

THE EFFECT OF BREAKING WAVES ON THE MIXING
OF LIQUID POLLUTANTS INTO THE SEA.

by

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ABSTRACT.

Experience gathered during the last few years shows that a serious problem when fighting oil spills out in open waters, is the mixing of the oil into the sea by breaking waves. In this paper we propose a theoretical model that we believe represent a fruitful approach to the problem of determining the amount of oil mixed into the sea by breaking waves. Our model is based primarily on probabilistic methods, but with experimental observations and data as an essential foundation.

1. INTRODUCTION.

We shall in this paper consider the mixing of an oil spill into the sea caused by breaking waves, and the main topic will be a stochastic model for estimating the oil distribution with depth caused by this mixing. The answer to the question - How much oil can you expect to have left on the sea surface in a given sea state? - is rather crucial to planning and organizing oil pollution control. The reason is that oil being mixed into the sea is very difficult to recover.

The phenomenon that is the primary cause of dispersion is turbulence in the upper layers of the ocean. Now, the sea states of most immediate concern to us, are characterized by rather strong wind and breaking waves, which we shortly describe as rough seas. Under conditions as these, there are two mechanisms that completely dominate the generation of turbulence in the surface layers. One is breaking waves, the other is wind induced shear currents.

It is only in recent years that people have really begun to study breaking waves and their role in generating turbulence in the ocean. Of special interest to us would be a connection between sea state and turbulence level together with the penetration depth of the turbulent layer

beneath a breaking wave. However, in the literature there are no available data or analyses of a connection between sea state on the one hand and turbulence level and penetration depth on the other. In passing let me mention that some simple experiments we have done, indicated that the maximum penetration depth of the turbulence beneath a breaking wave is of the same order of magnitude as the wave height.

During conditions of rough seas, the strong down-welling turbulent water masses beneath breaking waves will constitute the main driving mechanism behind the dispersion of an oil slick into the sea. Generally speaking, we may say that breaking waves are two orders of magnitude more effective in dispersing surface oil than turbulence due to wind induced shear currents. This conclusion has also been drawn by other workers in this field, see reference 1.

However, it is not legitimate to conclude that turbulence due to dynamically instable shear currents in the surface region have little or no effect on dispersion when breaking waves dominate. While the turbulence generated by breaking waves is characterized by being highly non-stationary, i.e. the energy in the turbulence beneath a breaking wave dissipates rather quickly, we may with some confidence assert that the turbulence generated by wind induced shear currents has an approximately stationary character. This means that when the strong turbulence beneath the breaking waves dies out, a certain turbulence level is still present. How this turbulence will affect oil already being dispersed, is not well known. However, there is strong reason to believe that the residence time of an oil globule within the water column will depend significantly on this turbulence level.

When a breaking wave hits an oil slick, it will tear up part of it and drive a cloud of oil globules down into the sea. As long as the turbulent water motions beneath the breaker continues, the positions of the globules are mainly determined by this. But as soon as the turbulence has lost its energy, the buoyancy of the globules will drive them towards the surface. The rise velocity will certainly depend on globule size, but also probably on the background level of turbulence in the surface layers as we just mentioned.

How fast the amount of oil being mixed into the water will rise towards the surface again depend on how the amount of oil is distributed over different globule sizes, or more precisely, over different rise velocities and on how deep down into the water the oil globules are mixed. Every serious effort to estimate the dispersion of oil in open waters will consequently have to find estimates of rise velocity distribution and mixing depth due to breaking waves. And for this information to be of practical use, it has to be related to sea states by some suitable method.

In the first part of our paper we will describe the method or model chosen as a starting point in dealing with this problem. In the latter part a description of experiments and analyses we have done in order to estimate rise velocity distribution and mixing depth will be given.

However, before we enter the details of our paper, we shall list some excellent reports and papers dealing with the dispersion of oil. In reference 1 is presented a mathematical model of dispersion. In reference 2, detailed discussions of most of the phenomena playing a part in the dispersion of oil are given, and theoretical models for some of these are presented. This report is a highly valuable addition to the literature. A short presentation of some of the work contained in reference 2 is given in reference 3. Reports on work having a strong bearing on the experimental part of this paper are given in references 4 and 5.

2. THEORETICAL MODEL.

2.1 Basic assumptions.

In order to construct a manageable theoretical model to estimate the vertical distribution of oil in the sea due to the mixing induced by breaking waves, it is obvious that quite a few simplifying assumptions have to be made.

The basic assumptions on which we build our model, are as follows: Firstly, we shall assume that the time points when breaking waves pass a fixed location on the sea surface are randomly distributed according to some probability law that may, in principle, be determined for each environmental condition.

Assume then that initially we have an oil slick of surface density σ at our fixed location. With each passing of a breaking wave, a mixing of the oil into the sea will occur. In agreement with experimental observations, we will assume this mixing process to be instantaneous, i.e. relative to the time intervals between passing breakers. Further, we assume that with each breaking wave we may associate a quantity, called the "mixing depth", which gives a measure of the depth to which oil is carried by the mixing created by the breaking wave, and that this quantity is a random variable whose probability distribution may, in principle, be determined for each environmental condition. From our experimental observations, we may conclude that the "mixing depth" is largely determined by the depth to which the more or less turbulent water motion created by the breaking wave extends, and much less on the type of oil present. We shall, accordingly, assume that the probability distribution of "mixing depth" is independent of the particular type of oil present.

