BOUNDARY CONDITIONS FOR ANALYSIS OF FLOW IN TIDAL INLETS

T. C. Copalakrishnan[1] and J. L. Machemehl[2]

INTRODUCTION

One-dimensional gradually varied flow analysis of flood routing and inlet flows (Mahmood and Yevjevich, 1975; Amein, 1975 and Hinwood and Wallis, 1975) has been a subject of study in the last few decades and many mathematical models have been developed based on those studies. A common feature in all these models is that the boundary conditions at the ends of the channel reaches are supplied from measured values of stage, discharge or velocity. These boundary conditions form an integral part of the mathematical models. In the case of implicit schemes, without the supply of these boundary conditions there will be more unknowns than equations. Even though in the explicit schemes they are not required in order to supply sufficient equations, it is obvious that the flow will not be properly simulated without imposing proper end conditions of flow.

Normally two end conditions will be required, the upstream and downstream conditions, even though in a network of channels there will be more than two end conditions. Of these two the upstream condition is usually the forcing function and the downstream one is the result of the flow due to the forcing function. The downstream condition depends on what happens to the flow outside the system. In other words, it depends on the shallow water wave reflections from the continuation of the channel beyond the downstream end of the system considered. These reflections are characterized by the expansion or contraction of the channel, the rate of change of the side slopes and other channel characteristics. As mentioned earlier, the downstream end condition is supplied from measured values of flow parameters so that the channel features (outside the system) mentioned above are automatically simulated. However, if it is required to know the response for any given forcing function, the corresponding measured downstream values may not be available. This means that the downstream boundary condition cannot be imposed in the usual way. This paper describes a method by which the downstream boundary condition can be imposed in the absence of measured downstream response to a given forcing function. However, the method presupposes

[1]Assistant Professor, Ocean Engineering Centre, Indian Institute of Technology, Madras, India 600 036.

[2]Associate Professor, Department of Marine Science and Engineering, North Carolina State University, Raleigh, N. C. 27650.

that measured values for at least one forcing function are known as part of the features of the channel or channel system.

FLOW PARAMETERS

The rate of change of velocity or stage at a point is a function of geometric and dynamic parameters. The geometric parameters include the shape of the channel cross section and the rate of change of that section beyond the point under consideration. The stage and velocity themselves at the point form the dynamic parameters, in addition, of course, to the acceleration due to gravity. However, when looking for the rate of change of velocity or stage at an end section, it should be recognized that such rates can be different for the same stage and velocity depending on whether the flow is accelerating or decelerating and whether the flow is inward or outward of the inlet. Thus, the following four situations are:

1) Flood tide - accelerating flow
2) Flood tide - decelerating flow
3) Ebb tide - accelerating flow
4) Ebb tide - decelerating flow

By grouping the geometric and dynamic parameters to form appropriate non-dimensional numbers that govern the flow, it is possible to predict with a degree of accuracy the time rate of change of velocity or stage at the end sections of a channel system. In the present study, the rate of change of velocity, $\partial U/\partial t$, has been used as the downstream boundary condition.

Application of the Π - Theorem

A functional relationship of the dynamic parameters involved can be written as follows, remembering that we are looking for, $\partial U/\partial t$.

$$F(\dot{U}, U, g, \eta) = 0$$

where, U, is the velocity, $\dot{U}$, is time-derivative, g, the acceleration due to the gravity and, η, the water level.

Using, η, and, g, as the repeating variables, the non-dimensional numbers, Π_1, and, Π_2, are:

$$\Pi_1 = \eta^a g^b \dot{U}$$

$$\Pi_2 = \eta^{a_1} g^{b_1} U$$

On applying the dimensional equations, it is found that $a = 0$, $b = -1$, $a_1 = -\frac{1}{2}$ and $b_1 = -\frac{1}{2}$. Therefore:

$$\Pi_1 = \dot{U}/g$$

$$\Pi_2 = U/\sqrt{g\eta}$$

As espected, the Froude number of flow is a governing factor.

