

## CHAPTER 50

### INTERACTIONS OF WAVES WITH SUBMARINE TRENCHES

by Jiin-Jen Lee,<sup>1</sup> Robert M. Ayer,<sup>2</sup> Wen-Li Chiang<sup>3</sup>

#### ABSTRACT

An analysis is presented for the propagation of water waves past a submarine trench of irregular shape. Two dimensional, linearized potential flow is assumed. The fluid domain is divided into two regions along the mouth of the trench. Solutions in each region are expressed in terms of the unknown normal derivative of the potential function along this common boundary with the final solution obtained by matching. Reflection and transmission coefficients are found for various submarine geometries. The accuracy of the technique employed is demonstrated by comparing with previously published results for a rectangular trench. In addition, results from limited laboratory experiments were included for comparison. The result shows that for a particular flow configuration, there exists an infinite number of discrete wave frequencies at which waves are completely transmitted.

#### INTRODUCTION

A class of problems involving the propagation of water waves in a fluid of variable depth is one in which the depth is constant except for variations over a finite interval. Interest in these problems is largely due to the phenomena associated with the passage of waves over submarine trenches in the ocean and wave propagation across navigational channels, where changes in water depth are commonly the case. A common method employed in the solution of problems involving changes in water depth is that of matching the solution along a geometrical boundary that separates the regions of different depths. Such an approach is found in the work of Bartholomeusz (1958) and Miles (1967). It has also been found by Newman (1965) and Black, et. al. (1971) that for wave propagation over submarine obstacles there exists an infinite set of wavelengths such that the incident wave is totally transmitted.

Lassiter (1972) solved for the transmission and reflection coefficients in the case of monochromatic plane progressive surface waves over a rectangular submarine trench where the water depths before and after the trench are constant but not necessarily equal. Lassiter formulated the problem in terms of complementary variational integrals and solved for the velocity potential by matching the solution along vertical lines before and after the trench.

In this present study, the problem considered is two-dimensional motion of linear periodic water waves over an arbitrarily shaped sub-

---

<sup>1</sup>Assoc. Prof. of Civil Engrng., Univ. of Southern Calif., Los Angeles, Calif., 90007 USA.

<sup>2,3</sup>Graduate Research Asst., Dept. of Civil Engrng., Univ. of Southern Calif., Los Angeles, Calif., 90007 USA.

marine trench where the water depths before and after the trench are equal and constant. By drawing a horizontal line, the authors have separated the domain into two subregions, namely an infinite rectangular region of constant depth and a finite region of irregular shape representing the trench itself.

An analytic solution for each region is then found explicitly in terms of an unknown velocity distribution along the trench -- constant depth boundary. By superimposing a linear periodic incident wave of specified frequency in the infinite constant depth region, the final solution is obtained by matching the solutions in each subregion along the common boundary.

#### THEORETICAL CONSIDERATIONS

Let  $(x,y)$  constitute a Cartesian coordinate system with  $y = 0$  coinciding with the impermeable boundary of the constant depth region as shown in the definition sketch in Fig. 1. Assuming a steady-state solution for the velocity potential in the form of

$$\Phi(x,y;t) = \phi(x,y)e^{-i\sigma t} \quad (1)$$

the potential function  $\phi(x,y)$  must satisfy Laplace's equation throughout the fluid domain and the following boundary conditions:

$$\frac{\partial \phi}{\partial y} = \frac{\sigma^2}{g} \phi \quad \text{on } y = h, \quad -\infty < x < \infty$$

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = 0, \quad x < 0$$

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = 0, \quad x > \ell \quad (2)$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on solid boundary in the trench}$$

$$\frac{\partial \phi}{\partial y} = q(x) \quad \text{on } y = 0, \quad 0 < x < \ell$$

(the function  $q(x)$  is to be determined by matching)

In Eqs. (1) and (2),  $\sigma$  represents the circular frequency,  $2\pi/\text{wave period}$ ;  $i$  is the complex number  $\sqrt{-1}$ .