A few more simplifying assumptions are needed to avoid an intractable model.

To be precise, we neglect the horizontal motion of the oil due to waves and current, so that the vertical distribution of the oil always adds up to σ at our fixed point. If on the average the oil slick is moving with a uniform velocity, we may introduce a reference frame moving with the same velocity and relate our analyses to this. The probability distribution must then, of course, be determined relative to this frame. The next simplification is to neglect the variation of the vertical distance between specific water particles caused by the wave motion.

We shall also neglect the influence of the oil on the breaking waves. Now it is a fact that oil under some conditions does have an influence on breaking waves, but what limitations that put on our model is hard to say at present. For instance, observations in connection with the blow-out on the Ixtoc 1 field in the Gulf of Mexico, seem to indicate that strongly emulgated oil have no or very little effect on breaking waves. The main reason for neglecting this effect is, however, that the state of knowledge in this field is such as to allow no quantitative conclusion.

2.2 Probabilistic model.

With the simplifying assumptions made above in mind, we shall now construct our theoretical model. Starting with the occurrence of breaking waves, we then have a sequence of random time points

$$\dots, T_1, T_2, T_3, \dots, T_n, \dots$$

describing each passing of a breaking wave past a fixed location. With each T_k there is associated a random variable D_k , called the "mixing depth", and we assume that all the D_k 's are mutually independent and identically distributed with probability density f_D . The D_k 's are also assumed to be independent of the timepoints T_k . The last assumption is not essential.

Due to the mixing process, the oil will be broken up into droplets of different sizes, which we characterize by the parameter u equal to the terminal rise velocity of a droplet through calm water, cfr. section 4. For the present discussion, we will neglect the influence on the vertical distribution of oil of all but the last breaking wave. More precisely, we assume that the vertical distribution at an arbitrary time is determined by the last breaking wave to pass before that time. This approximation seems satisfactory if the ratio of "mixing depth" to the time interval between breakers is "small enough". It is clear that when this ratio increases the need to include the effect of earlier breakers will sooner or later become mandatory. Now, there is nothing in our model that prevents us from taking into consideration the effect of any number of preceding breakers, the only limitation being the unwieldiness of the expressions obtained.

To get on with our present model, assume that the density of droplets with rise velocity u as a function of depth z immediately after a breaker has passed, i.e. at time T_k for some k , is given by the function $\phi(D_k, z; u)$. For each definite value of the random variable D_k , this is an ordinary function of the real variable z . Letting $C(t; z, u)$ denote the density of droplets with rise velocity u at depth z at an arbitrary time t , we may write

$$C(T_k; z, u) = \phi(D_k, z; u) \text{ for every } k \dots\dots\dots(1)$$

and

$$C(t; z, u) = \phi(D_k, z + u(t - T_k); u); T_k \leq t < T_{k+1} \dots\dots\dots(2)$$

for every k.

We now want to determine the probability density of $C(t; z, u)$ for an arbitrary fixed time t . To this effect, let f_W denote the probability density function of the time interval W between t and the time of the last preceding breaker. Having fixed t, z and u , we suppress the variables from $C(t; z, u)$ and write simply C . We now have

$$C = \phi(D, z + uW; u) \dots\dots\dots(3)$$

As the next step, we shall exploit our experimental observations which indicate that the mixing created by the breaking wave gives a nearly uniform distribution down to a rather well defined depth, which we have called the "mixing depth". Accordingly, we shall assume that we may write

$$\phi(D, z; u) = \frac{p(u)}{D}, \text{ for } 0 \leq z \leq D \text{ (= 0 elsewhere)} \dots\dots\dots(4)$$

Here $p(u)$ denotes the fraction of droplets with rise velocity u , cfr. section 4. For simplicity, we shall assume $\sigma = 1$ so that

$$\int_0^{u_{\max}} p(u) du = 1 \dots\dots\dots(5)$$

From the above we may proceed to calculate the probability distribution function F_C of the random $C = C(t; z, u)$. Instead of entering the details of this calculation, we refer the interested reader to reference 6 where the details may be found. Here we shall just give the final answer, which is:

$$F_C(c) = \int_0^Z f_D(x) dx + \int_Z^\infty \int_{\frac{x-Z}{u}}^\infty f_W(w) f_D(x) dw dx + \int_0^\infty \int_0^{\frac{x-Z}{u}} f_W(w) f_D(x) dw dx \dots\dots\dots(6)$$

for $0 \leq c \leq \frac{p(u)}{Z}$ (the singular point $c = 0$ causes no difficulty here) while for c outside this interval:

$$F_C(c) = 0 \dots\dots\dots(7)$$

Before we continue to make specific assumptions about the probability laws, a few words on the probability density function f_W are appropriate. Now, it is obvious that to determine the probability law of W directly from observations of passing breakers is a rather cumbersome process. In trying to connect W to a random variable more easily determined, it is natural to investigate the connection between

W and the time intervals between passing breakers. Define, then, the random variables $Y_k = T_k - T_{k-1}$, which we assume to be identically distributed with probability density f_Y . It turns out that for most practical purposes we have the following connection between f_W and f_Y .

$$f_W(w) = \frac{1}{E(Y)} \int_w^\infty f_Y(y) dy; w \geq 0 \quad (=0; w < 0) \dots\dots\dots(8)$$

Consequently, an estimate of f_Y , which we may obtain directly from the observational data (with the proper assumptions), allow us quite easily to estimate f_W .

