

SEDIMENTATION IN CHANNELS AND TRENCHES

E.W. Bijker*

1. Introduction

In this paper the siltation in approach channels and trenches due to cross currents and waves is discussed. It is in this respect not necessary that the current crosses the channel at a right angle. When the current crosses the channel obliquely simply a greater distance over which the water flows over the greater depth is introduced. Deviations in the flow pattern - see figure 1 - due to the channel are neglected. When the flow pattern is known, either from measurements in nature or in model this effect can easily be introduced. The influence of the waves is introduced through the introduction of an increased bed shear and subsequently higher diffusion coefficient (see Bijker, 1971, [3]).

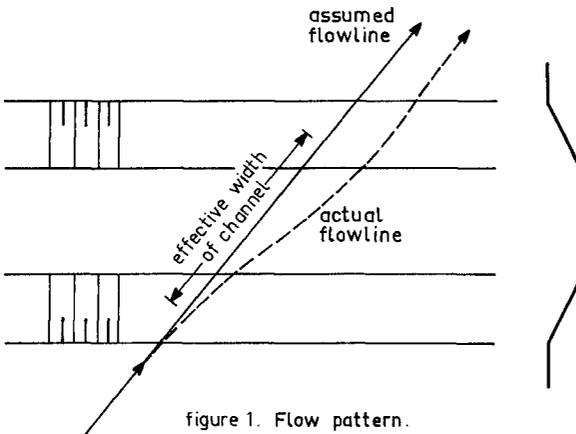


figure 1. Flow pattern.

Although computer programs are available to compute the siltation under the above described circumstances, an attempt will be made to come to a relatively simple method which enables a quick estimate of the siltation to be expected without the requirement of a big computer. This method could be especially usefull for the engineer in the field who has to make the first appraisal for the various solutions.

* Professor of Coastal Engineering, Delft University of Technology

2. Basic equations

For the water motion a logarithmic velocity profile and a first order linear wave theory are applied. The vertical distribution of the suspended sediment is described by:

$$w c + \epsilon dz/dz = 0, \quad (1)$$

with w = fall velocity of the sediment in still water, c = concentration at height z above the bed and ϵ = diffusion coefficient for the suspended material at height z . For ϵ the same value as for the diffusion coefficient for the momentum is used, which is acceptable for relative fine material ($D \leq 300 \mu\text{m}$). Applying the logarithmic velocity distribution and the linear shear stress distribution the following expression of ϵ is obtained.

$$\epsilon = \kappa v_* z(1-z/h), \quad (2)$$

in which v_* = the bed shear velocity, κ = the constant of Von Karman and h = the total water depth.

This expression of the diffusion coefficient gives the well known concentration gradient which is after Einstein also used by Bijker (1971, [3]). However, experiments by Coleman (1970, [4]) indicate that with reasonable accuracy a constant diffusion coefficient for the whole depth can be applied. Kerssens and Van Rijn (1979, [7]) use a constant value for the upper half of the depth and a logarithmic distribution for the lower half. Although this represents the available data certainly better, in this case nevertheless one constant value is assumed in order to keep the procedure as simple as possible. This gradient of ϵ determines the vertical sediment concentration. Since in this case the decrease of this vertical sediment concentration with the distance over the channel is, rather than the actual form, the main item, this inaccuracy is accepted. For the constant value of ϵ is assumed:

$$\epsilon = \kappa v_* h/2,5 = 0,16 v_* h = 0,16 \bar{v} h \sqrt{g/C}, \quad (3)$$

in which \bar{v} = the mean value of the uniform flow, g = acceleration of the earth gravity and C = the resistance coefficient after Chezy and equal to $18 \log(12h/r)$ with r as apparent bed roughness. With this value of ϵ the following expression for the vertical sediment concentration is obtained:

$$c = c_b \exp(-w z/\epsilon), \quad (4)$$

with c_b = concentration at the bed.
The average silt concentration is

$$\bar{c} = (c_b \epsilon/w h) [1 - \exp(-w h/\epsilon)] \quad (5)$$

With this it is implicitly assumed that the total suspended load is $S_s = \bar{c} h$. This assumption is checked against the strict computation $S_s = \int_0^h v c dz$. Although for current alone the errors are less

