Relationship Between Alongshore Wave Energy and Littoral Drift in the Mid-West Coast at Taiwan

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ABSTRACT

Based on the wave pattern, the geographical location and the disposition of rivers, the littoral drift moves predominantly from NE to SW direction in section II as shown in Fig. 1. Seven rivers of rapid stream bring tremendous amount of sediments from the high mountain to the nearshore of this section in typhoon season (i.e. from June to September). But for the winter monsoon season, i.e. from October to the next April, the waves induced by NE monsoons migrate littoral drift from North toward South.

Applying the energy approach for unidirectional steady flow derived by Bagnold(1963), the theoretical relationship between the littoral immersed weight transport rate and the alongshore breaking wave energy is found out. It reveals that the relationship is not strictly linear, i.e. the larger part of the alongshore breaking wave energy is supplied for transporting the sediment as the former increases. But for a coast having a steady oceanographical condition, the relationship could be considered as linear relation since the alongshore breaking wave energy is not varying very much.

In this paper, the study of littoral drift vs wave energy at the Taichung Coast from the Ta-Chia River to the Ta-Tu River will be carried out. Using the wave records gained by the ultrasonic wave gauge at 19m depth and the littoral drift quantity obtained from long-term observation, the relationship between alongshore breaking wave energy and littoral immersed weight transport rate is found out.

First, the waves which have the same direction are summed up. Then from "THE WAVE CHARACTER COMPUTING PROGRAM", the incident directions of these wave groups at 19m depth are determined. Then the alongshore breaking wave energy per unit time per unit length of beach could be calculated by the same PROGRAM. Finally the relationship between alongshore breaking wave energy and littoral immersed weight transport rate of this coastal condition is obtained as $I_l = 0.55\ (P_l)_b$

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1. Introduction

The littoral transport rate is defined as the transport rate passing through a cross section which is perpendicular to the shoreline or contour line and the range of movement is considered between the threshold depth of sediment and the height of wave run-up.

To evaluate the littoral transport rate are classified into four different methods as follows:

(1). Using the littoral transport rate of neighbor coast, and considering the local conditions such as the shoreline configuration, the grain size and the source of sand, then modifying the neighbor littoral transport rate as the interest littoral transport rate.

(2). Using the nearshore bathmetry to evaluate the volume change of the total sediment along the shore, and estimate the littoral transport rate by groin or spit activity.

(3). Using the longshore current to estimate the littoral transport rate.

(4). Using the alongshore breaking wave energy to compute the littoral transport rate.

Galvin(1972) derived the formula $Q_l = 2H^2$, where Q_l in unit 10^5 yd^3/year and H(ft) is the mean breaking wave heigh of one year. But the formula is good only for suspended load due to its hypothesis. In fact, the quantity of the bed load is much larger than that of the suspended load.

Method (1) is more practical than the others, but the data of littoral transport rate are so few, therefore, the other methods are need to try. If the study area has not groin, breakwater or spit to block completely the littoral transport, then the littoral transport rate is not easily to

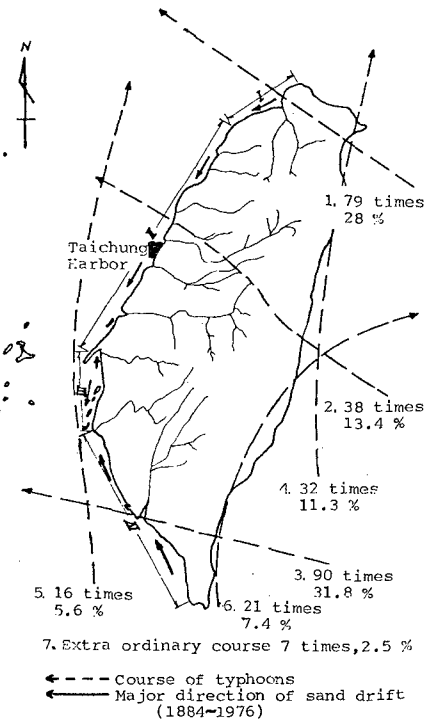

Figure 1. The sketch of the prevailing direction of littoral drift at Taiwan sandy coast.

estimate. The alongshore current is changing from place to place, from time to time, it is very difficult to find out a dominate value of alongshore current. Therefore , it is more often to use the alongshore breaking wave energy to estimate the littoral transport rate. If there is no wave records then the wave characters could be predicted from wind records.

Bagnold (1963) use the littoral immersed weight transport rate I_l to represent the littoral transport rate, where I_l has the same unit with the alongshore breaking wave energy $(P_l)_b$.