In order to solve for  $\phi(x,y)$  in an efficient manner, the fluid domain is divided into two regions, Region I and Region II, by the common boundary  $\Gamma$  shown in Fig. 1.

The strategy used herein is to solve for  $\phi(x,y)$  in each respective region in terms of the unknown  $\partial\phi/\partial y$  along the common boundary  $\Gamma$ . Thus, by matching the solutions in each region at  $\Gamma$ , one is able to obtain the final solution.

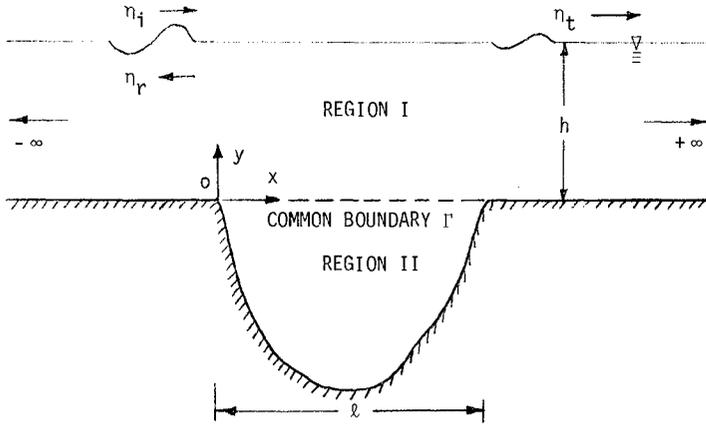


FIGURE 1. Definition sketch of the trench with regions of consideration.

Region I Solution

The solution for the velocity potential in Region I assuming that the unknown normal derivative of the velocity potential  $\partial\phi_I/\partial y$  is equal to  $q(x)$  (along  $y = 0, 0 < x < l$ ) has been obtained by Lee and Ayer (1980). This solution can be summarized as follows:

- (1) If  $x > x_j$  for all  $j$ ,

$$\phi_I(x,y) = \sum_{j=1}^N Q_j \left\{ \frac{\left[ e^{ik_r(x-x_j)} - e^{ik_r(x-x_{j-1})} \right]}{k_r^2} \cdot S_r(k_r,y) + \sum_{n=1}^{\infty} \frac{\left[ e^{-k_n(x-x_{j-1})} - e^{-k_n(x-x_j)} \right]}{k_n^2} \cdot S_n(k_n,y) \right\} \quad (3)$$

- (2) If  $x < x_j$  for all  $j$ ,

$$\phi_I(x,y) = \sum_{j=1}^N Q_j \left\{ \frac{\left[ e^{-ik_r(x-x_{j-1})} - e^{-ik_r(x-x_j)} \right]}{k_r^2} \cdot S_r(k_r,y) + \sum_{n=1}^{\infty} \frac{\left[ e^{k_n(x-x_j)} - e^{k_n(x-x_{j-1})} \right]}{k_n^2} \cdot S_n(k_n,y) \right\} \quad (4)$$

