CORRECT REPRODUCTION OF GROUP-INDUCED LONG WAVES

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ABSTRACT

In nature short period storm waves generate longer waves with periods corresponding to the wave group periods. The long waves are generally referred to as the wave set-down of water level. The set-down term is of second order in the height of the short waves. With first-order reproduction of natural storm waves in the laboratory, the set-down bound to the wave groups is not reproduced. As a result, various free waves are generated, propagate towards the model and reflect from the boundaries. These so-called parasitic waves cause an exaggeration of long wave phenomena, such as harbour resonance and slow drift oscillations of moored ships.

The parasitic waves can be eliminated by means of compensating free waves imposed on the system by second-order paddle motion reproducing the natural set-down. The control signal for this motion has been calculated and checked by testing. The agreement between calculated and measured results is found to be good.

Further, an alternative method for reducing the parasitic wave problem is presented. Utilizing the shoaling properties of the various waves, the influence of parasitic waves can be diminished by generating the waves in somewhat deeper water before they propagate into the shallower model area.

INTRODUCTION

Resonance conditions ('seiche') in harbours and bays are experienced all over the world. They can be a nuisance for the operation of ship terminals because, from time to time, they may induce unacceptably large movements of moored ships resulting in breaking of moorings. The periods of interest are usually in the range of 20 seconds to 5 minutes.

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Harbour resonance may be induced by various mechanisms, such as tsunamies, storm surges, land slides or calving glaciers (Greenland). Most frequently, however, the source will be meteorological pressure fluctuations or long period waves from distant storms or wave groups. In this paper long period waves induced by wave groups are considered. This is the most common mechanism of generation.

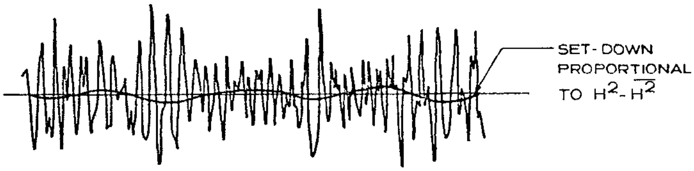


Fig. 1 Long period wave generated by wave groups.

Long waves generated by storm wave groups are generally referred to as the wave set-down, because they are characterized by having their troughs in the regions of the larger waves in the groups. Typically, a group contains from 5 to 20 waves (Fig. 1).

To each wave pertains an internal compressive force, the radiation stress (Longuet-Higgins and Stewart, 1960) or wave thrust (Lundgren, 1963), acting in the direction of wave propagation. For a regular wave this force is proportional to the square of the wave height. To balance this force a set-down in mean water level has to appear within the series of higher waves, and a corresponding set-up is produced within the series of smaller waves. This problem has previously been considered and quantified by Longuet-Higgins and Stewart (1964) and by Ottesen Hansen (1978).

An example of the effect of long waves generated by wave groups is the 'seiche' in the Bay of Telok Plan at Bintulu in Malaysia (Fig. 2a). The 'seiche' in the bay and the incoming waves were recorded simultaneously (Fig. 2b). It appeared that wave groups approaching from NW produced an edge wave between the head of the bay and the tip of the promontory Tanjong Kidurong with a period of 100 - 180 s. This long period wave normally occurred when the offshore significant wave exceeded 0.9 m.

PROBLEMS IN PHYSICAL MODELS

Mooring forces and ship movements are normally determined by physical model tests, which at present is the simplest and most reliable method. Physical model technique has developed considerably over the past decade. Presumably, the major improvement has been the introduction of natural, irregular waves in the model. The use of natural wave trains secures that important phenomena found in nature are reproduced accurately in the model. This applies particularly to:

(i) Direct wave action on ships.
(ii) The effect of wave groups or long waves on harbour resonance and slow drift ship motions.