2.The theoretical consideration

2.1.The relationshep between the bed load and the available power of fluid under wave action

Under wave action, the forces acting on the sand element (i.e. the free body m_b of containing suffucient sand particles) include the total driving force of fluid T_F, the tangential reactive force T_l due to collisions, the normal reactive force P_l due to collisions and its own gravity. T_F includes drag force, lift force and the presure gradient stress. T_l and P_l is defined as shown in Fig.2,where $\tan\phi' = T_l/P_l$ is the friction coefficient of solid phase. ϕ' is almost equal to the repose angle of sand, ϕ. Then

$$T_l = P_l \tan\phi \qquad (1)$$

As shown in Fig.3

$$T_l = \left[\left(\frac{\rho_s-\rho}{\rho_s}\right) g\, m_b \cos\alpha_B - T_F \sin\beta\right] \tan\phi$$

where β is the angle between T_F and the bed. Since the sand moves up and down. From Newton's second law, i.e.

$$\left[\frac{(\rho_s-\rho)+\rho C_M}{\rho_s}\right] m_b \frac{du_{bb}}{dt} = T_{FU}\cos\beta - \left(\frac{\rho_s-\rho}{\rho_s}\right) g m_b \sin\alpha_B$$

$$- \left[\left(\frac{\rho_s-\rho}{\rho_s}\right) g m_b \cos\alpha_B - T_{FU}\sin\beta\right] \tan\phi$$

and

$$T_{FU} = \left(\frac{\rho_s-\rho}{\rho_s}\right) g m_b \left\{\frac{1}{\cos\beta+\sin\beta\tan\phi}\left[\frac{1}{g}\left(1+C_M\frac{\rho}{\rho_s-\rho}\right)\right.\right.$$

$$\left.\left.\frac{du_{bb}}{dt} + \sin\alpha_B + \cos\alpha_B \tan\phi\right]\right\} \qquad (2)$$

Similarly

$$T_{FD} = \left(\frac{\rho_s-\rho}{\rho_s}\right) g\, m_b \left\{\frac{1}{\cos\beta+\sin\beta\tan\phi}\left[\frac{1}{g}\left(1+C_M\frac{\rho}{\rho_s-\rho}\right)\right.\right.$$

$$\left.\left.\frac{du_{bb}}{dt} - \sin\alpha_B + \cos\alpha_B \tan\phi\right]\right\} \qquad (3)$$

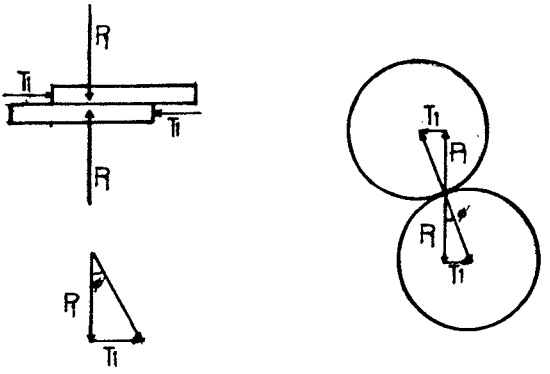

Figure 2. The diagram of definition of frictional coefficient in solid phase.

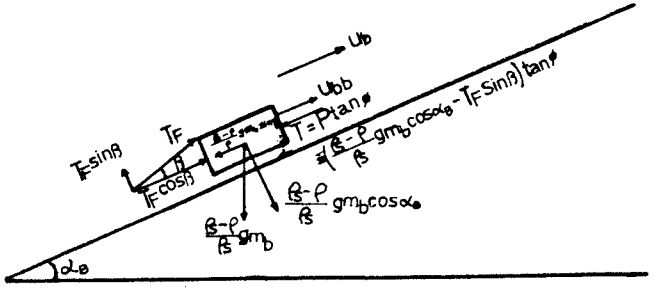

Figure 3. The force system acting on the bed load.

where T_{FU} is the upward driving force, T_{FD} is the downward driving force, u_{bb} is the velocity of sand and C_M is the virtual mass coefficient, while the volume concentration of sand is C_v.

$$C_M = \frac{1}{2}\left(1 + 1.59\,\frac{C_v}{\pi}\right) \qquad (4)$$

Then, as the sand element move up and down, the work done by fluid is

$$|T_{FU}\cos\beta|\,|(u_{bb})_U| + |T_{FD}\cos\beta|\,|(u_{bb})_D|$$

$$= \frac{\rho_s-\rho}{\rho_s}\,g m_b u_{bb}\left\{\frac{2\cos\beta}{\cos\beta+\sin\beta\tan\phi}\left[\frac{1}{g}\left(1+C_M\frac{\rho}{\rho_s-\rho}\right)\frac{du_{bb}}{dt}+\cos\alpha_B\tan\phi\right]\right\} \qquad (5)$$

where $(u_{bb})_U$, $(u_{bb})_D$ represent the upward and downward velocity of sand element respectively. Part of the force in Eq.(5) brings the sand away from the original place, i.e. to cause the net sediment transport, which is

$$\left(\frac{\rho_s-\rho}{\rho_s}\right) g m_b \bar{u}_{bb}\left\{\frac{\cos\beta}{\cos\beta+\sin\beta\tan\phi}\left[\frac{1}{g}\left(1+C_M\frac{\rho}{\rho_s-\rho}\right)\frac{du_{bb}}{dt}+\sin\alpha_B+\cos\alpha_B\tan\phi\right]\right\} \qquad (6)$$

$\bar{u}_{bb}$ is the net velocity of bed load. Set

$$i_b = \left(\frac{\rho_s-\rho}{\rho_s}\right) g\, m_b\, \bar{u}_{bb} \qquad (7)$$

where i_b is the immersed weight transport rate. If ω is the available power of fluid, ε_b is the effective coefficient which is used to cause bed load. Then from eq.(6) and eq.(7), the relationship between the immersed weight transport rate of the bed load and the available power of fluid is as

$$i_b\left\{\frac{\cos\beta}{\cos\beta+\sin\beta\tan\phi}\left[\frac{1}{g}\left(1+C_M\frac{\rho}{\rho_s-\rho}\right)\frac{du_{bb}}{dt}+\sin\alpha_B+\cos\alpha_B\tan\phi\right]\right\} = \varepsilon_b\omega \qquad (8)$$