From our experimental data, we are able to give estimates of $p(u)$ as a function of u , cfr. section 4. By taking this into consideration, we see from (6) that the remaining unknown factors are the probability density functions f_W and f_D . At present, however, there is no information available regarding these probability laws. In passing we mention that we intend to carry out research aiming at providing this necessary information.

Using (6) and (7) and writing $C = C(z, u)$ since the probability structure of C is independent of t , as it should be, we shall have

$$E(C(z, u)) = \int_0^{p(u)/z} \{1 - F_C(c)\} dc \dots\dots\dots(9)$$

(9) gives the expected density at depth z of droplets with rise velocity u . To find the expected density of oil at depth z , we just integrate (11) over u (expectation commutes with integration) to find

$$E(C(z)) = \int_0^{u_{max}} E(C(z, u)) du \dots\dots\dots(10)$$

where $C(z)$ denotes the density of oil at depth z .

Denoting by $M(z)$ the amount of oil below the depth z , we find that

$$E(M(z)) = \int_z^\infty E(C(x)) dx \dots\dots\dots(11)$$

If, initially, the surface density is σ , then we just multiply (12) and (13) by σ to obtain the appropriate answers.

A quantity of special interest for oil recovery operations is the mean amount of oil left on the sea surface under given weather conditions. Expressed in per cent of total amount, we denote it by P_S . It is now easy to see that P_S will be given as follows:

$$P_S = 100 \cdot \{1 - E(M(0))\} \dots\dots\dots(12)$$

3. EXPERIMENTS.

This section is an adaption of the work reported in reference 7.

3.1 Test conditions and instrumentation.

In order to analyze the break-up and turbulent mixing into the sea of an oil slick on the surface caused by a spilling breaker passing the slick, a series of experiments were carried out in a towing tank of main dimensions 25 x 2.5 x 1.5 meters (length x width x depth). The principal aims were to find an estimate of the droplet size distribution, which is caused by the break-up of the oil slick, and to determine the "mixing depth" produced by each breaker.

After some preliminary experimentation, it was decided to apply the following testing procedure:

To produce the breaking waves, the technique of impulse waves was applied. An impulse wave is brought about by the dispersive properties of deep water waves. By decelerating the wave maker in a proper manner a wave train is produced that will converge at a preassigned location in the test tank. For details, see for instance reference 8. Using this technique, it was possible to produce a rather well defined spilling breaker in a given region in the middle of an oil slick with a thickness of about 5-7 mm. The height of the breaking impulse wave was varied by changing the amplitude of the wave maker. During the test programme, a wave height of approximately 0.4 metres was used.

To get a measure of the amount of oil at different depths as a function of time after the breaker had passed, an apparatus as sketched in Fig. 1 was used. (Tables and figures are placed at the end of the paper). It consisted of six thin glass tubes of different length, each inserted into a larger reservoir pipe, as shown in Fig. 1. The six reservoir pipes ended in a common suction chamber, which was connected to a vacuum tank by a flexible suction hose. A manually operated magnetic valve was used to open and close this connection. The suction chamber with its six protruding glass tubes was secured to an aluminium frame on the top of two light float elements in such a manner that the length of the submerged part of the glass tubes were 2, 4, 7, 10, 15 and 20 cm, respectively, when the assembled rig was floating on calm water. With this floating rig it was desired to take a sample of the oil/water mixture at the depths given above and at different times after the breaker had passed. In order to obtain this, it is obvious that the wave riding properties of the rig had to be very good. By trial and error, float elements were found that gave a wave rider that followed the wave forms nicely. The rig was kept in an upright vertical position by long nylon strings, as shown in Fig. 2. The strings were made sufficiently long to ensure unhampered vertical motion of the rig.

The sample of the oil/water mixture was taken by opening the connection between the suction chamber and the vacuum tank for a few tenths of a second. By allowing the oil to settle in each reservoir pipe, the fraction of oil in the sampled mixture could be measured, as illustrated in Fig. 1.

The tests were performed with a pure mineral oil both in fresh and emulgated state, i.e. oil that had been on the water for several weeks. In the non-emulgated state, the oil has properties similar to those of fresh crude oil from the Norwegian Continental Shelf. The properties of the oil in the two states are given in Table 1. As we mentioned above, during all the tests there was a mean oil film thickness of about 5 - 7 mm in the test tank.

The test programme we refer to in this report is detailed in Table 2. Explanatory comments to the tests are given in the table.

