CHAPTER 81

SAND BED-FORM LENGTHS UNDER OSCILLATORY MOTION

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ABSTRACT

Tests were carried out in the oscillating water tunnel to determine the bed-form lengths of sand ranging from 0.1 mm to 0.55 mm subjected to oscillatory motion of various orbital diameters ranging from 0.15 m to 1.5 m and of various periods ranging from 3 to 15 s. The relationship for the bed-form length is given in the accompanying graph in dimensionless form.

INTRODUCTION

Oscillatory motion over an initially flat bed can, under certain conditions, cause the formation of relatively stable bed-forms of trochoidal shape having a definite "wave-length" from crest to crest of A m and wave-height from crest to trough of A m. The purpose of the tests described herein is to provide more information on the bed-form lengths for fine sands of median diameter D of 0.1 and 0.2 mm and of density $\rho_s$ (water density $\rho$) to compare these results with the results of previous tests. A test bed 15 m long and 0.15 m thick of sand was screeded off flat in the oscillating tunnel and the piston throw set to give a fixed value of orbital diameter A m. A starter from which the bed-forms would grow was created by heaping up a small quantity of the sand approx. 1 cm high across the tunnel normal to the oscillating motion.

This starter was not essential since the bed-forms would grow from any slight discontinuity, especially from the ends of the test section, but made the measurement of growth easier to record.

The oscillatory motion was then imposed on the bed with a very long period T s such that the maximum velocity just above the boundary

$$U_{\text{max}} \approx \frac{\pi A}{T} \text{ m/s}$$

was incapable of creating motion. As the period was slowly decreased striations of negligible height and of about 5 cm wave length appeared, giving a mottled appearance to the bed. Bed-forms do not form until the mobility number

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\[ \frac{u_{\text{max}}^2}{g\left(\frac{\rho_s}{\rho} - 1\right) D} \]

(which is essentially the ratio of drag force to inertia force or Shields' parameter) is greater than 3. Under such conditions the bed-form will grow on either side of the starter, spreading to both ends of the test section, gradually growing in length and height until a stable form is attained. The number of oscillatory cycles, \( n \), to achieve a sensibly constant length is a direct function of the value of the mobility number. The growth of the bed-form length with cycles is as shown on Fig. 1.

\[ \frac{A}{\text{A}} \]

\[ \text{High Mobility} \]

\[ \text{Low Mobility} \]

\[ \eta \]

**FIGURE 1**

When a bed-form has been created and is sensibly stable if the period is reduced, so increasing the mobility number, the bed-form can be wiped out such is the amount of turbulence and material in suspension above the bed.

The author's results have been plotted on Fig. 2, as have the test results of Mogridge and Kamphuis (1) and Lofquist (2) for differing densities and sizes of bed material using the dimensionless group

\[ \frac{\gamma_s D^3}{\rho \nu^2} \]

(proposed by Yalin and Russell (3)) to show why the bed-form length decreases with increasing mobility number for fine sands. Here \( \gamma_s \) is the submerged specific weight and \( \nu \) the kinematic viscosity of water. This group is essentially a fall velocity Reynolds Number.
With regard to the ratio of bed-form "wave height" $\Delta$ to bed-form "wave-length" $A$, this ratio is given by $(\Delta/A)_{\text{max}} = 0.14$ and the crest angle at this maximum steepness is $120^\circ$. These numbers are either remarkable or fortuitous — by water-wave theory for deep-water conditions the maximum water-wave steepness at breaking is 0.14 (i.e. $\pi - 3$) and Stokes' angle is $120^\circ$. Also, using typical soil mechanics figures for the natural angle of repose for loose sand, namely about $30^\circ$, this gives the internal angle of the crest as $(180^\circ - 2 \times 30^\circ) = 120^\circ$. The author leaves the explanation of this remarkable similarity of the two systems to erudite applied mathematicians such as Professor Longuet-Higgins.

In trying to relate this work to the extremely difficult task of modelling sea-bottom and beach processes one has firstly to keep in mind Fig. 1 where it is seen that the process is very time dependent. A 10 s prototype wave at a high mobility number will produce bed-forms in many less cycles than a 1 s model wave at perforce a smaller mobility number. Secondly, the model mobility number must be at least 3 to achieve any chance of bed-form formation — and hence material movement during the cycle. Lastly, for small sizes of sand (0.1 to 0.2 mm) which are very common on the world's best recreation beaches, a high prototype mobility number may result in no bed-form whatsoever and only massive amounts of sand in suspension above the bed which are easily moved by littoral or other currents. This may be impossible to reproduce in a model at lower mobility numbers unless one uses extremely lightweight material since the use of smaller model sizes of cohesionless material is not feasible. It is impossible for the same mobility number, by using say lightweight material in the model, to simultaneously have the same fall velocity Reynolds Number. Thus strict modelling of the motion of small size sand bed-forms under wave action is virtually impossible.

REFERENCES