Assume the sediment transport has completely developed, and the resultant force acts normal to the bed is in eguilibrium, i.e. $P_t = 0$. Then

$$T_F\sin\beta = \left(\frac{\rho_s-\rho}{\rho_s}\right) g\, m_b\cos\alpha_B$$

$$T_F = \frac{1}{\sin\beta}\left(\frac{\rho_s-\rho}{\rho_s}\right) g m_b\cos\alpha_B \qquad (9)$$

The work done by fluid to take sand element move up and down is

$$2\,T_F\cos\beta\,u_{bb} = 2\,\frac{\cos\alpha_B}{\tan\beta}\left(\frac{\rho_s-\rho}{\rho_s}\right) g\,m_b\,u_{bb} \quad \cdots\cdots(10)$$

But cause the net sediment transport is only equal to

$$T_F\cos\beta\,\bar{u}_{bb} = \frac{\cos\alpha_B}{\tan\beta}\left(\frac{\rho_s-\rho}{\rho_s}\right) g\,m_b\,\bar{u}_{bb} = \varepsilon_b\omega$$

or

$$i_b\,\frac{\cos\alpha_B}{\tan\beta} = \varepsilon_b\omega \quad \cdots\cdots(11)$$

It simply represents the relationship between the immersed weight transport rate of bed load and the available power of the fluid. Since T_F might change periodically, the equilibrium state of the assumption is existing temporally and the sediment move up and down occasionally. But when the sediment transport has completely developed, the particles move up and down is considered as the same rate. Therefore, eq.(11) is quite reasonable.

2.2.The relationship between the suspended load and the available wave power

Since the suspended load is away from bed, its wall effect is less than that of bed load. The transport of suspended load is mainly caused by the mass transport due to nonlinear waves and the alongshore current. The motion of suspended load is considered as the resultant of the horizontal velocity u_s and the settling velocity w. The collision effect between suspended particles is ignored since the concentration of suspended load is low. The only external force on the suspended load is the driving force of fluid including the drag force, the lift force and the pressure gradient stress. It is shown in Fig.4. and yields

$$T'_F\cos\beta' = \left(\frac{\rho_s-\rho}{\rho_s} + C_M\right.$$

$$\left.\frac{\rho}{\rho_s}\right) m_s\,\frac{du_s}{dt} \quad \cdots\cdots(12)$$

where m_s indicates suspended load while u_s is its horizontal velocity, T'_F is the driving force of the fluid, β' is the angle of T'_F from horizontal direction. Fluid transports the suspended load by doing two actions. One is to make m_s move in the horizontal direction, the other is to keep m_s from falling with settling velo-

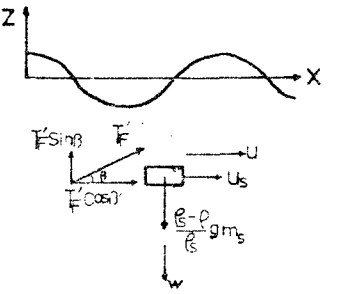

Fig.4 The force system on suspended load.

city w. Therefore, the power worked by fluid to transport the suspended load with net transport velocity $\bar{u}_s$ is

$$[T'_F \cos\beta'] \bar{u}_s + [T'_F \sin\beta'] w = \varepsilon_s (1-\varepsilon_b)\omega \quad \cdots(13)$$

If the sediment transport is fully developed, the suspended load may be considered statistically as move in the horizontal direction. Then

$$T'_F \sin\beta' = \left(\frac{\rho_s - \rho}{\rho_s}\right) g\, m_s \quad \cdots\cdots(14)$$

and

$$T'_F \cos\beta' = \left(\frac{\rho_s - \rho}{\rho_s}\right) g\, m_s \frac{1}{\tan\beta'} \quad \cdots\cdots(15)$$

From eq.(13), (14) and (15), then

$$\left(\frac{\rho_s - \rho}{\rho_s}\right) g m_s \bar{u}_s \left[\frac{1}{\tan\beta'} + \frac{w}{\bar{u}_s}\right] = \varepsilon_s (1-\varepsilon_b)\omega \quad \cdots\cdots(16)$$

is obtained, where is the suspension efficiency, and next by

$$i_s = \left(\frac{\rho_s - \rho}{\rho_s}\right) g m_s \bar{u}_s \quad \cdots\cdots(17)$$

as the immersed weight transport rate of suspended load. Therefore,

$$i_s \left[\frac{1}{\tan\beta'} + \frac{w}{\bar{u}_s}\right] = \varepsilon_s (1-\varepsilon_b)\omega \quad \cdots\cdots(18)$$

is the relationship between the suspended load and the available power of the fluid as the sediment transport is fully developed.