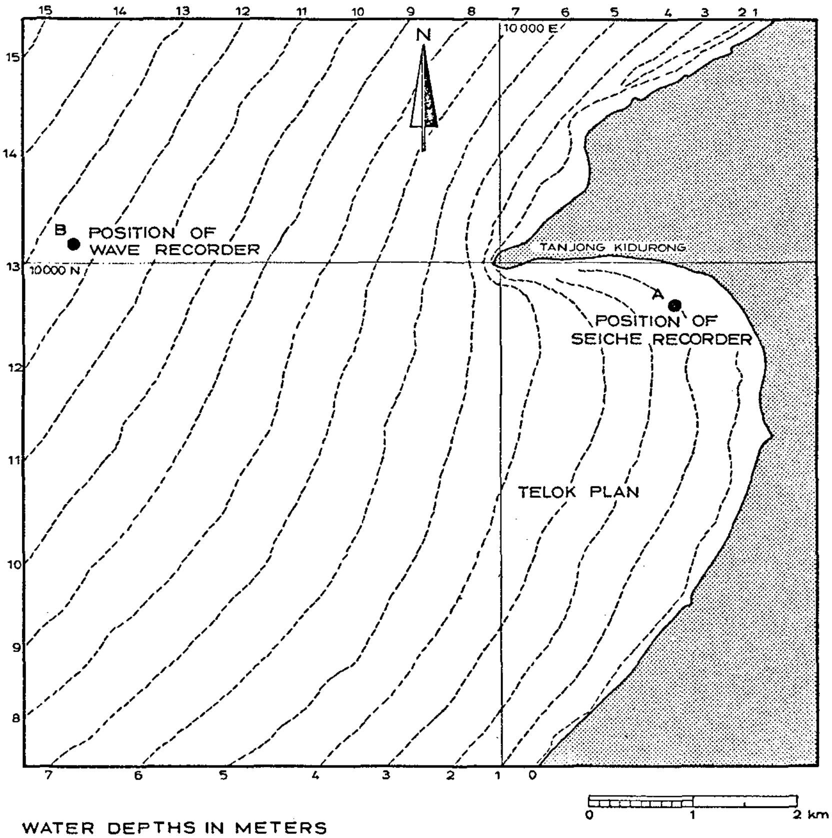
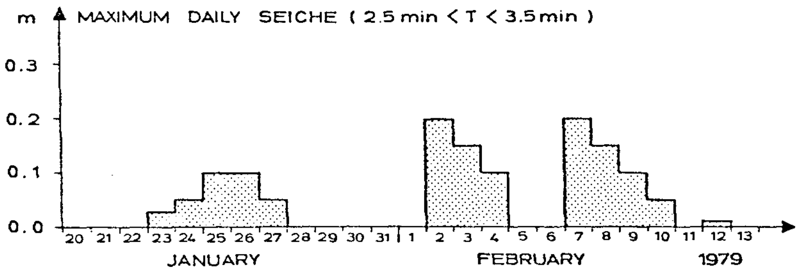


Fig. 2a Topography of the Bay of Telok Plan at Bintulu, Sarawak, Malaysia. A 'seiche' (long wave) recorder is placed in the bay, and a waverider buoy approximately 4 km offshore.



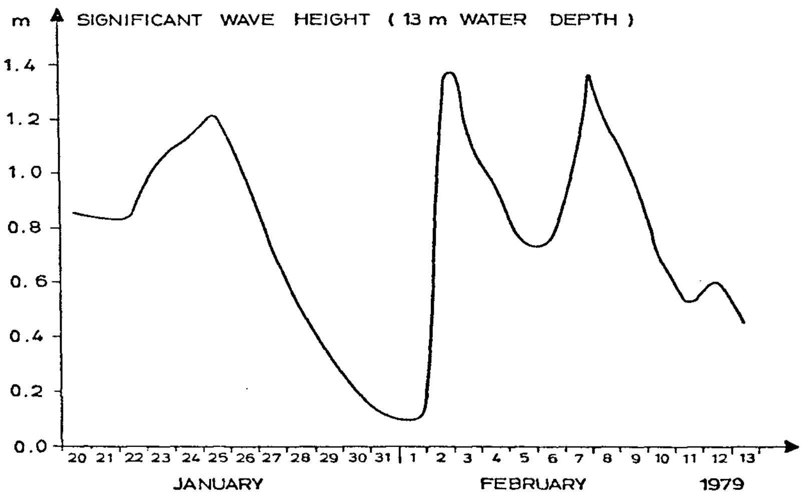


Fig. 2b The results of simultaneously recorded 'seiche' in the Bay of Telok Plan and the significant wave height 4 km offshore.

For problems with ships moored in harbours or movements of large vessels, the effect of long waves is particularly important due to the risk of resonance with the longer natural vessel periods pertaining to surge, sway and yaw.