3.2 Presentation of results.

We shall present the experimental results in the form of diagrams giving the arithmetical mean and standard deviation of the measurements for each condition.

Fig. 3 to 7 show the percentage of emulgated oil in the sample as a function of time after the breaker had passed the rig. During these tests the wave started to break about 1.5 meters ahead of the rig.

Fig. 8 to 11 show the percentage of fresh oil in the sample as a function of time after the breaker had passed the rig. The wave started to break about 1.5 meters ahead of the rig.

3.3 Discussion of results.

In order to assess the errors we may commit in using the described sampling procedure to estimate the "mixing depth" and distribution of the oil, we need to consider what happens when a breaking wave hits the oil slick. During the first second after the breaker has passed, there is a mixing of the oil into the water which gradually reaches its maximum depth. Then, immediately, the oil droplets will start to rise towards the surface and, for instance, during three tenths of a second the fastest droplets may have risen as much as 4 cm. From this it is clear that sampling at time intervals of one second leads to the following conclusion: The deeper the sample is taken, the more we are in error when using the sample to estimate the initial density of oil. Consequently, the applied sampling procedure will not allow a proper estimate of the "mixing depth". However, the measured evolution with time of the density at the different sampling depths will be approximately correct, except for the first second after the breaker has passed. Therefore, if by some other means we are able to give a reasonably correct estimate of the initial distribution with depth of the density of oil, we shall also be able to estimate the droplet size distribution.

To obtain an estimate of the density distribution, we shall exploit the visual observations made of the mixing process. These observations did indicate a rather well-defined depth to which the mixing of oil extends and also a nearly uniform distribution of oil droplets with depth. The corresponding assumption of a uniform distribution down to a well-defined depth, called the "mixing depth", has already been used in section 2 in constructing our theoretical model. We shall also apply it in the next section in order to determine the droplet size distribution. From both visual observations and later experiments (cfr. reference 9) we have estimated the "mixing depth" to be approximately 0.2 meters.

4. DRDPLET SIZE DISTRIBUTION.

Exploiting the experimental data given in section 3 and with the assumptions of section 2 still effective, we now want to find an estimate for the distribution of droplet sizes $p(u)$ characterized by the parameter u , which is the terminal rise velocity of a droplet through calm water. According to the assumptions we made in section 2, each passing breaker produce the same distribution $p(u)$ and a uniform depth distribution from the sea surface down to the "mixing depth". Assuming a unit surface density and letting $K_D(t, z; u)$ denote the density of droplets with rise velocity u at depth z as a function of time t after the breaker has passed, given the associated "mixing depth" D , we then have

$$K_D(0, z; u) = \frac{p(u)}{D} ; D < z \leq D \dots\dots\dots(13)$$

Under the given conditions there will be an upper bound to the rise velocities. We shall consequently assume a fixed upper limit u_{max} to the rise velocities u , so that $p(u) = 0$ for $u > u_{max}$. u_{max} will generally depend on the type of oil present. A more detailed discussion of u_{max} is deferred to the end of this section.

We define $K_D(t, z)$ by

$$K_D(t, z) = \int_0^{u_{max}} K_D(t, z; u) du ; 0 < z \leq D \dots\dots\dots(14)$$

$K_D(t, z)$ is then the density of oil at depth z at time t after the breaker has passed. From (13) and (14) we get

$$K_D(0, z) = \int_0^{u_{max}} K_D(0, z; u) du = \frac{1}{D} ; D < z \leq D \dots\dots\dots(15)$$

At a time t after the breaker has passed, all droplets with a rise velocity $u > (D-z)/t$ will have passed the depth z on their way to the

surface. This, of course, presupposes that $t \geq (D-z)/u_{\max}$. Accordingly, we shall have

$$K_D(t,z) = K_D(0,z), \quad 0 \leq t < \frac{D-z}{u_{\max}}, \dots\dots\dots(16)$$

$$K_D(t,z) = \int_0^{\frac{D-z}{t}} \frac{p(u)}{D} du ; \quad t \geq \frac{D-z}{u_{\max}} \dots\dots\dots(17)$$

The experimental results for $K_D(t, z)$ show an exponential decrease with time. In agreement with our assumptions, we therefore expect to find that

$$K_D(t,z) = \frac{1}{D} e^{-\kappa(D,z)(t-t_{\min})} ; \quad t \geq t_{\min} \dots\dots\dots(18)$$

where $t_{\min} = (D-z)/u_{\max}$ and $\kappa(D,z)$ is constant for each D and z. To determine the dependence of κ on D and z, we apply the following one-to-one correspondence between the rise velocity and time

$$u = \frac{D-z}{t} ; \quad t \geq t_{\min} \dots\dots\dots(19)$$

Differentiating (17) and (18) with respect to time, by direct comparison we obtain, using (19):

$$p(u) = \kappa(D,z)(D-z)u^{-2} e^{-\kappa(D,z)(D-z)\left(\frac{1}{u} - \frac{1}{u_{\max}}\right)} \dots\dots\dots(20)$$