The total transport rate i is expressed as

$$i = i_b + i_s \quad \cdots\cdots(19)$$

$$i = \left[\frac{\varepsilon_b}{\cos\alpha_B / \tan\beta} + \frac{\varepsilon_s (1-\varepsilon_b)}{1/\tan\beta' + w/\bar{u}_s}\right] \omega \quad \cdots\cdots(20)$$

Eq.(20) is derived from eq.(11) and (18). Let K be as follows.

$$K = \frac{\varepsilon_b}{\cos\alpha_B / \tan\beta} + \frac{\varepsilon_s (1-\varepsilon_b)}{1/\tan\beta' + w/\bar{u}_s} \quad \cdots\cdots(21)$$

Eq.(20) represents the relationship between the immersed weight transport rate and the available power of the fluid. Because the action between sand bed and water mass is different as breaking wave type is changed, ε_b and ε_s will be affected by the breaking wave parameters. It is believed that the plunging wave causes seriously erosion than that of the other type of breaking wave. The settling velocity increases as grain size increases, therefore K will decrease as sand grain size increases. It is shown in eq.(21). Besides that, K becomes large as the net transport rate of suspended load increases.

Therefore, the littoral transport will be strengthen in the direction of alongshore current if there is a permanent or semi-permanent alongshore current. The parameter β(orβ') reveals the magnitude of fluid acting on moving sediment, it grows up as the vertical fluctuation increases. Therefore, it will be larger in the shallower water or under the action of the bigger waves. And the K value will be larger in the shallower water and for the heigher wave energy. The above reasons show that the K value is not always a constant. But, for the long-term climate, if the weather is steady at the same coast, the K value could be considered as a constant.

2.3.The definition of the available wave power

By Using the Airy wave theory, the energy density of a wave with wave height H is E

$$E = 1/8\ \rho g H^2 \quad \cdots\cdots (22)$$

The power P transmitted between wave rays with distance Δb is

$$P = E \cdot C_g \cdot \Delta b \quad \cdots\cdots (23)$$

where C_g is group velocity, as shown in Fig.5, the angle between contour line and wave crest line is α and the power transmitted in the longshore direction is

$$P\sin\alpha = E \cdot C_g \cdot \Delta b \sin\alpha$$

where Δb is in term of the coast length Δl as

$$\Delta b = \Delta l \cdot \cos\alpha$$

Then, the alongshore wave energy per unit length of the coast is

$$P_l = E \cdot C_g \cdot \sin\alpha \cos\alpha = \frac{1}{8}\rho g H^2 \cdot C_g \cdot \sin\alpha \cdot \cos\alpha \quad \cdots\cdots (24)$$

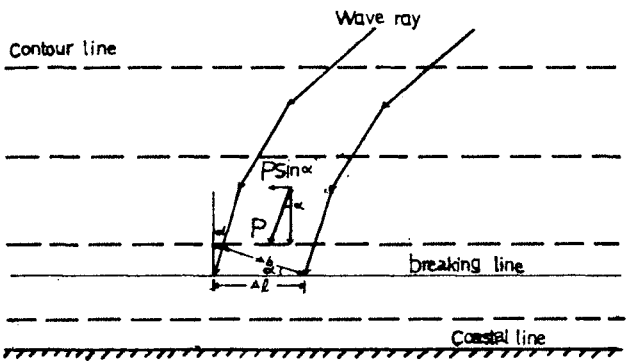

Fig.5. The diagram of the alongshore wave energy.

Eq.(24) is called the available alongshore wave power for transporting the coast sediment.

3.The experimental investifation

3.1.The estimate of littoral transport

At a straight sandy coast of infinite length, if characteristics of waves and characteristics of shore are kept the same along the straight sandy coast, then there is neither erosion nor accumulation. Because the movement of sediment transport is continuous. To estimate the littoral transport rate from the nearshore bathmetry, the groin or breakwater or spit is necessary installed to block the littoral transport. Therefore, the accumulation in the upstream area is the total littoral transport of this sandy coast.

The littoral transport rate of the Taichung coast is illustrated as follows.

There are the Ta-An and the Ta-Cha rivers in the north side of Taichung Harbor, while the Ta-Tu river is in the south side. In the typhoon season (From May to Oct.), Ta-Cha river bring about 1.13×10^6 m³ per year and Ta-Tu river bring about 1.20×10^6 m³/yr into the coast. The orientation of the coastline from the Ta-Tu river to the Ta-Cha river is N 21.5 E and that of the Ta-Cha river to the Ta-An river is in the direction of N 42.5 E. This means that the neighborhood of the mouth of Ta-Cha river act as a spit which block a part of the littoral transport.

The bottom slope of depth contour from -0 m to -20 m is 1/75 in the neighblrhood of the mouth of the Ta-Cha river and the slope of the Ta-Tu esturay is 1/100; the bottom slope from -20 m to -50 m is 1/80 in the mouth of the Ta-Cha ruver and it is 1/20 in the mouth of the Ta-Tu river. The width of the inter-tidal zone becomes wider from the Ta-Cha river to the Ta-Tu river. It reveals that the injecting flow of Ta-Tu river blocks the littoral transport and carries sediments toward offshore.

From the wind records, during winter season, wind blows from N to NE direction which is the predominant wind and occurs 80 % per year, and the speed is often beyond 10 m/sec. The induced wind wave is mainly H=1 ~ 2 m, T=6 ~ 6.4 sec. In summer, wind which blow in S to WSW direction occur 13 % Per year, and its speed is always below 5 m/sec unless in typoon. The induced wind wave is alway less than 0.7 m. Therefore, the littoral drift will migrate from north to south. The plot of the mean grain size distribution of the sediment confirms the result, as shown in Fig.6.

