of shoaling waves with the surf-similarity parameter

points to decrease away from 1 as the value of the surf-similarity parameter increases, although the computed results scatter considerably of numerical errors in the computed value of wave-celerity. As it is judged that the energy transport velocity cannot physically exceed wavecelerity, instability with regard to energy transfer is considered to occur when the ratio c_q/c exceeds 1. Accordingly, this instability of energy transfer occurs dependently on the parameter and becomes liable to occur as the value of the ratio approaches 1. This instability could be conjectured to be closely related to the mechanism of wave breaking, and, in particular, to the mechanism of the occurence of a spilling breaker, where excess energy not transported by stable waves is exhausted by partial wave breaking. The approach of the ratio to 1 depends on the parameter, as mentioned above, and the instability of energy transfer occurs dependently on the value of the parameter, becoming liable to arise with the approach of the ratio to 1. Thus, it might be said that the instability of energy transfer is closely related to the spilling breaker and that the beginning of the instability is the breaking inception of a spilling breaker.

CONCLUSION

Integral quantities of shoaling waves have been calculated by using the K-dV equation with variable coefficients and the stream function method satisfying the conservation laws of mass and energy, and some investigations have been made on their characteristics. The most significant conclusions of this paper are summarized as follows.

The maximum wave-height of shoaling waves computed by the K-dV equation is controlled by the upper limit of the curvature of wave profiles, where numerical instability arises. Thus, the theoretical breaker point is defined by the limit, and it corresponds reasonably to the actual breaker point defined by Goda's breaking inception. However, Rankine-Stokes' condition of greatest wave-height is not satisfied at the theoretical breaker point of the computed shoaling waves and the wave breaking of shoaling waves seems to occur independently from the condition.

Although the extrema are always found in the potential energy of shoaling waves before the theoretical and actual breaker points are attained and the potential energy of the highest waves is not maximum just as in the results shown by Longuet-Higgins et al., the extrema of the kinetic and total energies are not always found in the shoaling waves with asymmetrical wave profiles before both the breaker points are attained, in contrast to the case of potential energy. In addition, the existence of the extrema depends on the value of the surf-similarity parameter and the extrema occur in the value of the parameter under the critical value controlling the existence. It is found by comparison with the experimental results of Galvin and his examination of the change in the ratio of energy transport velocity to wave-celerity that the existence of the extrema of total energy closely relates to the breaking inception and breaker type, especially to those of the spilling breaker, because it causes the instability of energy transfer of shoaling waves which is conjectured to be the cause of the wave breaking of the spilling type.

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