Steps to Arrive at $\partial U/\partial t$

From a given set of velocity-stage relationship at a given end of a channel system a relationship between the two non-dimensional numbers, Π_1, and, Π_2, is obtained using a least squares fit of a fourth order polynomial. From this relationship one can compute, $\partial U/\partial t$, once the velocity and stage at the point for a given time is known. The, $\partial U/\partial t$, is then imposed as the downstream boundary condition. The steps involved in the procedure are shown in Figures 1 and 2.

APPLICATION TO THE CAROLINA BEACH INLET

The procedure mentioned was tested with the field data obtained for Carolina Beach Inlet, North Carolina, USA. The inlet (Fig. 3) has a small inlet-channel connecting the ocean to the Atlantic Intracoastal Waterway. That part of the channel system studied is shown in Fig. 3. The Galerkin finite element approach coupled with a Hermitian Cubic shape function (Zienkiwicz, 1971) was used to analyze the inlet. Details of this method have been reported independently. (However, the type of the numerical analysis is irrelevent at this point). The channel system requires the rate of change of velocity, i.e. $\partial U/\partial t$, at the two points Range 1 and Range 2. Independent relationships between Π_1 and Π_2 were obtained for these two points in order to impose the two downstream boundary conditions. The upstream boundary condition was the forcing function which is the tidal fluctuation at the mouth of the inlet.

The relationship between Π_1, and Π_2 for flood tide at Range 2 and ebb tide at Range 1 for both accelerating and decelerating trends are shown in Figs. 4 and 5. The figures shown indicate a fairly smooth relationship in all cases. The inlet was analyzed using these relations for a complete tidal cycle. The best way to test the method would be to use two sets of tidal data, one to construct the relationship between Π_1, and, Π_2, and other to compare measured computed values. As two sets of data were not available for the inlet, a hypothetical tidal input was used as a second forcing function, the first one being from the records. Figs. 6 and 7 show the measured and computed tidal fluctuations at Ranges 1 and 2. The hypothetical forcing function was taken as 0.8 times the tide used for the first set of computations.

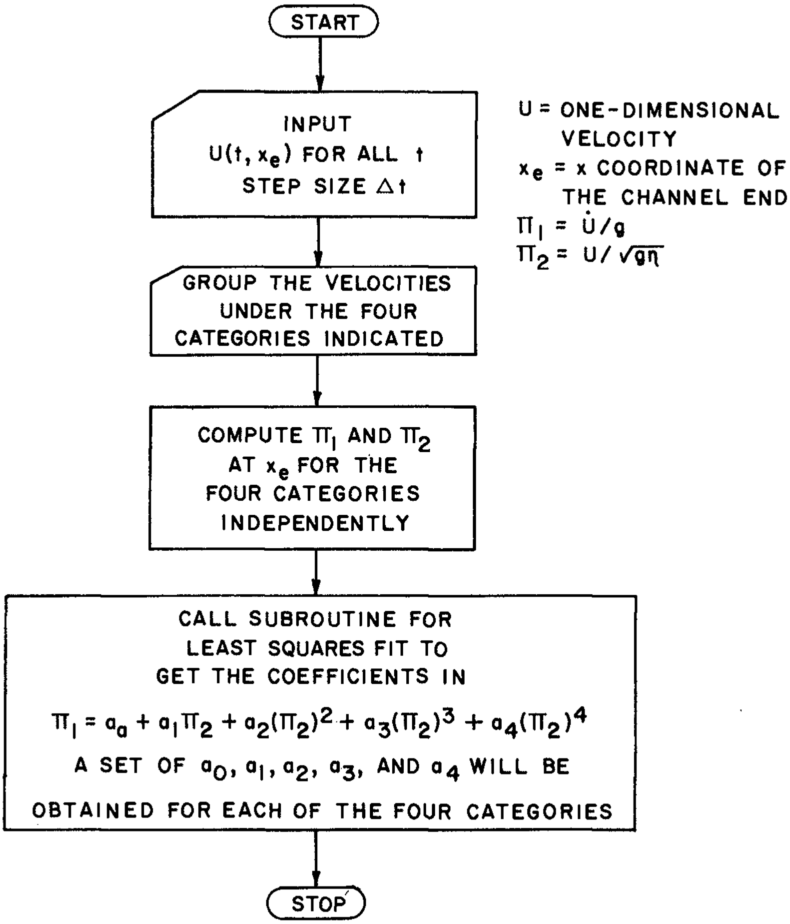

FIGURE 1. FLOW CHART NUMBER ONE.