From the continuous bathmetry, using the mesh method, the volume change of the total sediment in the study area is found out as

$$\triangle V = \frac{A}{4} \left[\sum h_1 + 2 \sum h_2 + 3 \sum h_3 + 4 \sum h_4 \right] \cdots\cdots\cdots (25)$$

where A is the grid area and h_i is the fluctuation of water depth at the point co-related to the i number of grids, as shown in Fig.7.

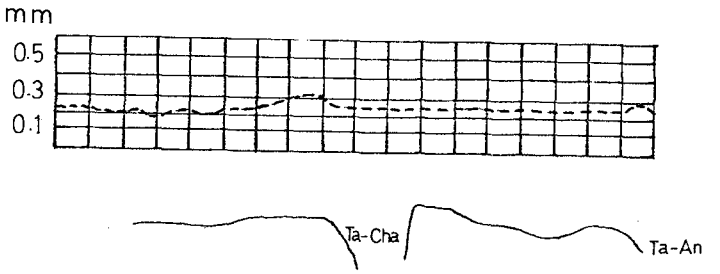

Fig.6. The distribution of the mean grain size.

After 1975, the north breakwater of Taichung Harbor has extended to the water depth of -5m, which is usually deeper than that of wave breaking,the north breakwater is completed at the October of 1976, the head of the north breakwater reached to the depth of -20m, as shown in the Table1.

From Fig.8, it is found out that sea bottom fluctuated seriously before the head of the north breakwater reached -20m. This means that the littoral drift still passes through the north breakwater and moves southward before the time of July 1976. After the length of the north breakwater is prolonged, the area effected by the moving sediment to the place about 6Km south from south breakwater, which is always the change of erosion and accumulation, the littoral transport rate of the study area is equal to the accumulation between the Ta-Chia river and the place which is 6Km south from the south breakwater before July, 1976. The littoral transport rate is equal to the accumlated sand volume of the north side of the north breakwater after July,1976.

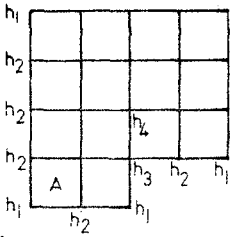

Fig. 7. The plot of the meth method.

Table 1. The progress of the north groin and the north breakwater of Taichung Harbor.

Date		4, 1974	10,1974	10,1975	10,1976	10,1977
north breakwater	progress	+ 0 m	+ 142 m	+ 479 m	+ 462 m	+ 310 m
	the head's depth	± 0 m	− 3 m	− 5 m	− 10 m	− 20 m
north groin	progress	+ 0 m	+ 319 m	+ 6 m	+ 287 m	+ 200 m
	the head's depth	± 0.4 m	− 0.8 m	− 2 m	− 4 m	− 6 m

For computing the sand volume change rate by the mesh method, the yearly littoral drift of this area is shown as follows.

1971, 6 - 1972, 7	:	1,340,000	m/yr
1972, 7 - 1973, 7	:	1,636,000	"
1973, 7 - 1974, 7	:	1,458,000	"
1974, 7 - 1975, 7	:	1,406,000	"
1975, 7 - 1976, 7	:	1,386,000	"
1976, 7 - 1977, 8	:	2,028,000	"
1977, 8 - 1978, 8	:	1,313,000	"
1978, 8 - 1979, 8	:	1,307,000	"

(26)

By representing the above results as the immersed weight of littoral transport rate, and using the formula

$$I_l = (\rho_s - \rho)\, g a' Q_l \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (27)$$

then the quantity of I_l is, $(a' = 0.6)$

1971, 6 - 1972, 7	:	1.277 x 10	Kg m /sec/m-yr
1972, 7 - 1973, 7	:	1.588 x 10	"
1973, 7 - 1974, 7	:	1.389 x 10	"
1974, 7 - 1975, 7	:	1.339 x 10	"
1975, 7 - 1976, 7	:	1.320 x 10	"
1976, 7 - 1977, 8	:	1.929 x 10	"
1977, 8 - 1978, 8	:	1.251 x 10	"
1978, 8 - 1979, 8	:	1.245 x 10	"

(28)

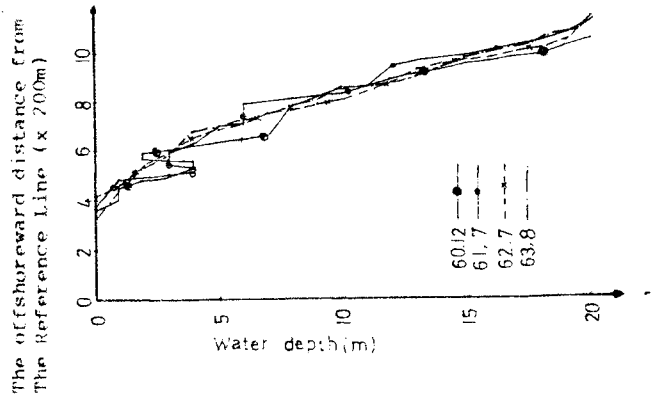

Figure 8.1. The change of the bottom slope at the north side of the north breakwater.