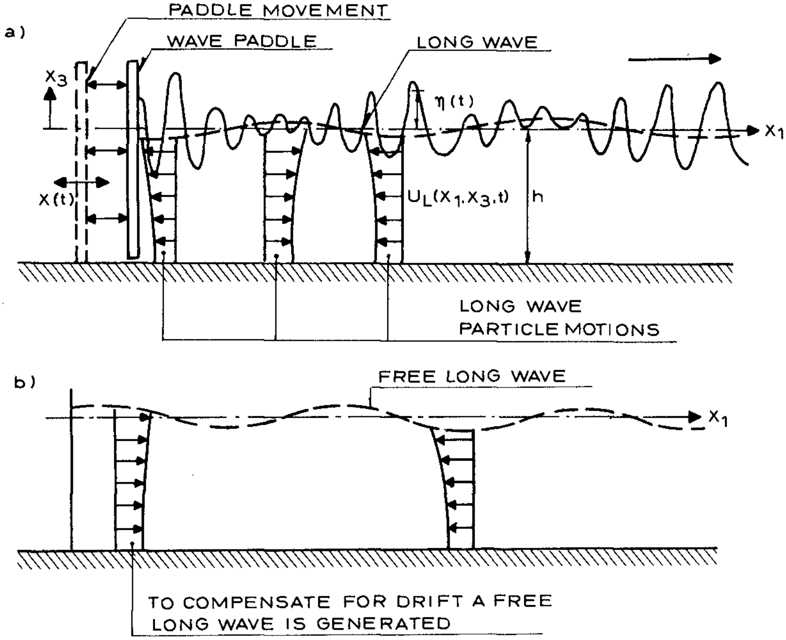


Fig. 3 Origin of parasitic long wave when generating irregular waves in the laboratory.

When natural waves are reproduced in the laboratory, the paddle is normally controlled by a 1st order signal. The wave groups are accompanied by a long wave corresponding to the set-down (Fig. 3a). The set-down behaves as a normal progressive wave with forward orbital velocities underneath the crests and backward orbital velocities under the troughs, also called drift velocities.

At the paddle, however, the desired 2nd order (long period) drift velocities are not produced by the 1st order (short period) signal. With no flow trough the paddle the natural drift velocities will be compensated by identical velocities of opposite sign (Fig. 3b). The latter velocities generate a progressive long wave that is free, i.e. not bound to the groups. This phenomenon is called a *parasitic long wave*, and it results in an exaggeration of the long wave effects mentioned under (ii) above.

In order to eliminate the parasitic long wave, the 1st order signal must be superposed by a 2nd order long period signal that will produce the drift velocities required by the set-down wave. In principle, these drift velocities must be produced at the mean piston position, X = 0. In practice, the long period signal will control the piston velocity, and hence the drift velocity, at the instantaneous piston position X, which is displaced somewhat from X = 0. This piston displacement gives a correction to the long period signal, see discussion below.

As will be seen from the formulae below, the long period signal is a sum over differences between each pair of frequencies in the short wave spectrum. Therefore, the long period signal may be designated as a sum of subharmonics.

The control of the paddle by the 1st order short wave signal also gives rise to a displacement. The corresponding 2nd order correction would contain a number of terms, each term corresponding to the sum of a pair of frequencies in the short wave spectrum. Hence, this correction would be a sum of superharmonics, cf. the second harmonic introduced in the generation of regular waves (Buhr Hansen et al., 1975). In the present paper the superharmonic correction has not been considered.

METHOD OF SOLUTION

The mathematical description of the long wave and the parasitic wave problem depends on the description used in the wave generation. In all modern laboratories, models are tested with irregular waves. Often a Fourier approach with random or specified phase spectrum is used for the generation, but at present a time-domain generation must be preferred (Lundgren and Sand, 1979).

The direct Fourier approach with specified phases is the simplest one for the problem discussed in this paper and will be used in the following to illustrate the principles.

A regular wave group with water surface elevation η_{nm} and frequency Δf_{nm} consists of two regular waves with water surface elevations η_n, η_m and frequencies f_n, f_m, respectively, where

$$\eta_{nm} = \eta_n + \eta_m \qquad \text{and} \qquad \Delta f_{nm} = f_n - f_m \tag{1}$$

n and m being indices indicating the numbers of the waves considered.