Our assumptions imply that $p(u)$ should not exhibit any dependence on D and z, which lead to the following equality

$$\kappa(D, z) = \frac{\kappa}{D-z} \dots\dots\dots(21)$$

where κ should depend only on the properties of the oil. Substituting (21) into (18) we obtain

$$K_D(t,z) = \frac{1}{D} e^{-\frac{\kappa}{D-z}(t-t_{\min})} ; \quad t \geq t_{\min} \dots\dots\dots(22)$$

Inserting (21) into (20) we get

$$p(u) = \kappa u^{-2} e^{-\kappa \left(\frac{1}{u} - \frac{1}{u_{\max}} \right)}, \quad 0 < u \leq u_{\max} \dots\dots\dots(23)$$

In order to test out the relations we have obtained on the basis of our assumptions, we plotted $K_D(t, z)$ taken from Fig. 4 -7 and 9 - 11. As mentioned at the end of the previous section, the "mixing depth" was estimated to be approximately 0.2 m. For fresh oil we found $\kappa = 0.17$ from Fig. 9, $\kappa = 0.20$ from Fig. 10 and $\kappa = 0.22$ from Fig. 11. This supports our assumptions rather well, and by putting $\kappa = 0.2$ in (22) we shall find that this expression fits the experimental results nicely. Similarly, we find that (22) with $\kappa = 0.02$ agrees tolerably well with the experimental results for the emulgated oil.

The distribution $p(u)$ that we have determined above is the result of one breaking wave. In reality, however, it is clear that the appropriate rise velocity distribution induced by a passing breaker is not determined solely by that wave, but also by preceding breakers. The reason for this is that the smaller oil globules may not have resurfaced when the next breaker comes along. The "equilibrium" rise velocity distribution will therefore contain a larger portion of small rise velocities than the $p(u)$ determined previously. In order to obtain a better approximation to the correct distribution, we shall assume a constant frequency of passing breakers and a constant mixing depth. It is natural to choose the corresponding mean values \bar{Y} and \bar{D} , respectively. We then have a limit velocity $u_1 = \bar{D}/\bar{Y}$, because all the oil with rise velocity greater than u_1 will have resurfaced before the next breaker comes. However, in many cases u_{\max} is smaller than u_1 , and as this simplifies the analysis below, we shall assume this to be the case.

Let us now study the effect of a sequence of breaking waves, equidistant in time, passing a fixed location with an initial surface density of oil $\sigma_0 = 1$. The rise velocity distribution induced by the first passing breaker we have already determined to be given by $p(u)$ as in (23).

With the assumption $u_{\max} \leq u_1$ in mind, we find that the amount of oil of rise velocity u having surfaced when the second breaker passes, i.e. after a time \bar{Y} , is given by

$$\sigma_1(u) = p(u) \frac{u}{u_1}; \quad 0 < u \leq u_{\max} \dots\dots\dots(24)$$

The surface density at that instant is given by

$$\sigma_1 = \int_0^{u_{\max}} \sigma_1(u) du \dots\dots\dots(25)$$

The distribution over rise velocity due to surface oil induced by the second breaker is $\sigma_1 p(u)$. The distribution due to oil being

in the water column will be $p(u) - \sigma_1(u)$. The rise velocity distribution, $p_1(u)$, due to the second breaker will then be given by

$$p_1(u) = (1 + \sigma_1) p(u) - \sigma_1(u) \text{ or } (\sigma_0 + \sigma_1)p(u) - \sigma_1(u) \dots\dots\dots(26)$$

We have, of course, tacitly assumed a uniform distribution with depth, as implied by the considerations of section 2.

The amount of oil of rise velocity u having surfaced when the third breaker passes, is given by

$$\sigma_2(u) = p_1(u) \frac{u}{u_1} = [(\sigma_0 + \sigma_1)p(u) - \sigma_1(u)] \frac{u}{u_1} \dots\dots\dots(27)$$

The corresponding surface density is

$$\sigma_2 = \int_0^{u_{\max}} \sigma_2(u) du \dots\dots\dots(28)$$

Similarly as above, we find that the rise velocity distribution $p_2(u)$, due to the third breaker is given by

$$p_2(u) = \sigma_2 p(u) + p_1(u) - \sigma_2(u) \\ = (\sigma_0 + \sigma_1 + \sigma_2)p(u) - (\sigma_1(u) + \sigma_2(u)) \dots\dots\dots(29)$$

Continuing this process, we shall find generally for every $n = 1, 2, 3, \dots$

$$\sigma_n(u) = [\sum_{i=0}^{n-1} \sigma_i p(u) - \sum_{i=1}^{n-1} \sigma_i(u)] \frac{u}{u_1} \dots\dots\dots(30)$$

$$\sigma_n = \int_0^{u_{\max}} \sigma_n(u) du \dots\dots\dots(31)$$

$$p_n(u) = \sum_{i=0}^n \sigma_i p(u) - \sum_{i=0}^n \sigma_i(u) \dots\dots\dots(32)$$

Here $\sigma_n(u)$ denotes the surface density of oil of rise velocity u having surfaced just prior to the $(n+1)$ th breaker, σ_n is the total density of surfaced oil at that instant and $p_n(u)$ is the rise velocity distribution induced by the $(n+1)$ th breaker.