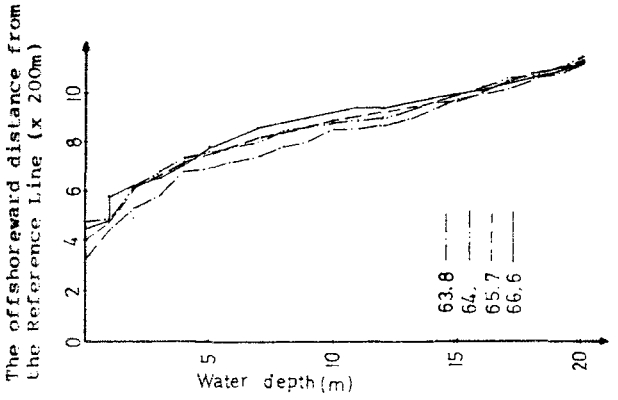

Figure 8.2. The change of the bottom slope at the north side of the north breakwater.

3.2 The calculation of the alongshore breaking wave energy :

The wave data are recorded by the ultrasonic wave gauge at the depth -19m per 2 hours from July 1972 to June 1976.

The wave is affected by the effects of shoaling, refraction, bottom friction and percolation. By neglecting the effects of bottom friction and percolation as wave is propagating toward the surf zone, there the breaking wave height is equal to

$$H = H_o K_r K_s$$

where K_r and K_s are refraction coefficient and shoaling coefficient respectively. The mathematical representation is as follows

$$k_r = (B_o / B)^{1/2}$$

$$k_s = (C_{g_o} / C_g)^{1/2}$$

where B is the separation of the wave rays and prefix "o" represents the characters of deep water. As the bottom slope is smaller than 1/10, then K_r and K_s could be calculated from the four equations derived by Chao,Y.Y.(1970) as follows

$$C^2 = \left(\frac{g}{k}\right) \tanh kh$$

$$\frac{d\theta}{ds} = \frac{1}{c}\left(\sin\theta \frac{\partial c}{\partial x} - \cos\theta \frac{\partial c}{\partial y}\right) = -\frac{1}{c}\frac{dc}{dB}$$

$$H_o (C_g)_o B_o = H^2 \cdot C_g \cdot B = \text{constant}$$

$$\frac{d^2B}{ds^2} - P^{(1)} \frac{dB}{ds} + P^{(2)} B = 0$$

$$P^{(1)}=\frac{1}{c}\left(\cos\theta\frac{\partial c}{dx}+\sin\theta\frac{\partial c}{\partial y}\right)$$

$$P^{(2)}=\frac{1}{c}\left(\sin^2\theta\frac{\partial^2 c}{\partial x^2}-2\sin\theta\cos\theta\frac{\partial^2 c}{\partial x\partial y}+\cos^2\theta\frac{\partial^2 c}{\partial y^2}\right)$$

Where D is water depth, θ is the angle between X axis and wave direction, S is the distance along wave ray and C is the phase velocity. Use numerical method to get

$$D_{n+1}=D_n+\left(\frac{\partial D}{\partial x}\right)_n dx+\left(\frac{\partial D}{\partial y}\right)_n dy+\frac{1}{2}\left(\frac{\partial^2 D}{\partial x\partial y}\right)_n dx\,dx$$
$$+\left(\frac{\partial^2 D}{\partial x\partial y}\right)_n dx\,dy+\frac{1}{2}\left(\frac{\partial^2 D}{\partial x\partial y}\right)_n dy\,dy$$

$$P_{n+1}=[(4-2P_n^{(2)}\Delta s^2)/(2-P_n^{(1)}\Delta s)]B_n-[(2+P_n^{(1)}\Delta s)/(2-P_n^{(1)}\Delta s)]B_{n-1}$$

The subscript "n" represents the value of the nth calculation , as shown in Fig.9, where X axis is taken parallel to the shoreline. Developing these numerical calculations to get "THE WAVE CHARACTER COMPUTING PROGRAM", the wave characters such as K_r, K_s, H, C_g and θ could be found out at any water depth h=D. Since $\theta=\frac{\pi}{2}-\alpha$ as shown in Fig.9, then

$$(P_l)_b=\frac{1}{8}\rho g H_b^2(c_g)_b\cos\theta_b\sin\theta_b$$

is the alongshore breaking wave energy.

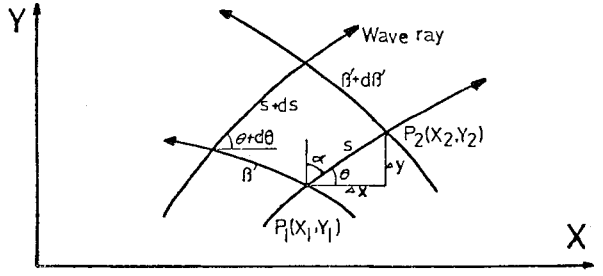

Figure 9.1. The diagram of the wave refraction.

Since there is no records of wave direction at -19m depth, the incident wave direction is found out by the above program using the deep water incident wave direction which is the wind direction as the wave gauge serves.