then 10%, it can be expected that for conditions with waves and currents the errors could be more. In that case the form of the decrease of the sediment load and therefore the mechanism of the siltation will, nevertheless, be the same. The actual values of the transport can be calculated then with the more strict formula, while for the decrease the method described in this paper is used. This will be demonstrated in the example with waves and currents. When the suspended load is related to the bed load, the concentration c_b at the bed is calculated from this bed load. Bijker (1971, [3]) found for this relation via the velocity distribution close to the bed:

$$c_b = S_b / 6,34 r \sqrt{\tau_c / \rho} = S_b C / 6,34 \bar{v} r \sqrt{g} \quad (6)$$

in which S_b = bed load, τ_c = bed shear and ρ = density of the water. As well the bed load as the concentration are expressed in volumes of deposited material with the normal porosity coefficient. The wave influence is taken into account through an increased bed shear as derived by Bijker originally in 1966 and 1967 ([1] and [2]). According to this derivation

$$v_{*cw} = v_{*c} \left[1 + \frac{1}{2} (\xi u / \bar{v})^2 \right] \quad (7)$$

in which v_{*cw} and v_{*c} are the bed shear velocities due to waves and current and current alone, \hat{u} = amplitude of the horizontal orbital velocity at the bed and

$$\xi = C \sqrt{f_w / 2g}, \quad (8)$$

with f_w = friction coefficient as derived by Jonsson (1966, [6]). (See Swart, 1974, [8], and Van de Graaff and Van Overeem, 1979 [5]). This friction factor can be written as:

$$f_w = \exp \left[-5,977 + 5,213 (a_o / r)^{-0,194} \right] \quad (9)$$

in which a_o = amplitude of horizontal orbital motion at the bed. This adjusted value of u_{*} is introduced in the formula for ϵ .

Since, as has been demonstrated by Van de Graaff and Van Overeem (1979, [5]), the transport formula as given by Bijker in 1971 [3] gives for the combination of waves and current better results than the adapted Ackers and White and Engelund and Hansen formulae, this formula is also used here. The bed load is then written according to the adjusted Kalinske-Frijlink formula as:

$$S_b = (B D \bar{v} \sqrt{g} / C) \exp \left[-0,27 \Delta D C^2 / \mu \bar{v}^2 \left(1 + \frac{1}{2} (\xi \hat{u}_o / \bar{v})^2 \right) \right] \quad (10)$$

in which B = coefficient ≈ 5 , D is the grainsize of the bed material, Δ = relative density of the bed material and μ = a ripple coefficient = $(C_{D90} / C)^{3/2}$, with C_{D90} = bed resistance coefficient for a flat bed consisting of grains with a diameter of D_{90} ; that is that grainsize which is exceeded by only 10% of the bed material.

3. Sedimentation

The sedimentation in the channel is determined by an equation of the type of eq. (1).

So the first thing to do is to study the behaviour of ε and dc/dz above the channel. Since c is a property of the water, the value of c will remain constant in the first instance. The vertical gradient of the sediment concentration will, however, immediately decrease by a factor h_1/h_2 , h_1 and h_2 being the depth upstream and in the channel. (See figures 2 and 3).

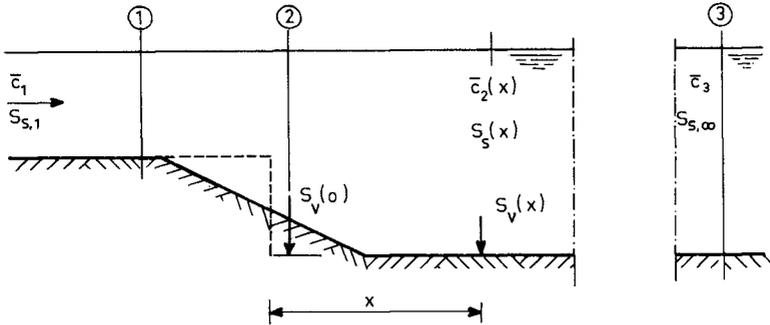


figure 2. Mechanisme of sedimentation.