(3) If  $x > x_{j-1}$  and  $x < x_j$  for some  $j$ ,

$$\begin{aligned}
 \phi_I(x,y) = & \sum_{j=1}^{j-1} Q_j \left\{ \left[ \frac{e^{ik_r(x-x_j)} - e^{ik_r(x-x_{j-1})}}{k_r} \right] \cdot S_r(k_r,y) \right. \\
 & + \left. \sum_{n=1}^{\infty} \left[ \frac{e^{-k_n(x-x_{j-1})} - e^{-k_n(x-x_j)}}{k_n} \right] \cdot S_n(k_n,y) \right\} \\
 & + Q_j \left[ \frac{1 + \frac{\sigma^2}{g}(y-h)}{\sigma^2/g} - \frac{[e^{ik_r(x-x_{j-1})} + e^{-ik_r(x-x_j)}]}{k_r} \right] \cdot S_r(k_r,y) \\
 & + \sum_{n=1}^{\infty} \left[ \frac{e^{-k_n(x-x_{j-1})} + e^{k_n(x-x_j)}}{k_n} \right] \cdot S_n(k_n,y) \Bigg] \\
 & + \sum_{j+1}^N Q_j \left\{ \left[ \frac{e^{-ik_r(x-x_{j-1})} - e^{-ik_r(x-x_j)}}{k_r} \right] \cdot S_r(k_r,y) \right. \\
 & + \left. \sum_{n=1}^{\infty} \left[ \frac{e^{k_n(x-x_j)} - e^{k_n(x-x_{j-1})}}{k_n} \right] \cdot S_n(k_n,y) \right\} \tag{5}
 \end{aligned}$$

In Eqs. (3) - (5) the functions  $S_r$  and  $S_n$  are defined by

$$S_r(k_r,y) = \frac{k_r [\cosh k_r(y-h)] + \sigma^2/g \sinh [k_r(y-h)]}{k_r h \operatorname{sech} k_r h + \sinh k_r h}$$

and

$$S_n(k_n,y) = \frac{k_n [\cos k_n(y-h)] + \sigma^2/g \sin [k_n(y-h)]}{k_n h \sec k_n h + \sin k_n h}$$

The value of  $k_r$  and the values of  $k_n$  are determined by

$$\frac{\sigma^2}{g} = k_r \tanh (k_r \cdot h)$$

$$\frac{\sigma^2}{g} = -k_n \tan (k_n \cdot h) \quad .$$

The value of  $Q_j$  is the average value of  $q(x)$  in the  $j^{\text{th}}$  subinterval whose midpoint is defined as  $(x_j,0)$  within the common boundary of the trench mouth.

### Region II Solution

Again, the solution for the velocity potential in Region II has the steady-state form

$$\phi_{II}(x,y;t) = \phi_{II}(x,y) e^{-i\sigma t} \quad (6)$$

and the potential  $\phi_{II}$  must satisfy Laplace's equation subject to the boundary condition of  $\partial\phi_{II}/\partial n = 0$  along the solid boundary of the trench.

As the trench shape is considered arbitrary it can be conveniently solved by boundary integral method. The potential function along the boundary of the trench can be expressed as follows:

$$\phi_{II}(\vec{x}_j) = \frac{1}{\pi} \int \left[ \ln\left(\frac{1}{r}\right) \frac{\partial\phi_{II}}{\partial n} - \phi_{II} \frac{\partial}{\partial n} \ln\left(\frac{1}{r}\right) \right] ds \quad (7)$$

The term  $\partial\phi_{II}/\partial n$  in Eq. (7) is zero except along the mouth of the trench which is equal to  $q(x)$  according to Eq. (2). This integral equation can be approximated by a matrix equation as used in Raichlen and Lee (1978).

Therefore, the value of  $\phi_{II}$  along the trench mouth can be expressed in terms of the unknown vector,  $Q_j$ . This unknown vector  $Q_j$  can then be solved by matching the value of  $\phi_{II}(x_j)$  and  $\phi_I(x_j) + \phi_{in}(x_j)$  (where  $\phi_{in}(x_j)$  is a specified potential function of an incident wave along the trench mouth with the coordinate of  $(x_j, 0)$ ).

Once the value of  $Q_j$  is obtained, the velocity potential at any position in Region I can be computed by Eqs. (3) - (5) along with the superposition of the incident wave potential.