From (30) we easily establish the following recursion relation valid for every $n = 1, 2, \dots$

$$\sigma_{n+1}(u) = \sigma_n p(u) \frac{u}{u_1} + (1 - \frac{u}{u_1}) \sigma_n(u) \dots\dots\dots(33)$$

Using this recursion relation, we find that for $n = 1, 2, \dots$

$$\sigma_{n+1}(u) = p(u) \frac{u}{u_1} \sum_{i=0}^n \sigma_{n-i} \left(1 - \frac{u}{u_1}\right)^i \dots\dots\dots(34)$$

We have for every $n = 1, 2, \dots$

$$\sigma_{n+1}(u) = p_n(u) \frac{u}{u_1} \dots\dots\dots(35)$$

Combining (34) and (35), we get

$$p_n(u) = p(u) \sum_{i=0}^n \sigma_{n-i} \left(1 - \frac{u}{u_1}\right)^i \dots\dots\dots(36)$$

The appropriate rise velocity distribution to use in the model developed in section 2 would be $\lim_{n \rightarrow \infty} p_n(u)$ provided, of course, that

this limit exists. It turns out that with $p(u)$ as given by (23), $p_n(u)$ converges very fast for all relevant values on u_1 , and we may assume a stationary situation after just a few waves (~ 10).

To know $p(u)$ we see that we also need to know u_{max} . Unfortunately, we are not able to estimate u_{max} from our experiments, as is easily realized by recollecting the discussion in section 3.3. According to reference 10, the maximum rise velocity through calm water for oil droplets of fresh crude oil with properties similar to our mineral oil is approximately 0.1 m/sec. This corresponds to droplets with diameter about 2 - 3 mm. A further increase of the diameter leads to a decrease of the rise velocity. As we have nothing else to rely on at the present, we shall assume $u_{max} = 0.1$ m/sec. in the next section, where we try to illustrate our theory by some numerical examples. The value of u_{max} that enters the distribution $p(u)$ cannot be expected constant for each type of oil. For instance, u_{max} will most certainly depend on the surface density σ and the intensity of the mixing process. The assumptions $u_{max} = 0.1$ m/sec. may therefore prove to be a rather crude one.

For the sake of completeness we mention that some simple experiments we performed did corroborate a maximum rise velocity of the mineral oil of about 0.1 m/sec. More precisely, we found a maximum rise velocity of 0.13 - 0.15 m/sec, and there was no significant difference between fresh and emulgated oil.

Regarding the maximum rise velocity for emulgated oil, it turns out not to be easily accessible. This is due to the fact that the interfacial tension between the water phase and the emulgated phase is a key parameter in the mechanism that determines the maximum rise velocity, and this interfacial tension is almost impossible to measure.

5. CONCLUDING REMARKS.

In this paper we have constructed a theoretical framework which we believe will enable us to determine with reasonable accuracy the vertical distribution of oil in the sea under given environmental conditions. But this, of course, depends on the availability of sufficiently accurate input data. These, however, do not exist today.

At the Ship and Ocean Laboratory in Trondheim we are at present engaged in research aiming at developing suitable experimental methods that will allow us to establish the correct input data to our model by full scale measurements out at sea. Especially, we want to establish the statistical laws of breaking frequency and "mixing depth" which are the two most important inputs to the model.

The importance to pollution control of the problem we have considered in this paper should warrant a considerable effort towards the establishing of a reasonably correct solution.

ACKNOWLEDGEMENTS.

The author would like to express his appreciation of the many helpful discussions with Prof. E. Huse of the Ship and Ocean Laboratory. I also stress my debt to Mr. S. Eggen and Mr. E.G. Kringeland for supplying the experimental data given in this paper.

7. NOMENCLATURE.

- t - arbitrary, fixed time point
- T_k - random time points of passing breakers
- Y_k - $Y_k \stackrel{\text{def.}}{=} T_k - T_{k-1}$
- f_Y - probability density function of the random variables Y_k
- W - random variable equal to the time interval between t and time of the last preceding breaker
- f_W - probability density function of the random variable W (W_1)
- D_k, D - random "mixing depth" as induced by passing breakers
- f_D - probability density function of the random variables D_k and D
- u - rise velocity of an oil droplet
- $p(u)$ - fraction of droplets with rise velocity u
- $p_n(u)$ - fraction of droplets with rise velocity u just prior to (n+1)th breaker (section 4)

- $\sigma_n(u)$ - Surface density due to oil of rise velocity u just prior to $(n + 1)$ th breaker (section 4)
- δ_n - Surface density of oil just prior to $(n + 1)$ th breaker (section 4)
- z - Depth below the water surface
- $\phi(D,z;u)$ - Density of droplets with rise velocity u as a function of depth z immediately after a breaker inducing a "mixing depth" D has passed
- $C(t;z,u)$ - Density of droplets with rise velocity u at depth z at an arbitrary time t
- F_C - Probability distribution function of the random variables $C(t;z,u)$
- $M(z)$ - Amount of oil below depth z
- $K_D(t,z;u)$ - Density of droplets with rise velocity u at depth z as a function of time t after the breaking impulse wave has passed (section 4)
- $K_D(t, z)$ - Density of oil at depth z at time t after the breaking impulse wave has passed (section 4)
- P_S - Defined by $P_S = 100(1-E(M(D)))$, i.e. $P_S\%$ of the oil is expected on the surface
- κ - Parameter entering $p(u)$