According to the previous outline of the physical principles, the wave group η_{nm} generates a wave set-down ξ_{nm}. Each pair n,m of components in the wave spectrum contributes to the set-down. This means that the wave set-down ξ in an irregular wave train can be calculated by summing up the contributions from all pairs, i.e.

$$\xi = \sum_{n-m=1}^{\infty} \sum_{m=m^*}^{\infty} \xi_{nm} \qquad \text{with} \quad m^* = f^*/f_0 \tag{2}$$

in which f_0 is the interval of discretization of the short wave spectrum, and f^* is the lowest frequency in the short wave spectrum.

Hence, the first step in the problem is to solve the simpler problem of two regular, interacting waves. In this analysis the following assumptions are made:

(1) The short period waves can be described by first order Stokes' theory.
(2) The long period waves constitute a second order phenomenon, i.e. of the order $O(H^2/h^2)$.
(3) The bottom is plane.
(4) The waves are progressive and propagate in only one direction.
(5) Frictional effects can be neglected.

With these assumptions the following solution for the wave set-down ξ_{nm} under a regular wave group has previously been found (Ottesen Hansen, 1978)

$$\frac{\xi_{nm}}{h} = G_{nm}\, h \left(\frac{a_n a_m + b_n b_m}{h^2} \cos(\Delta\omega_{nm} t - \Delta k_{nm} x_1) + \frac{-a_n b_m + b_n a_m}{h^2} \sin(\Delta\omega_{nm} t - \Delta k_{nm} x_1) \right) \quad (3)$$

The notation in this paper is:

(x_1, x_3)	=	horizontal/vertical coordinates
t	=	time
n,m	=	indices indicating numbers of the waves considered
L_n	=	length of the n'th wave
k_n	=	$2\pi/L_n$ = wave number of the n'th wave
Δk_{nm}	=	$k_n - k_m$ = long wave number
σk_{nm}	=	$k_n + k_m$
c_n	=	phase velocity of the n'th wave
T_n	=	period of the n'th wave
f_n	=	$1/T_n$ = frequency of the n'th wave
Δf_{nm}	=	$f_n - f_m$ = long wave frequency
ω_n	=	$2\pi f_n$ = cyclic frequency of the n'th wave
$\Delta\omega_{nm}$	=	$\omega_n - \omega_m$
h	=	water depth
η_n	=	$a_n \cos(\omega_n t - k_n x_1) + b_n \sin(\omega_n t - k_n x_1)$ = elevation of water surface for the short wave
ξ	=	elevation of water surface for the long wave
ρ	=	density of water
U_L	=	long wave horizontal velocity
u_s	=	short wave horizontal velocity
u	=	total, instantaneous horizontal velocity
X	=	paddle position

The function G_{nm}, which is shown graphically in Fig. 4, is a transfer function given by (Ottesen Hansen, 1978)

$$G_{nm} = \left(\frac{g}{2} \Delta\omega_{nm} \Delta k_{nm} \left(\frac{1}{c_n} + \frac{1}{c_m}\right) \Delta k_{nm} h \coth(\Delta k_{nm} h) - \frac{1}{2} \Delta k_{nm}^2 \Delta\omega_{nm}^2 h + \frac{\omega_n \omega_m \Delta k_{nm}^2 h \cosh(\Delta k_{nm} h)}{\cosh(\sigma k_{nm} h) - \cosh(\Delta k_{nm} h)} \right) \Bigg/ \left(\Delta k_{nm} h \coth(\Delta k_{nm} h)\, \Delta\omega_{nm}^2 - g\, h\, \Delta k_{nm}^2 \right) \quad (4)$$

The asymptotic expressions for the shallow and deep water cases of Eq. (4) are presented in Fig. 4 on the left hand and right hand sides, respectively.