### PRESENTATION AND DISCUSSION OF RESULTS

The effect of the trench on the propagation of waves can be demonstrated most easily by the transmission characteristics. In order to ensure that the present analysis for Region II using the boundary integral method can provide a reliable result, the method is applied to a rectangular trench, where in a separate study, the solution has been obtained in terms of an Eigenfunction expansion, therefore providing a basis for comparison. Figure 2 shows the transmission coefficient,  $K_t$ , as a function of the relative wave length. The ordinate is the ratio of the transmitted wave amplitude divided by the incident wave amplitude, while the abscissa is the ratio of the water depth,  $h$ , in Region I divided by the incident wave length,  $\lambda$ . The wave length  $\lambda$  is computed from the dispersion relationship,  $\lambda = (gT^2/2\pi) \tanh(2\pi h/\lambda)$ , where  $T$  is the incident wave period.

The present theoretical results (as can be seen from Figure 2) come within 3% of the results of Lee and Ayer (1980). The result of

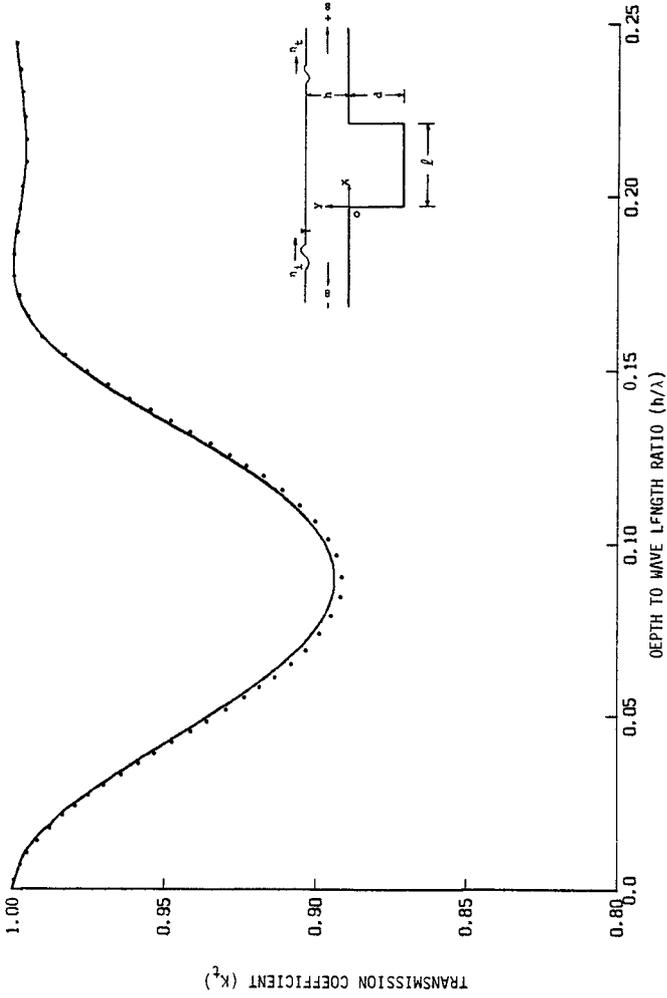


Figure 2 Transmission coefficient as a function of relative wave length ( $h = 4''$ ,  $d = 26\frac{1}{2}''$ ,  $\lambda = 21 \frac{1}{8}''$ ). — theoretical results of Lee & Ayer (1980), ..... present results.

Lee and Ayer (1980) was obtained by using an analytic solution in Region II. The fact that the present solution is so close to the more exact theory demonstrates that the approximate method used can provide a fairly accurate result. It is seen from Figure 2 that for  $h/\lambda > 0.18$ , the incident waves are almost fully transmitted. At  $h/\lambda = 0.09$ , the transmission coefficient is approximately 0.89. To understand the trench effect further, one can compute the value of  $l/\lambda$  at these critical points. At  $h/\lambda = 0.18$ , it corresponds to  $l/\lambda = 0.95$  while at  $h/\lambda = 0.09$ , it corresponds to  $l/\lambda = 0.475$ . It appears that for a relatively short trench length, the maximum reduction of transmitted wave occurs as  $l/\lambda$  approaches 0.5. As the wave period is decreased to where  $l/\lambda$  approaches 1, the effect on wave transmission due to the trench is negligible.