8. REFERENCES.

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TABLE 1

Type of oil	Kinematic viscosity cSt (20 °C)	Density kg/m ³
Fresh mineral oil	2.75	795
Emulgated mineral oil	21.00	829

TABLE 2

Type of oil	Wave height of breaker (m)	Comments
Emulgated mineral oil	0.4	Samples taken at different points of time after the breaker had passed the rig. The rig was placed approx. 1.5 m downstream from where the wave started to break. 4 samples were taken for each condition. Fig. 3 - 7.
Fresh mineral oil	0.4	Same comments as above. 7 samples taken for each condition. Fig. 8 - 11.

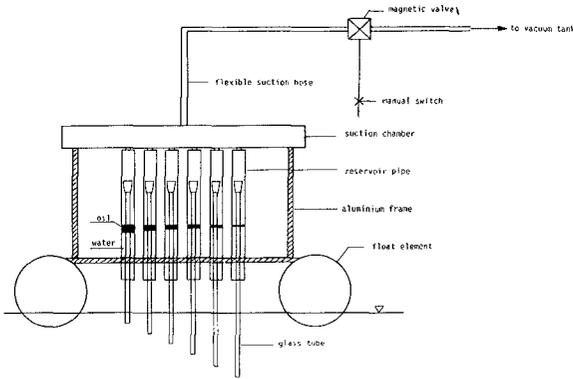


Fig. 1 - Rig for sampling the oil/water mixture.

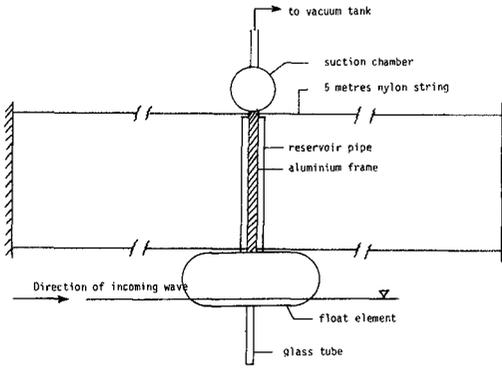


Fig. 2 - Side-view of rig showing crawl-foot arrangement to keep rig in vertical position.

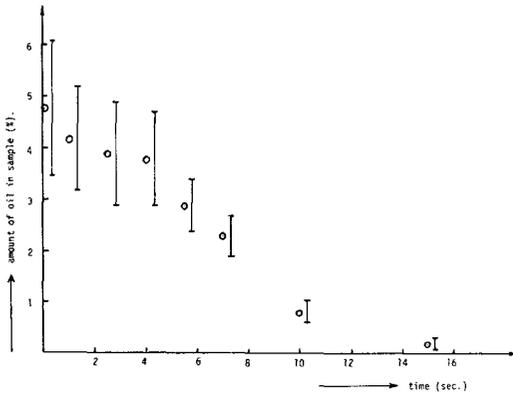


Fig. 3 - Sample taken at depth 2 cm. Emulgated oil.

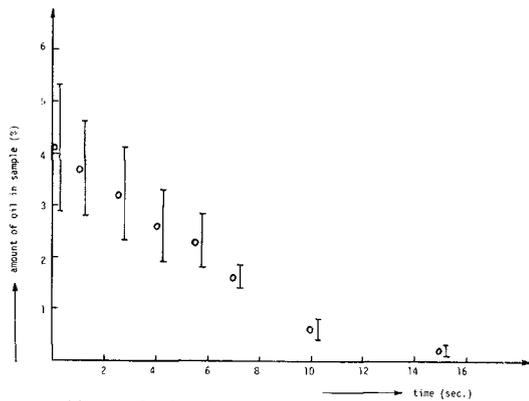


Fig. 4 - Sample taken at depth 4 cm. Emulgated oil.

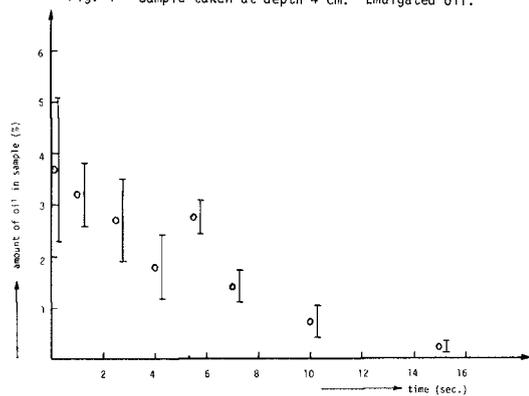


Fig. 5 - Sample taken at depth 7 cm. Emulgated oil.