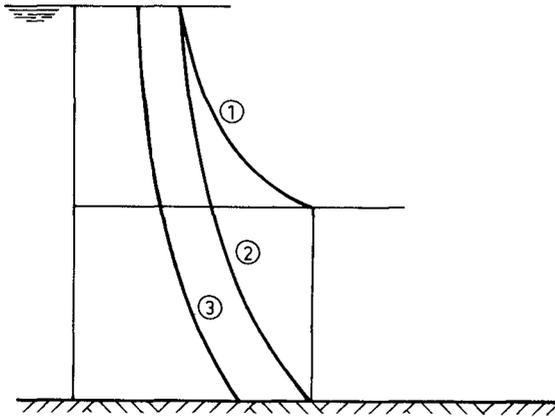


figure 3 Concentration verticals.

Since the actual sedimentation, or vertical transport is determined by the condition just above the bed this vertical transport reads then

$$S_v = w c_{b2} + \epsilon (dc/dz)_b, \quad (1)$$

in which the subscript b indicates the situation at the bed and ϵ is not yet determined. Due to the greater depth ϵ will eventually decrease. It is not known with certainty how fast this will happen. Since it is assumed that with normally occurring slopes of the channel banks the current will follow the bed, it is assumed that also ϵ will adapt immediately to the new situation. The value to be used above the channel will be then

$$\epsilon_2 = 0,16 v_{*2} h_2 = 0,16 \bar{v}_2 h_2 \sqrt{g/C_2} = 0,16 q \sqrt{g/C_2} \quad (11)$$

The vertical sediment concentration distribution at the most upstream side of the channel, before any sedimentation has taken place, is:

$$c = c_b \exp(-w h_1 z / \epsilon_1 h_2). \quad (12)$$

The vertical gradient is in this case:

$$(dc/dz) = -(h_1 w / h_2 \epsilon_1) c_b \exp(-w h_1 z / \epsilon_1 h_2). \quad (13)$$

The average sediment concentration is at that moment:

$$c_2 = (\epsilon_1 / w h_1) c_{b,1} \left[1 - \exp(-w h_1 / \epsilon_1) \right] \quad (5)$$

which is - as it should be - equal to the average sediment concentration just upstream of the channel. Since it is assumed that the value of the diffusion coefficient would adjust itself rather soon to that for the situation above the channel, the sedimentation - or vertical transport - at the most upstream side of the channel is:

$$\begin{aligned} S_v &= w c_{b,2} + \epsilon_2 (dc/dz)_b = \\ &= w c_{b,2} - w \frac{h_1 \epsilon_2}{h_2 \epsilon_1} c_{b,2} \exp(-w h_1 z (=0) / \epsilon_1 h_2) = \\ &= w c_{b,2} \left(1 - \frac{h_1 \epsilon_2}{h_2 \epsilon_1} \right) \end{aligned} \quad (14)$$

With $w h / \epsilon = h_{*}$, this equation reads

$$S_v = w c_{b,2} (1 - h_{*,1} / h_{*,2}) \quad (14)$$

The vertical transport S_v is expressed in m^2/s per unit of bed surface. The suspended load of the stream will decrease as result of the sedimentation until a new equilibrium is obtained.

This equilibrium is determined again by equation (1) which reads

$$w c_3 + \varepsilon_2 (dc/dz)_3 = 0 \quad (15)$$

with subsequently:

$$c_3 = c_{b,3} \exp(-w z/\varepsilon_2). \quad (16)$$

$$\bar{c}_3 = (c_{b,3} \varepsilon_2/w h_2) \left[1 - \exp(-w h_2/\varepsilon_2) \right] = (c_{b,3}/h_{x,2}) \left[1 - \exp(-h_{x,2}) \right] \quad (17)$$

$$(dc/dz)_{b,3} = -(w/\varepsilon_2) c_{b,3} \quad (18)$$

The vertical transport at that moment is

$$S_{v,3} = w c_{b,3} - \varepsilon_2 (w/\varepsilon_2) c_{b,3} = 0 \quad (19)$$

A positive value for S_v indicates a downward transport, that means sedimentation. The value of $c_{b,3}$ is for the moment unknown, but it is assumed that in the equilibrium situation it is again determined by the relationship of equation (6), and it will be written as

$$c_{b,3} = \alpha c_{b,2} = \alpha c_{b,1} \quad (20)$$

The equation of motion for the suspended load is

$$dS_s(x)/dx + S_v(x) = 0 \quad (21)$$

in which x = distance along the flow line from the upstream side of the channel.