As the trench length increases, the effect of the trench on the transmission characteristics of the incident waves becomes more interesting. This is shown in Figure 3. The trench length for this case is three times that shown in Figure 2 with other dimensions held constant. Two theoretical curves, one obtained by the present analysis and the other obtained from Lee and Ayer (1980) are shown for comparison. The experimental data obtained by Lee and Ayer (1980) in a laboratory wave tank of 12 inches wide, 48 feet long and 18 inches deep are also included for comparison. Again, it is seen that the two curves agree very well in every peak and trough (the results are within 3% of each other) for the range of  $h/\lambda$  presented. In the range of  $0 < h/\lambda < 0.25$ , there are six different wave periods at which waves are fully transmitted. The results indicate that the trench does exert a greater influence on wave transmission characteristics in that the transmission coefficient at  $h/\lambda = 0.042$  is only about 0.70. It is also seen that the experimental data in general tend to confirm the theoretical prediction. However, due to experimental errors and the unavoidable wave reflections from both ends of the wave tank, the experimental data show considerable scattering as evident in the figure.

An example for wave transmission over an even longer rectangular trench is shown in Figure 4. The length of the rectangular trench is now four times that shown in Figure 2. Again, the present results using the boundary integral method agree well with the result of Lee and Ayer (1980) further confirming that the method designed for an irregular shaped trench can be used for a trench of rectangular shape. The number of wave periods at which waves are fully transmitted is now increased to nine for the same range of  $h/\lambda$ . For each of the troughs in the response curve, the effect of the trench is further dramatized. For example, at the first trough ( $h/\lambda = 0.034$ ), the transmission coefficient is reduced to 0.68, while at the second trough ( $h/\lambda = 0.081$ ), the transmission coefficient is about 0.84. These are clearly smaller values than those shown in Figure 3.

An example of the transmission of waves over an irregularly shaped trench is presented in Figure 5. The dimension of the trench shape is shown in the insert of the figure. The transmission coefficient is very close to one, showing that the trench does not effect the wave transmission drastically. However, it is clear that there are a number of peaks and troughs in the transmission curve (a similar feature as