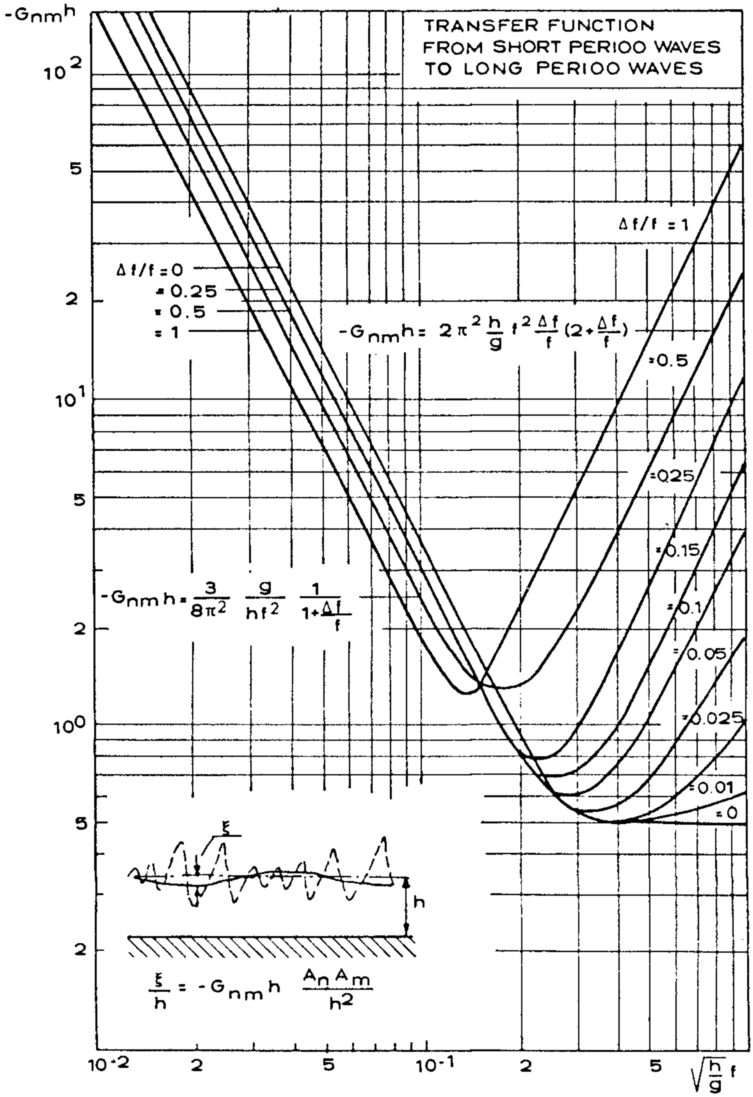


Fig. 4 Transfer function for long waves generated by two regular waves with amplitudes A_n, A_m and frequencies $f_n = f + \Delta f$ and $f_m = f$, respectively.

PRINCIPLE OF SOLUTION

A natural wave train is generated by a piston type wave paddle in a flume of depth h, cf. Fig. 3a. The paddle position as function of time is X(t) with paddle amplitude X_a. The short wave horizontal velocity is $u_S(x_1,x_3,t)$, and the corresponding long wave velocity is $U_L(x_1,x_3,t)$. The total, instantaneous horizontal velocity in the flume is denoted $u(x_1,x_3,t)$. Further, the velocity potential ϕ with $\phi_{x_1} = u(x_1,x_3,t)$ must satisfy Laplace equation.

The general boundary condition at the paddle expresses the equality between paddle velocity and horizontal orbital velocity, i.e.

$$\phi_{x_1} = u(x_1,x_3,t) = \frac{\partial X(t)}{\partial t} = X_t(t) \qquad \text{for} \begin{cases} x_1 = X(t) \\ -h \le x_3 \le 0 \end{cases} \tag{5}$$

A perturbation method is applied to the problem. This implies that the variables u and X are written

$$\begin{aligned} u &= \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots\ldots \\ X &= \varepsilon X^{(1)} + \varepsilon^2 X^{(2)} + \ldots\ldots \end{aligned} \tag{6}$$

It is not practical to deal with a boundary condition at $x_1 = X(t)$. Therefore it is transferred to $x_1 = 0$ by means of the following Taylor expansions

$$\begin{aligned} u^{(1)}(X,x_3,t) &= u^{(1)}(0,x_3,t) + X\,u^{(1)}_{x_1}(0,x_3,t) + \ldots\ldots \\ u^{(2)}(X,x_3,t) &= u^{(2)}(0,x_3,t) + X\,u^{(2)}_{x_1}(0,x_3,t) + \ldots\ldots \end{aligned} \tag{7}$$

The 1st and 2nd order boundary conditions now appear to be

$$\begin{aligned} u^{(1)}(0,x_3,t) &= X^{(1)}_t \\ u^{(2)}(0,x_3,t) &= -X^{(1)}u_{x_1} + X^{(2)}_t \end{aligned} \qquad \text{for} \quad x_1 = 0 \tag{8}$$

Then the problem is reduced to the following:

(i) For a given $X^{(1)}(t)$ (with $X^{(2)} = 0$) determine the corresponding $u^{(2)}(0,x_3,t)$ in the flume. This includes contributions from both the real wave set-down and parasitic waves.