For the solution of the continuity equation (21) two possibilities exist.

a). Assume that $S_s(x)$ decreases exponentially with x , so

$$S_s(x) = f (\exp(-\beta x)) \quad (22)$$

With the boundary conditions:

$$S_s(0) = S_{s,1} \quad (23)$$

$$S_s(\infty) = S_{s,3} \quad (24)$$

equation (22) can be written as

$$S_s(x) = (S_{s,1} - S_{s,3}) (\exp(-\beta x) - 1) + S_{s,1} \quad (25)$$

When this is substituted in the continuity equation (21) the following expression for β in S_s and S_v is obtained:

$$-\beta (S_{s,1} - S_{s,3}) \exp(-\beta x) + S_v(x) = 0 \quad (26)$$

$$-\beta (S_s(x) - S_{s,3}) + S_v(x) = 0 \quad (27)$$

$$\begin{aligned} \text{For } x = 0 & \quad S_v = S_v(0) \\ \text{For } x = \alpha & \quad S_v = 0 \end{aligned}$$

This results in the following expression for β .

$$\beta = S_v(0)/(S_{s,1} - S_{s,3}) \quad (28)$$

- b). Assume that the sedimentation, that is the vertical transport S_v , is linearly proportional to the ratio between the differences between the actual and equilibrium concentration and the original and equilibrium concentration.

Then the following equation will hold:

$$S_v(x) = S_v(0) \left[\frac{(\bar{c}(x) - \bar{c}_3)}{(\bar{c}_1 - \bar{c}_3)} \right] \quad (29)$$

And with $S_s = \bar{c} q$ this equation can be written as

$$S_v(x) = S_v(0) \left[\frac{(S_s(x) - S_{s,3})}{(S_{s,1} - S_{s,3})} \right] \quad (30)$$

Equation (30) is also obtained after substituting eq (28) in eq (27), so both approaches give the same result.

When $S_v(0)$, $c(x)$, \bar{c}_1 and \bar{c}_3 are expressed in the bed concentration the following expressions are obtained.

$$\begin{aligned} S_{v,0} &= w c_{b,1} (1 - h_{*,1}/h_{*,2}) \\ \bar{c}(x) &= (c_b(x)/h_{*,2}) \left[1 - \exp(-h_{*,2}) \right] \\ \bar{c}_1 &= (c_{b,1}/h_{*,1}) \left[1 - \exp(-h_{*,1}) \right] \\ \bar{c}_3 &= (c_{b,3}/h_{*,2}) \left[1 - \exp(-h_{*,2}) \right] \\ c_{b,3} &= \alpha c_{b,1} \end{aligned}$$

$$\begin{aligned} S_v(x) &= w c_{b,1} (1 - h_{*,1}/h_{*,2}) \frac{\bar{c}(x) - \bar{c}_3}{(c_{b,1}/h_{*,1})(1 - \exp(-h_{*,1})) - (\alpha c_{b,1}/h_{*,2})(1 - \exp(-h_{*,2}))} \\ &= w (\bar{c}(x) - \bar{c}_3) \frac{h_{*,1}(h_{*,2} - h_{*,1})}{h_{*,2}(1 - \exp(-h_{*,1})) - \alpha h_{*,1}(1 - \exp(-h_{*,2}))} = E w (\bar{c}(x) - \bar{c}_3) \end{aligned} \quad (31)$$

Equation (27) can be also written as follows:

$$S_v(x) = \beta q (\bar{c}(x) - \bar{c}_3)$$

from which follows that $\beta = E w/q$, (32)
and the equation (25) for $S_3(x)$ reads than

$$(S_{s,1} - S_s(x))/(S_{s,1} - S_{s,3}) = 1 - \exp(-E w x/q) \quad (33)$$

Or also

$$(S_s(x) - S_{s,3})/(S_{s,1} - S_{s,3}) = \exp(-E w x/q) \quad (34)$$

These equations make it possible to compute in a quick and simple manner the sedimentation above the channel or trench.

4. Results

In this chapter two cases will be computed and the results will be compared by those computed with the program "Susleuf" of the Delft Hydraulics Laboratory, which is described by Kerssens, Prins and van Rijn [7].