Consider only the waves which are moving onshore could cause littoral transporting. The waves which is the same incident direction is summed up. The root mean square value of their heights and the mean value of their periods are found out. Therefore, the total onshore acting waves are compiled to 8 equivalent waves which have the "rms" wave height H_{rms}, mean wave period $\overline{T}$. This is because

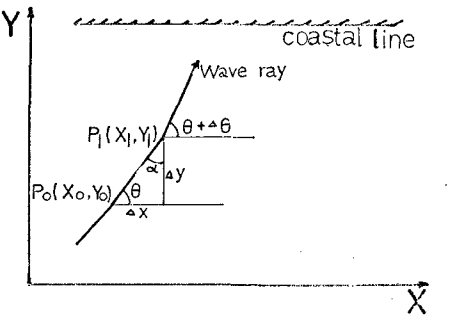

Figure 9.2. The diagram of the wave refraction.

$$(P_l)_b \propto H^2 \cos\theta \sin\theta \propto H^2 \sin 2\theta$$

$$\sum_{i=1}^{M} (P_l)_{bi} \propto \left(\sum_{i=1}^{M} H_i^2\right) \sin 2\theta = M H_{rms}^2 \sin 2\theta$$

Where M_j is the number of the waves which have the same wave direction θ_j, and H_{rms} is represented as

$$H_{rms}^2 = \frac{1}{M} \sum_{i=1}^{M} H_i^2$$

To compute the alongshore breaking wave energy, the input data including the water depths of the grid points, the water depths and the coordinates of the incident points, the H_{rms} values and the $\overline{T}$ values of the equivalent waves with the incident wave direction θ at the depth -19m, are all considered.

The output results are $(SUM)_j$ which is the summation of the alongshore breaking wave energy of MN wave rays. Then the total alongshore breaking wave energy per unit beach length of one year is expressed as

$$(P_l)_b = \frac{7200}{MN} \sum_{j=1}^{8} [M_j (SUM)_j]$$

where 7200 sec is the 2 hr wave acting duration.

The result of each year from July 1972 to June 1976 is shown as the Table 2,3,4 and 5.

3.3 The relationship between I_l and $(P_l)_b$

Equation (21) shown that the relationship between I_l and $(P_l)_b$ is not exact linearly proportional to each other, this is proved by Fig.10. which is originally prepared by Komar and Inman (1970) there is a upper limit K=0.77. Adding the data of equation(28) and Table 2 - 5 into Fig. 10, re-analyze the total data by the least square method to get a regression line which is expressed as

$$I_l = 0.154 (P_l)_b^{1.0695}$$

Table 2. The resulting wave data and the alongshore breaking wave energy during July 1, 1972 to June 30, 1973.

MN=19	NNE	N	NNW	NW	WNW	W	WSW	SW
M_j	1802	657	248	120	76	86	123	502
H_{rms}	1.62	1.23	0.80	0.74	0.79	0.57	0.62	0.74
$\overline{T}$	5.6	5.4	5.2	5.0	5.0	5.1	4.8	5.1
θ_j at - 19.m	14.5°	35.6°	57.5°	80°	102.5°	125°	147.5°	170°
$(SUM)_j \times 10^4$	4.67	2.26	0.74	0.05	-1.06	-1.08	-0.77	-0.82
$(P_l)_b$	3.59×10^{10} kg m²/ sec²/m-yr							

Table 3. The resulting wave data and the alongshore breaking wave energy during July1, 1973 to June 30, 1974.

MN= 11	NNE	N	NNW	NW	WNW	W	WSW	SW
M_j	1830	682	226	159	153	126	194	234
H_{rms}	1.45	1.22	0.93	0.90	0.85	0.82	0.8	0.75
$\overline{T}$	6.3	5.7	5.5	5.2	5.2	4.5	4.6	4.8
θ_j at - 19.m	16°	35.6°	57.5°	80°	102.5°	125°	147.5°	170°
$(SUM)_j \times 10^4$	1.64	1.12	0.47	0.08	-0.91	-0.59	-0.91	-0.35
$(P_l)_b$	2.24×10^{10} kg m²/ sec²/m-yr							

Table 4. The resulting wave data and the alongshore breaking wave energy during July 1, 1974 to June 30, 1975.

MN=11	NNE	N	NNW	NW	WNW	W	WSW	SW
M_j	1021	617	105	61	57	71	79	127
H_{rms}	1.77	1.6	0.8	0.68	0.54	0.4	0.52	0.53
$\overline{T}$	6.1	5.9	5.0	5.6	5.3	4.0	4.2	4.4
θ_j at - 19.m	16°	35.6°	57.5°	80°	102.5°	125°	147.5°	170°
$(SUM)_j \times 10^4$	2.79	1.74	0.56	-0.03	-0.08	-0.17	-0.24	-0.21
$(P_l)_b$	2.56×10^{10} Kg m²/ sec²/m-yr.							

Table 5. The resulting wave data and the alongshore breaking wave energy during July 1, 1975 to June 30, 1976.

MN=11	NNE	N	NNW	NW	WNW	W	WSW	SW
M_j	1375	455	120	97	83	76	106	196
H_{rms}	1.52	1.33	0.95	0.71	0.4	0.38	0.48	0.56
$\bar{T}$	5.8	5.5	5.5	5.2	5.3	5.2	5.3	5.4
θ_j at -19 m	14.5°	35.6°	57.5°	80°	102.5°	125°	147.5°	170°
$(SUM)_j \times 10^4$	1.67	1.82	0.89	0.6	-0.07	-0.12	-0.24	-0.17
$(P_l)_b$	2.11×10^{10} Kg m²/ sec²/ m–yr							

where I_l and $(P_l)_b$ are in units of cgs system.