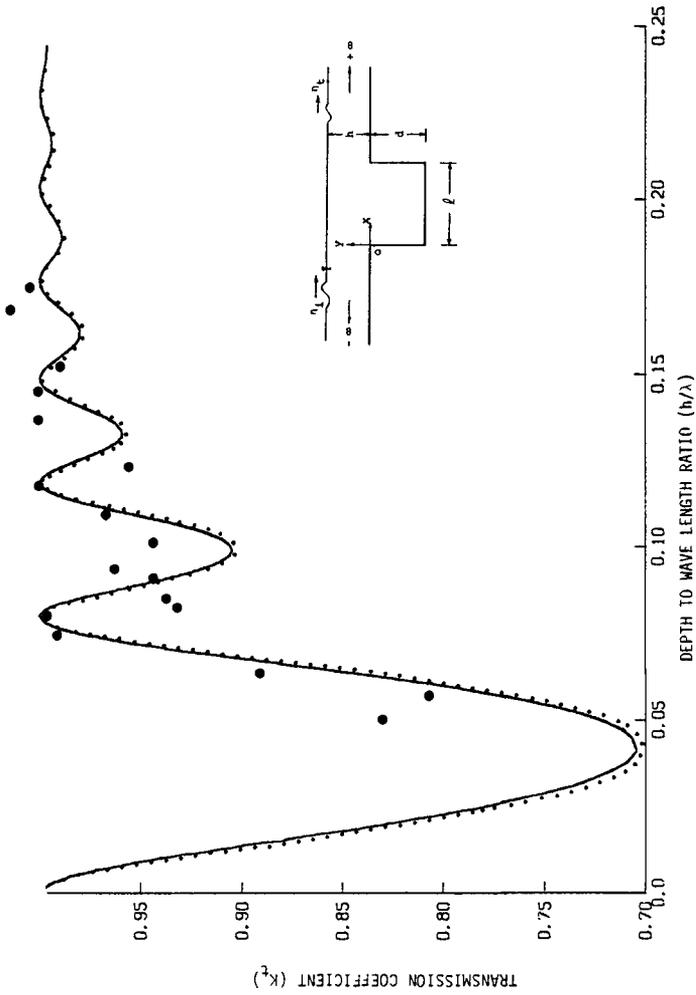


Figure 3 Transmission coefficient as a function of relative wave length ( $h = 4''$ ,  $d = 26\frac{1}{2}''$ ,  $\lambda = 63\frac{5}{8}''$ , — theoretical results of Lee & Ayer (1980), ..... present results using boundary integral method).

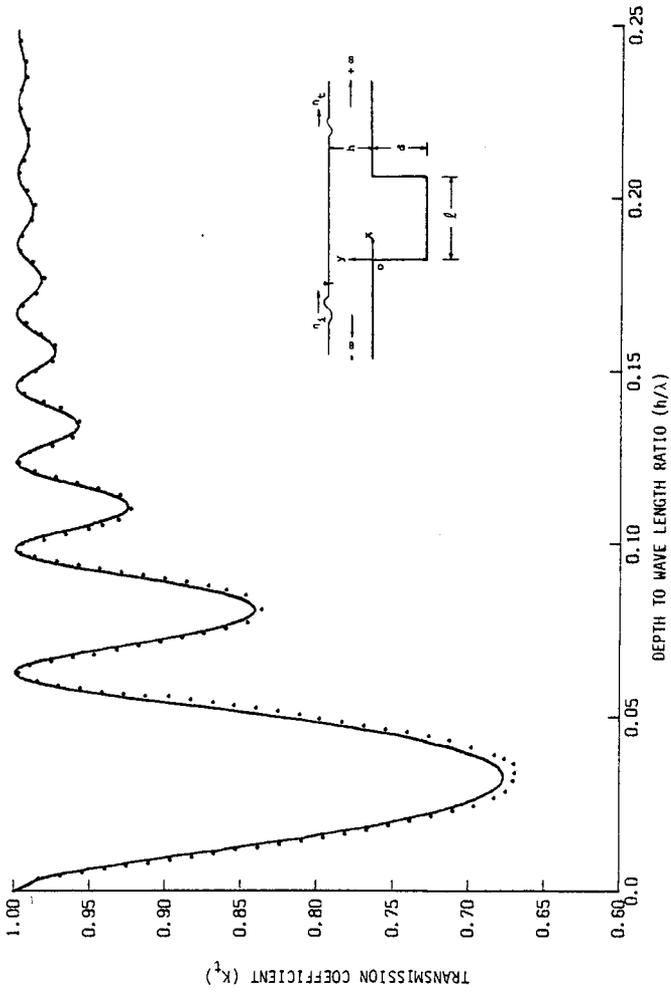


Figure 4 Transmission coefficient as a function of relative wave length ( $h = 4''$ ,  $d = 26\frac{1}{2}''$ ,  $\lambda = 85''$ , — theoretical results of Lee & Ayer (1980), ..... present results using boundary integral method).