(ii) Determine the signal $X^{(2)}(t)$ so that the progressive long waves correspond to the set-down, i.e. $u^{(2)} = U_L$, and all parasitic waves are eliminated.

In step (i) Laplace equation has to be solved with all the boundary conditions, including Eq. (8) written as

$$\begin{aligned} \phi^{(1)}_{x_1} &= X^{(1)}_t \\ \phi^{(2)}_{x_1} &= -X^{(1)}\phi^{(1)}_{x_1x_1} \end{aligned} \qquad \text{for} \quad x_1 = 0 \tag{9}$$

SHALLOW WATER SOLUTION

In the shallow water case the horizontal velocity distribution matches the paddle profile, and therefore no local disturbances will appear. Thus, Laplace equation will give the usual, well-known solution. Hence, step (ii) above may be solved directly from the 2nd order boundary condition Eq. (8) or through mass transport considerations at the paddle. This leads to

$$u^{(2)}(0,x_3,t) = U_L(0,x_3,t) = -X^{(1)}u^{(1)}_{x_1} + X^{(2)}_t \qquad \text{for} \quad x_1 = 0 \qquad (10)$$

i.e.

$$X^{(2)}_t = U_L(0,x_3,t) + X^{(1)}u^{(1)}_{x_1} \qquad (11)$$

For a regular wave group, $\eta_{nm}(t)$, produced by two short waves as defined by Eq. (1), the 1st order horizontal velocity, $u^{(1)}_{nm}$, is found from the usual expressions for progressive waves. The paddle movement $X^{(1)}_{nm}$ is calculated by application of the asymptotic transfer function kh, which is easily derived from the 1st order equations (cf. Biésel, 1951). The long wave velocity $U_L(0,x_3,t)$ is calculated from Eq. (3). Hence, the necessary 2nd order piston positions $X^{(2)}_{nm}(t)$ may be found from Eq. (11) as

$$X^{(2)}_{nm}(t) = \frac{k_n^2 + k_m^2 - 3/h^2}{2h^2k_nk_m(k_n-k_m)}\left[\left(a_na_m + b_nb_m\right)\sin\Delta\omega t + \left(a_nb_m - a_mb_n\right)\cos\Delta\omega t\right] \qquad (12)$$

In the numerator the term $-3/h^2$ is the dominant one and corresponds to elimination of the parasitic wave. The term $k_n^2 + k_m^2$ originates from the paddle displacement.

For short shallow water waves the transfer function is kh, i.e. the ratio between generated wave amplitude and 1st order paddle amplitude is $A/X_a = kh$. Similarly, a 2nd order transfer function can be defined as the ratio between the amplitude of the generated long waves, ξ_a, and the 2nd order paddle amplitude, $X^{(2)}_a$. These transfer functions are shown in Fig. 5. A good approximation to the 2nd order transfer function is $\xi_a/X_a \approx \Delta k_{nm}h$.

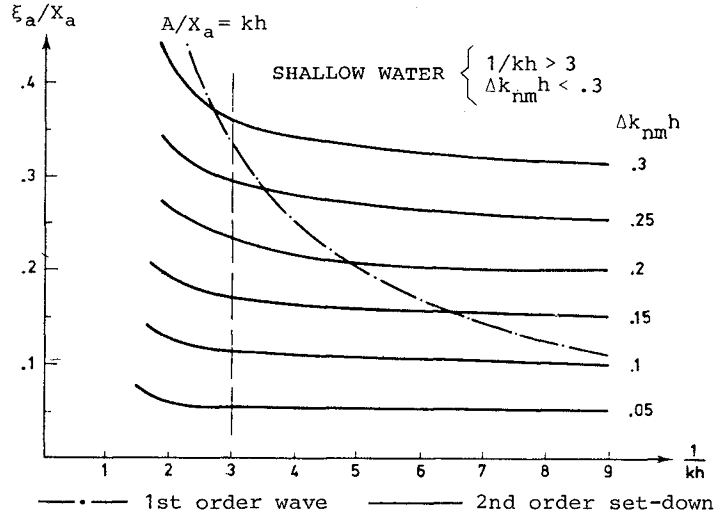


Fig. 5 Shallow water transfer functions.