But for the Taichung coast where the climate is so steady that the alongshore breaking wave energy fluctuate slightly, the relationship between I_l and $(P_l)_b$ could be expressed as

$$I_l = K(P_l)_b$$

where K is constant and is egual to 0.55. The equation is suitable for any unit system.

4. Conclusion and Discussion

Applying the energy approach for unidirectional steady flow (Bagnold, 1963), derive out the relationship between the alongshore breaking wave energy and the littoral immered weight transport rate as $I_l = K(P_l)_b$. K is function of wave height, bottom slope, the grain size and the sediment transport pattern. It increases as the grain size decreases or it does either there exists an ocean current in the predominant littoral transport direction or under the action of the bigger waves. This reveals that the larger part of wave energy is supplied to transport sediment as the wave energy becomes larger. This is shown by the empirical relationship $I_l = 0.154 (P_l)_b^{1.0605}$. But for a coast, such as the Taichung coast, where the oceanographic condition is so steady that the alongshore breaking wave energy fluctuates slightly, the relationship between I_l and $(P_l)_b$ could be written as $I_l = K(P_l)_b$, where K is constant. Then the Taichung coast has the relation of $I_l = 0.55 (P_l)_b$. This eguation could be applied for the coast of similar oceanographic conditions and beach characteristics to estimate the littoral transport rate. Such that the harbor planning and the shore protection could be based on.

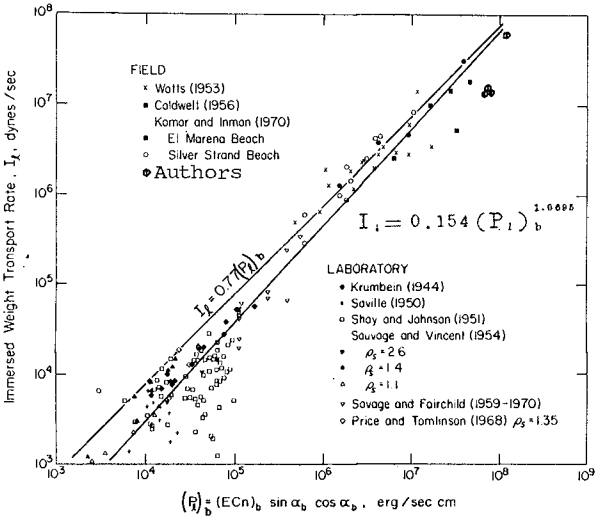

Figure 10. The relationship of the alongshore breaking wave energy and the immersed weight transport rate.

5.Acknowledge

The authors are grateful to the Taichung Harbor Bureau, Taiwan, R.O.C. for the observational wave records. Also the appreciation is given to Mr. Sen-Ho Chen, M.S., Harbor and Oceanic Engineering Institute, National Taiwan College of Marine Science and Technology, for his assistance to the computer program, and working personnel of The Computer Center at N.T.C.M.S.T.

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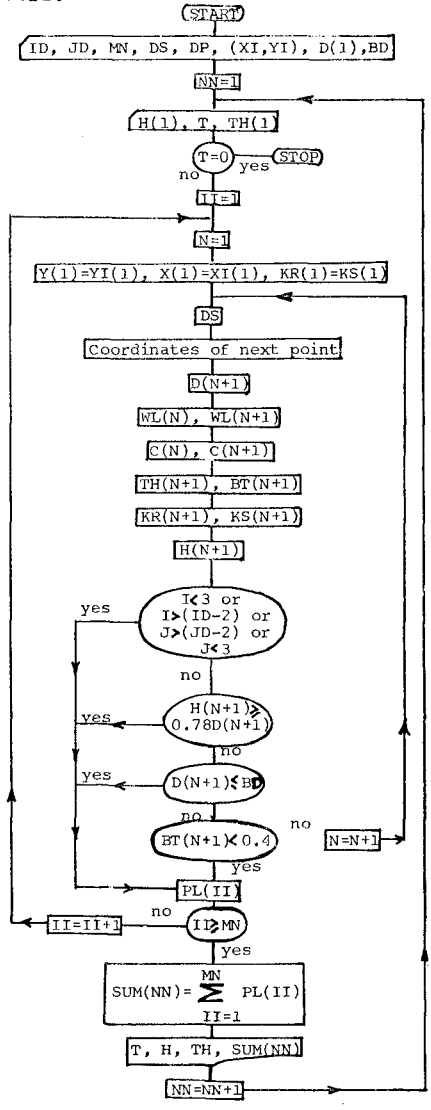
Appendix: Flow Chart of THE PROGRAM OF COMPUTING WAVE CHARACTORS.



List of Notations

ID,JD: number of the grids for x-axis, y-axis, respectively
MN : number of the incident wave rays
DP : water depth of the grid point
XI,YI: the initial coordinates of x and y axis respectively
X,Y : the coordinates of the position for calculating wave character
D : water depth
BD : water depth to output
BT : the separation of wave direction ray
NN : the index of waves
H : wave height
T : wave period
WL : wave length
C : celerity
CG : wave group velocity
PL : wave energy of each incident wave ray
SUM : the total wave energy of all beach for each wave
TH : the angle between x axis and the wave direction
II : the order of incident wave rays
N : the index of calculating step
KR,KS: the refraction coefficient and shoaling coefficient respectively