For only current the following situation has been computed:

$$\begin{aligned} D_{50} &= 0.2 \cdot 10^{-3} \text{ m} & r &= 0.1 \text{ m} & w &= 2.5 \cdot 10^{-2} \text{ m/s} & D_{90} &= 0.3 \cdot 10^{-3} \text{ m} \\ h_1 &= 5 \text{ m} & \bar{u}_1 &= 1 \text{ m/s} & q &= 5 \text{ m}^2/\text{s} & h &= 10 \text{ m} & \bar{v}_2 &= 0.5 \text{ m/s} \\ C_1 &= 50 \text{ m}^{1/2}/\text{s} & C_{D90} &= 95 \text{ m}^{1/2}/\text{s} & \mu &= 0.38 \\ v_{*1} &= 6.3 \cdot 10^{-2} \text{ m/s} & \epsilon_1 &= 0.16 v_{*1} & h_1 &= 5 \cdot 10^{-2} \text{ m}^2/\text{s} & h_{*1} &= w h/\epsilon_1 = 2.5 \\ C_2 &= 55 \text{ m}^{1/2}/\text{s} & C_{D90} &= 101 \text{ m}^{1/2}/\text{s} & \mu &= 0.40 \\ v_{*2} &= 28 \cdot 10^{-2} \text{ m/s} & \epsilon_2 &= 0.16 v_{*2} & h_2 &= 4.5 \cdot 10^{-2} \text{ m}^2/\text{s} & h_{*2} &= 5.56 \end{aligned}$$

With $B = 5$;

$$\begin{aligned} S_{b,1} &= 3.486 \cdot 10^{-5} \text{ m}^2/\text{s} & \text{and } c_{b1} &= 0.88 \cdot 10^{-3} \\ S_{b,3} &= 0.192 \cdot 10^{-5} \text{ m}^2/\text{s} & \text{and } c_{b3} &= 0.106 \cdot 10^{-3} \\ \alpha &= 0.12 & E &= 1.59 \end{aligned}$$

For the Susleuf program the following distribution of the diffusion coefficient has been applied.

$$\begin{aligned} z/h < \frac{1}{2} & \epsilon_1 = v_*^4 z(1-z/h) \left[0.13 + 0.20(w/v_*)^{2.12} \right] \\ z/h \geq \frac{1}{2} & \epsilon_2 = v_*^4 h \left[0.13 + 0.20(w/v_*)^{2.12} \right] \end{aligned}$$

Although this expression for ϵ gives an illogical trend for ϵ from upstream of the channel to above the channel, the most important value of ϵ , namely ϵ_2 is in reasonable agreement with the ϵ applied in the method of this paper.

In the following tabel the values of $(S_{s,1} - S_s(x))/(S_{s,1} - S_{s,3})$ as follow from the Susleuf computation and this method are given.

Deposit of suspended material as fraction maximum possible deposit as function of distance from upstream edge of channel.

x (m)	Susleuf program	This method
25	.22	0.18
50	.37	.33
75	.47	.45
100	.55	.55
125	.62	.63
150	.67	.70
175	.71	.75

The correspondence is reasonable.

For the combination of waves and current the following conditions are applied.

$$D_{50} = 0.2 \cdot 10^{-3} \text{ m} \quad r = 0.05 \text{ m} \quad w = 2.5 \cdot 10^{-2} \text{ m/s} \quad D_{90} = 0.27 \cdot 10^{-3} \text{ m}$$

$$h_1 = 5 \text{ m} \quad \bar{v}_1 = 1 \text{ m/s} \quad q = 5 \text{ m}^2/\text{s} \quad h_2 = 10 \text{ m} \quad \bar{v}_2 = 0.5 \text{ m/s}$$

$$C_1 = 55.4 \text{ m}^{1/2}/\text{s} \quad C_{D90_1} = 96.2 \text{ m}^{1/2}/\text{s} \quad \mu = 0.44$$

$$C_2 = 60.8 \text{ m}^{1/2}/\text{s} \quad C_{D90_2} = 101 \text{ m}^{1/2}/\text{s} \quad \mu = 0.46$$

$$H = 1 \text{ m} \quad T = 6 \text{ s}$$

For the Susleuf program the following distribution for ϵ has been applied this time

$$z < h/2 \quad \epsilon = \kappa \bar{v}_* z(1-z/h)$$

$$z \geq h/2 \quad \epsilon = 0.25 \kappa \bar{v}_* h$$

This results in $\bar{\epsilon} = 0.0883 \bar{v}_* h$ which is about half the value which is suggested by the author. In order to be able to compare the results of this method with that of Susleuf here also this lower value of $\bar{\epsilon}$ will be applied. The various values are given upstream and above the channel.