Assuming $L > 20\,h$ for shallow water waves, the term $k_n^2 + k_m^2$ in the numerator of Eq. (12) is less than 6% of the term $3/h^2$. If the smaller term is neglected, Eq. (12) becomes

$$X^{(2)}(t) \approx \frac{-3}{2h^4 k_n k_m (k_n - k_m)} Z_{nm}(t) \qquad (13)$$

where $Z_{nm}(t)$ is the expression in the brackets.

GENERAL SOLUTION FOR THE PADDLE MOVEMENT

In the general (non-shallow) case the horizontal orbital velocity of the waves diminishes hyperbolically towards the bottom. As this velocity profile is not compatible with that of the paddle, local disturbances, damped as $e^{-k_j x_1}$, arise both in 1st and 2nd order. These disturbances and the general x_3-dependence make it imperative to solve the Laplace equation with the appropriate boundary conditions. Thus the problem is as follows

$$\phi^{(2)}_{x_1 x_1} + \phi^{(2)}_{x_3 x_3} = 0 \qquad (14)$$

$$g\,\phi^{(2)}_{x_3} + \phi^{(2)}_{tt} = g\,\eta^{(1)}_{x_1}\phi^{(1)}_{x_1} - g\,\eta^{(1)}\phi^{(1)}_{x_3 x_3} - \eta^{(1)}_{t}\phi^{(1)}_{t x_3} - \eta^{(1)}\phi^{(1)}_{t x_3 t} + \phi^{(1)}_{x_1}\phi^{(1)}_{x_1 t} + \phi^{(1)}_{x_3}\phi^{(1)}_{x_3 t} \quad \text{for} \quad x_3 = 0 \qquad (15)$$

$$\phi^{(2)}_{x_3} = 0 \quad \text{for} \quad x_3 = -h \qquad (16)$$

$$\phi^{(2)}_{x_1} = -X^{(1)}\phi^{(1)}_{x_1 x_1} + X^{(2)}_{t} \quad \text{for} \quad x_1 = 0 \qquad (17)$$

The precise form of the local disturbances pertaining to the 1st order equations is (cf. Biésel)

$$\phi_{x_1,\text{disturbance}} = X_a \sum_{j=1}^{\infty} C_j\, \omega \cos\big(k_j (x_3 + h)\big)\, e^{-k_j x_1} \cos \omega t \qquad (18)$$

where $C_j = \dfrac{2 \sin k_j h}{\sin k_j h \cos k_j h + k_j h}$ and $\omega^2 = -g\,k_j \tan k_j h$.

Firstly, step (i) of the preceding section is considered, viz. $X^{(2)} = 0$. A paddle controlled by a 1st order signal will generate in 2nd order

$$\phi^{(2)}_{x_1} = -U_{L,\text{free}} + a_L \cos(\Delta\omega\, t - \Delta k\, x_1) + b_L \sin(\Delta\omega\, t - \Delta k\, x_1) + \sum_i C_{L,i}\, e^{-k_i x_1} \qquad (19)$$

when the input is the regular wave group given by Eq. (1). The functions $C_{L,i}$ represent the (x_3,t)-dependent local disturbances of 2nd order, which can be accepted in model tests because they are strongly damped away from the paddle. The first three terms in Eq. (19) are illustrated in Fig. 6 together with the short wave groups (full line) and the dash-dotted long wave (set-down) that follows the groups, and which it is desired to reproduce. The latter is characterized by the velocities $U_L(x_1,x_3,t)$. The first term, $-U_{L,\text{free}}$, is the free parasitic long wave shown as a fat dotted line and designated '180° out of phase with U_L' because it cancels U_L at $x_1 = 0$. The terms with a_L and b_L represent free long waves and contain the contribution from the paddle displacement shown as a thin dotted line and designated 'from the $X^{(1)}\phi^{(1)}_{x_1x_1}$ term'. These terms also include a complicated 2nd order free-wave effect from the 1st order local disturbances, cf. the formula for $F_{3,n}$ below.