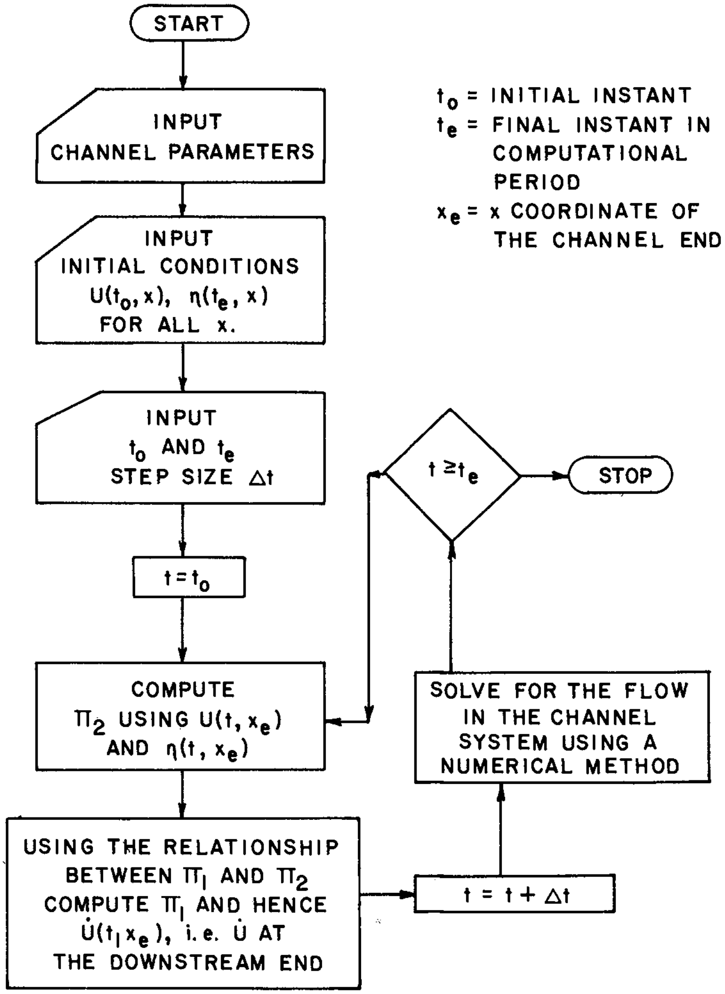

FIGURE 2. FLOW CHART NUMBER TWO.