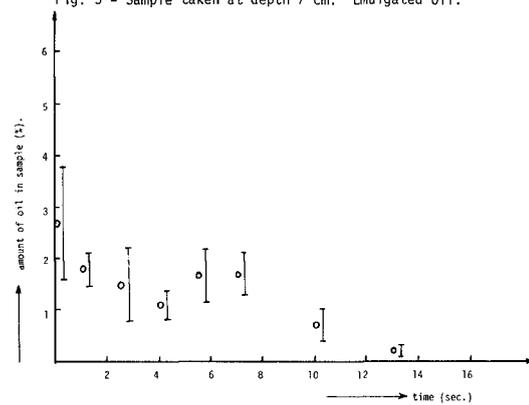


Fig. 6 - Sample taken at depth 10 cm. Emulgated oil.

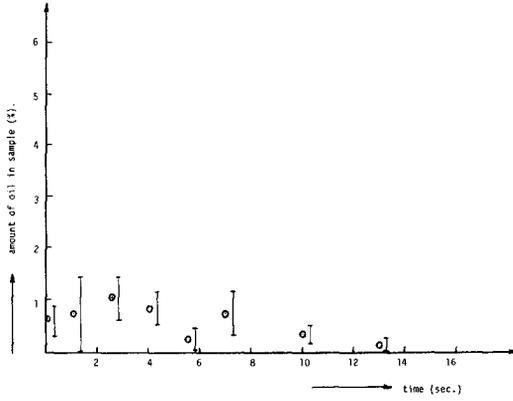


Fig. 7 - Sample taken at depth 15 cm. Emulgated oil.

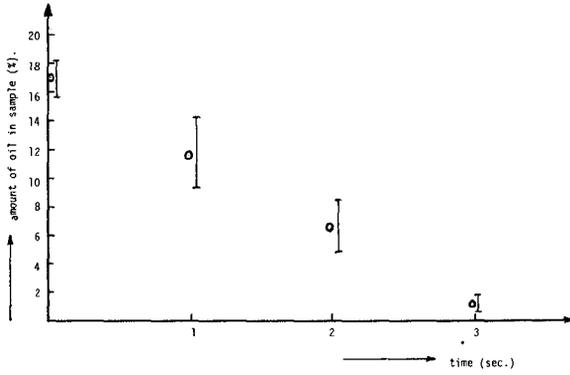


Fig. 8 - Sample taken at depth 2 cm. Fresh oil.

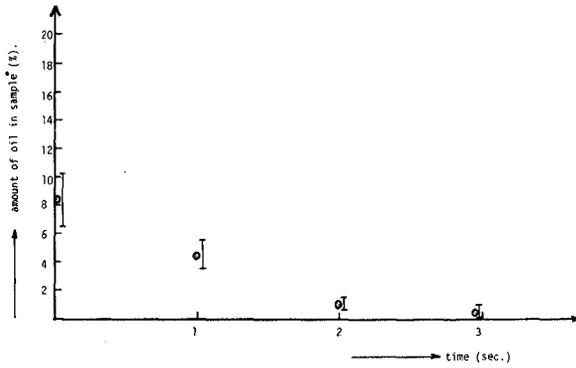


Fig. 9 - Sample taken at depth 4 cm. Fresh oil.

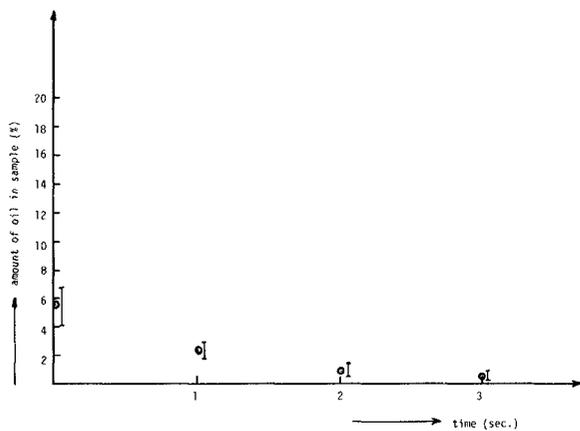


Fig. 10 - Sample taken at depth 7 cm. Fresh oil.

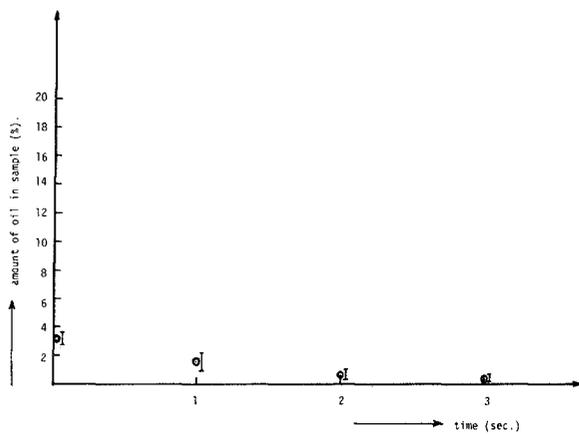


Fig. 11 - Sample taken at depth 10 cm. Fresh oil.