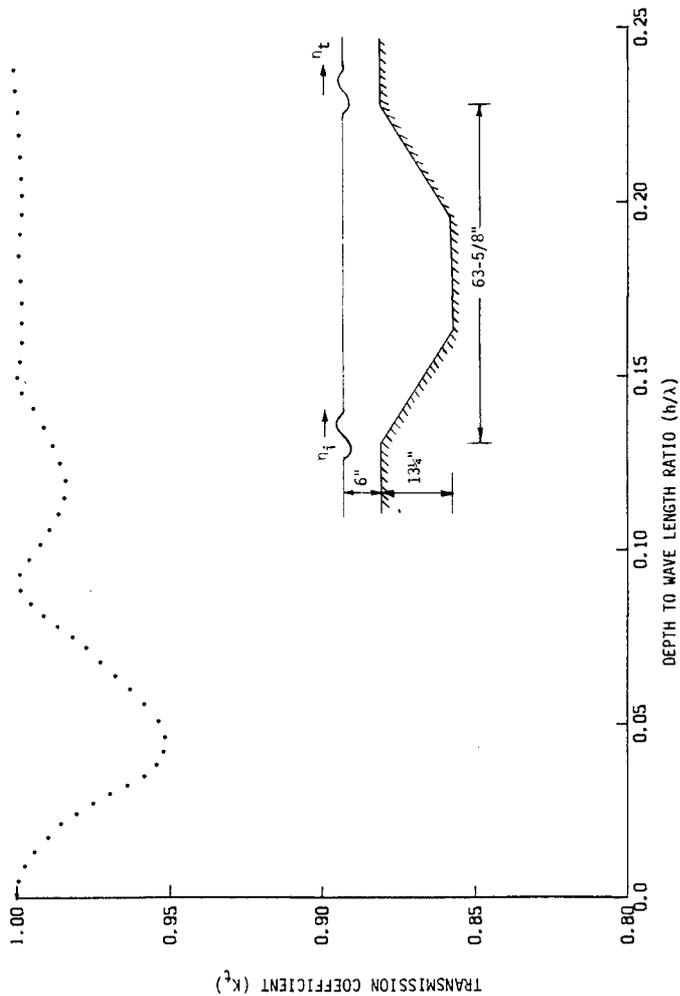


Figure 5 Transmission coefficient as a function of relative wave length

that shown in Figures 2 - 5).

#### CONCLUDING REMARKS

The methods outlined in this paper for analyzing the effect of an arbitrarily shaped trench on the propagation of periodic incident waves has been shown to be quite effective as illustrated by comparison with the solution techniques for a rectangular trench and with experiments. From the results on wave transmission and reflection, it is seen that there exists an infinite number of wave periods at which waves are fully transmitted, and that the effect of the trench on wave transmission is progressively smaller for higher wave frequencies (the larger values of  $h/\lambda$ ).

#### ACKNOWLEDGEMENT

The work reported herein is supported by the National Science Foundation under Grant No. ENV 77-01599 and by N.O.A.A. Sea Grant under Grant No. 04-80M01-186.

#### LIST OF REFERENCES

1. Bartholomeusz, E.F., "The Reflection of Long Waves at a Step," Proc. Camb. Phil. Soc., Vol. 54, 1958, p. 106.
2. Black, J.L., Mei, C.C., and Bray, M.C.C., "Radiation and Scattering of Water Waves by Rigid Bodies," Journal of Fluid Mechanics, Vol. 46, 1971, p. 151.
3. Kreisel, H., "Surface Waves," Quarterly Applied Math., Vol. 7, 1949, p. 21.
4. Lassiter, J.B., "The Propagation of Water Waves over Sediment Pockets," Ph.D. Thesis, Mass. Inst. of Tech., July 1972.
5. Lee, J.J., and Ayer, R.M., "Wave Propagation over Rectangular Trench," submitted to Journal of Fluid Mechanics, 1980.
6. Miles, J.W., "Surface-Wave Scattering Matrix for a Shelf," Journal of Fluid Mechanics, Vol. 28, 1967, p. 755.
7. Newman, J.N., "Propagation of Water Waves Past Lone Two-Dimensional Obstacles," Journal of Fluid Mechanics, Vol. 23, 1965, p. 23.
8. Newman, J.N., "Propagation of Water Waves over an Infinite Step," Journal of Fluid Mechanics, Vol. 23, 1965, p. 399.
9. Raichlen, F., and Lee, J.J., "An Inclined-Plate Wave Generator," Proc., 16th Int'l. Coastal Engrng. Conf., Hamburg, Germany, August 1978, pp. 388-399.