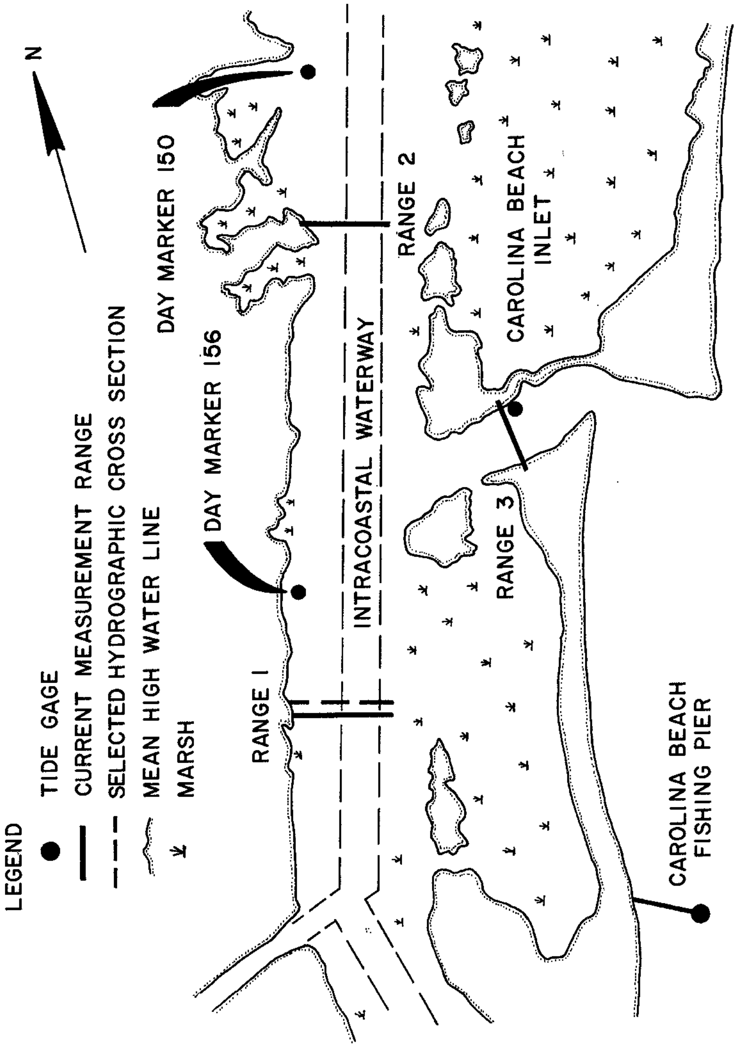

FIGURE 3. CAROLINA BEACH INLET

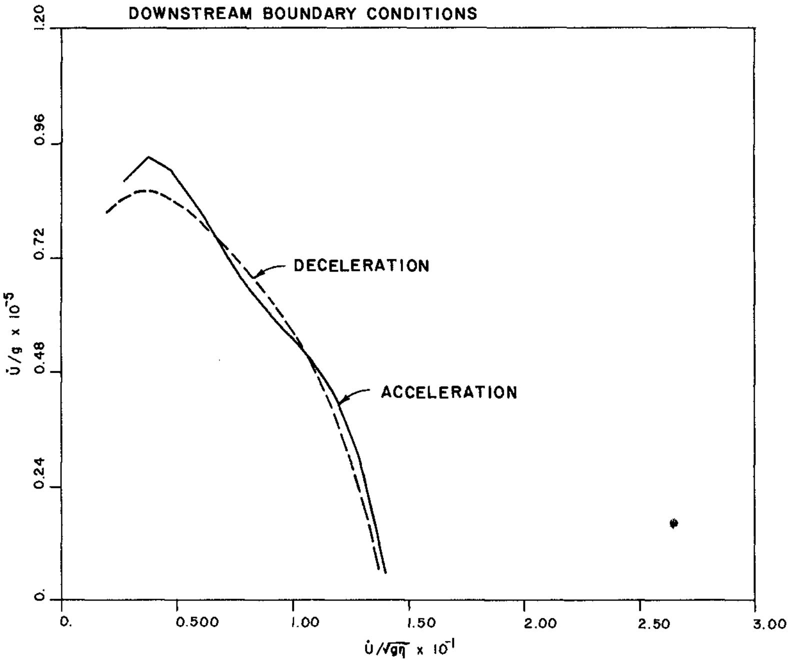

FIGURE 4. FLOOD TIDE AT RANGE NUMBER 2

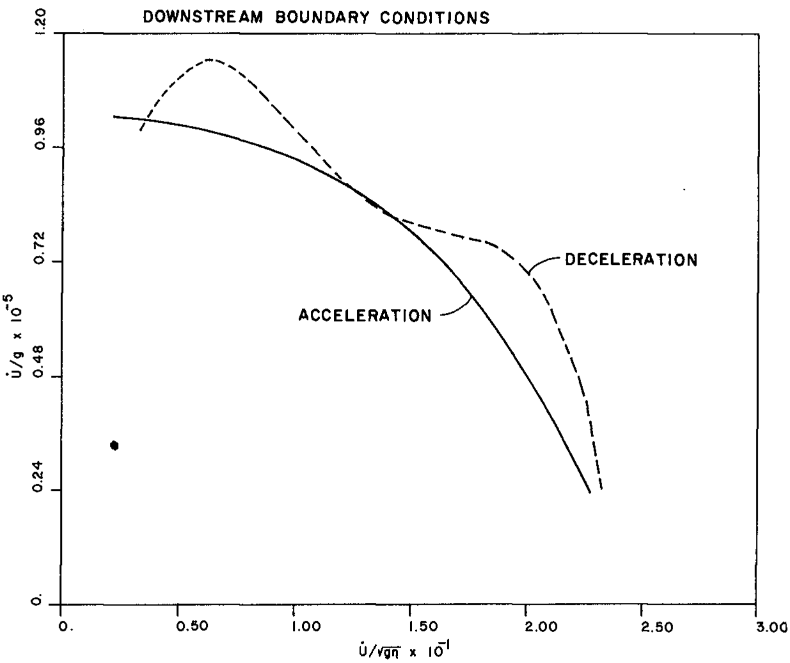

FIGURE 5. EBB TIDE AT RANGE NUMBER I

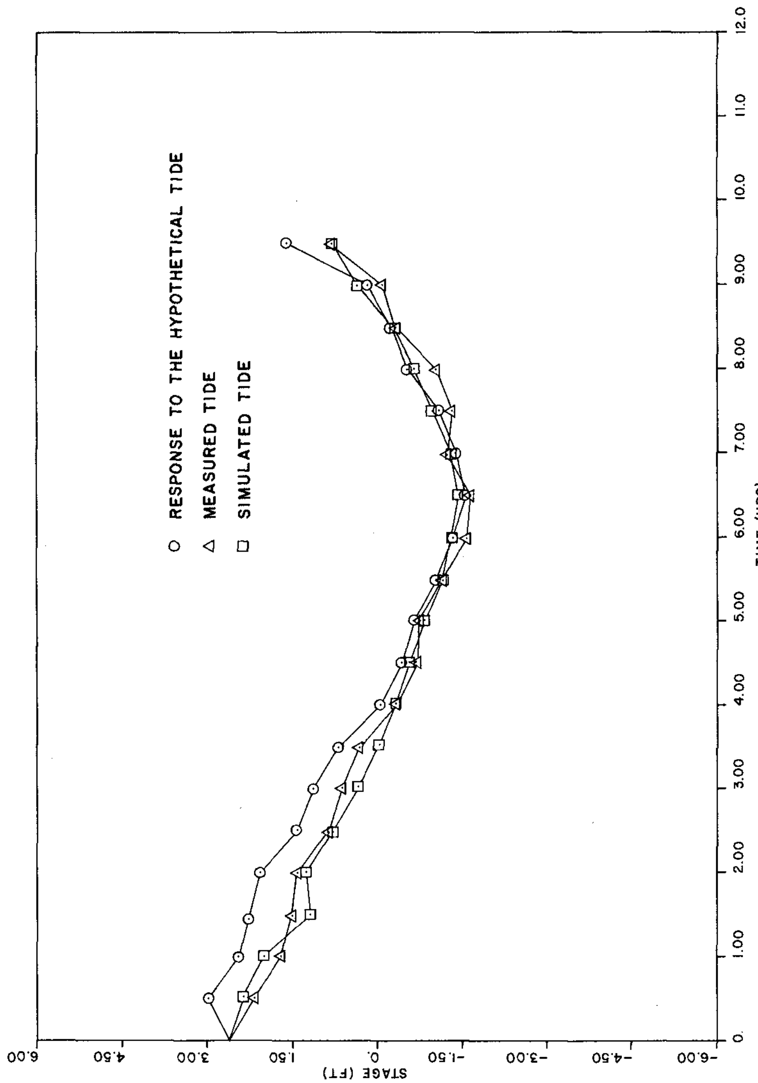

FIGURE 6. TIDAL FLUCTUATIONS AT RANGE NUMBER 1

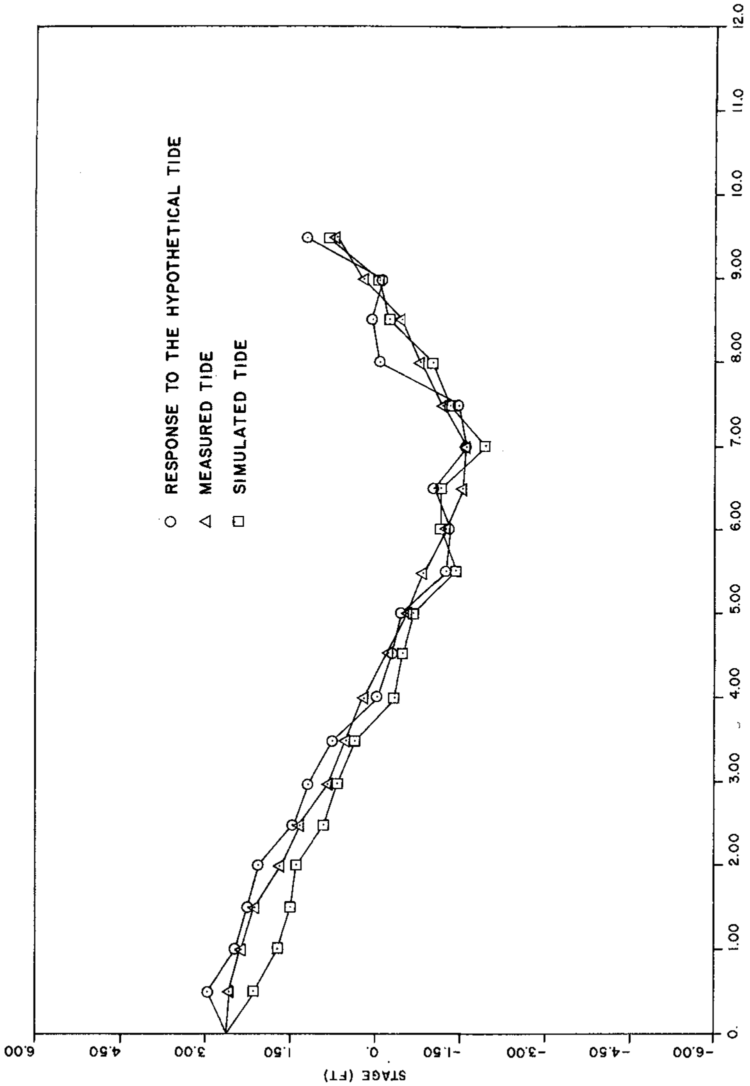

FIGURE 7. TIDAL FLUCTUATIONS AT RANGE NUMBER 2

CONCLUSIONS

A method for supplying downstream boundary condition in the absence of measured data for any given forcing function has been developed. This method is based on non-dimensional numbers which govern the dynamics of flow. These numbers were determined by a simple application of the Π - Theorem to the dynamic parameters of flow.

The above method was applied to the Carolina Beach Inlet, N. C. analysis. As shown by Figs. 6 and 7 the method yield values which are in good agreement to the measured tide at the two points considered. This, no doubt, depends on the numerical method of analysis also. The non-dimensional numbers can be used to yield the downstream boundary conditions for any numerical method chosen to analyze the flow provided. $\partial U/\partial t$, is the downstream condition needed for all time. If it is, $\partial \eta/\partial t$, then a different set of non-dimensional numbers are required.

The advantage of the method developed here is that one can know the response of a channel system to a hypothetical but possible forcing function for which measured downstream values are not available. This situation is especially relevant to power canals and inlet channels where the flow is affected by man-made alterations in river discharge or construction, etc.

ACKNOWLEDCEMENT

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APPENDIX I - References

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APPENDIX II - Notation

The following symbols are used in this paper:

a, b, a_1, b_1 = powers of variables while applying the Π-Theorem

g = acceleration due to gravity

x = independent variable, space

t = independent variable, time

U = the one dimensional velocity in the x direction

$\dot{U}$ = $\partial U/\partial t$

η = water level with regard to a horizontal datum

Π_1, Π_2 = the non-dimensional numbers $\dot{U}/g$ and $U/\sqrt{g\eta}$ respectively.