upstream of the channel
depth 5 m

$$u_o = 0.568 \text{ m/s}$$

$$a_o = 0.542 \text{ m}$$

$$v_{*cw} = 0.093 \text{ m/s}$$

$$C_1 = 55.4 \text{ m}^{1/2}/\text{s}$$

$$C_{D90_1} = 96.2 \text{ m}^{1/2}/\text{s}$$

$$\mu_1 = 0.44$$

$$\bar{\epsilon}_1 = 0.0388 \text{ m}^2/\text{s}$$

above the channel
depth 10 m

$$u_o = 0.309 \text{ m/s}$$

$$a_o = 0.295 \text{ m}$$

$$v_{*cw} = 0.0557 \text{ m/s}$$

$$C_2 = 60.8 \text{ m}^{1/2}/\text{s}$$

$$C_{D90_2} = 101.1 \text{ m}^{1/2}/\text{s}$$

$$\mu_2 = 0.46$$

$$\bar{\epsilon}_2 = 0.0464 \text{ m}^2/\text{s}$$

$$\begin{aligned}
 h_{*1} &= 3.22 & h_{*2} &= 5.39 \\
 S_{b1} &= 4.49 \cdot 10^{-5} \text{ m}^2/\text{s} & S_{b2} &= 1.4 \cdot 10^{-5} \text{ m}^2/\text{s} \\
 c_{b1} &= 2.51 \cdot 10^{-3} & c_{b2} &= 1.71 \cdot 10^{-3} \\
 \alpha &= 0.78 \\
 E &= 2.33
 \end{aligned}$$

The suspended load is computed with the original procedure as suggested by Bijker in 1971 [3] and described also by Van de Graaff and Van Overeem [5].

The following values are then obtained

$$\begin{aligned}
 w/\kappa v_{*wc1} &= 6.72 \cdot 10^{-1} & w/\kappa v_{*wc2} &= 1.12 \\
 S/S_b &= 19 & S/S_b &= 7 \\
 S_{s,1} &= 8.53 \cdot 10^{-4} \text{ m}^2/\text{s} & S_{s,3} &= 0.98 \cdot 10^{-4} \text{ m}^2/\text{s} \\
 S_{s,1} - S_{s,3} &= 7.55 \cdot 10^{-4} \text{ m}^2/\text{s} \\
 S_{tot}(x) &= S_s(x) + S_{b,2}
 \end{aligned}$$

In order to calculate the transport in ton/ms, a porosity of 0.4 and a density of the dry material of 2.65 is assumed.

Deposit of material as fraction of maximum possible deposit as function of distance from upstream edge of channel.

This method			Susleuf		
$S_{tot}(x)$	$\frac{S_{tot}(x)}{S_{tot,1}}$	Fraction of deposit	$S_{tot}(x)$	$\frac{S_{tot}(x)}{S_{tot,1}}$	Fraction of deposit
in ton/m's					
$1.43 \cdot 10^{-3}$	1.00		$1.5 \cdot 10^{-3}$	1.00	
$1.13 \cdot 10^{-3}$.79	.21	$1.20 \cdot 10^{-3}$.75	.25
$0.93 \cdot 10^{-3}$.65	.35	$0.92 \cdot 10^{-3}$.58	.42
$.78 \cdot 10^{-3}$.55	.45	$0.72 \cdot 10^{-3}$.45	.55
$.65 \cdot 10^{-3}$.45	.55	$0.61 \cdot 10^{-3}$.38	.62
$.55 \cdot 10^{-3}$.38	.62	$0.53 \cdot 10^{-3}$.33	.67
$.48 \cdot 10^{-3}$.34	.66	$0.46 \cdot 10^{-3}$.29	.71
$.42 \cdot 10^{-3}$.29	.71	$0.41 \cdot 10^{-3}$.26	.74
$.36 \cdot 10^{-3}$.25	.75	$0.36 \cdot 10^{-3}$.23	.77

The agreement is again reasonable.

In the case the transport is calculated via $S = q \bar{c}$ higher values for the actual transport are obtained. This last approximation is apparently - as was expected - acceptable for the computation of the rate of sedimentation, but not for the actual transports.

Acknowledgement:

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