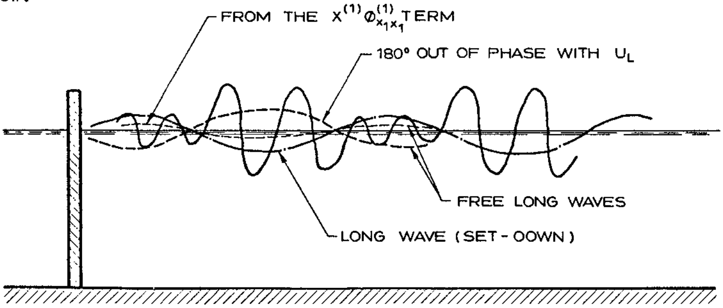


Fig. 6 Long waves for 1st order control signal.

Secondly, the appropriate $X^{(2)}_{nm}(t)$ must be determined as a part of the solution of the 2nd order Laplace equation in such a manner that:

(a) The long wave following the group is generated.
(b) All free long waves are eliminated.

The lengthy calculations result in the following solution:

$$X^{(2)}_{nm}(t) = \Big((a_nb_m - a_mb_n)\ F_1 + (a_na_m - b_nb_m)\ F_2\ (F_{3,m} - F_{3,n})\Big)\ \cos\Delta\omega t$$
$$+ \Big((a_na_m + b_nb_m)\ F_1 + (a_mb_n - a_nb_m)\ F_2\ (F_{3,m} - F_{3,n})\Big)\ \sin\Delta\omega t \quad (20)$$

in which

$$F_1 = \frac{\omega_m \Delta k_{nm} k_m (1+G_n)\Big\{(k_n - 2k_m)\sinh(k_nh) + k_n\sinh\big([k_n - 2k_m]h\big)\Big\}}{8(k_n^2 - 2k_nk_m)\Delta\omega_{nm}\sinh(\Delta k_{nm}h)\tanh(k_nh)\sinh(k_mh)}$$
$$+ \frac{\omega_n \Delta k_{nm} k_n (1+G_m)\Big\{k_m\sinh\big([2k_n - k_m]h\big) + (2k_n - k_m)\sinh(k_mh)\Big\}}{8(2k_nk_m - k_m^2)\Delta\omega_{nm}\sinh(\Delta k_{nm}h)\tanh(k_mh)\sinh(k_nh)}$$
$$+ \frac{G_{nm}(1 + \Delta G_{nm})}{2\tanh(\Delta k_{nm}h)} \quad (20a)$$

$$F_2 = \frac{\Delta k_{nm}(1+G_n)(1+G_m)}{8\,\Delta\omega_{nm}\tanh(k_nh)\tanh(h_mk)} \quad (20b)$$

$$F_{3,n} = \omega_n \sum_{j=1}^{\infty} \frac{2k_j\sin(k_jh)\Big\{k_j\sin(k_jh)\coth(\Delta k_{nm}h) + \Delta k_{nm}\cos(k_jh)\Big\}}{(k_j^2 - \Delta k_{nm}^2)\Big(\sin(k_jh)\cos(k_jh) + k_jh\Big)} \quad (20c)$$

Here k_j is found as the solution to

$$\omega_n^2 = -g k_j \tanh k_jh \qquad \text{with} \qquad (j - \tfrac{1}{2})\pi < k_jh < j\pi \quad (20d)$$

Analogously for $F_{3,m}$. Further, G_{nm} is given by Eq. (4) and

$$G_n = \frac{2\, k_n h}{\sinh(2\, k_n h)} \qquad \Delta G_{nm} = \frac{2\, \Delta k_{nm} h}{\sinh(2\, \Delta k_{nm} h)} \tag{20e}$$

The summation in Eq. (20c) is less troublesome than would appear at first glance because $\sin(k_j h)$ converges rapidly towards zero with increasing j.

REPRODUCTION OF NATURAL LONG WAVES

The 2nd order paddle control signal, $X^{(2)}_{nm}(t)$, given by Eq. (12) for the shallow water case and by Eq. (20) for the general case, is valid for a regular short wave group characterized by the pair of indices $_{nm}$. For the reproduction of natural long waves (without parasitic, paddle displacement and progressive disturbance waves) it is necessary to carry out a summation over all pairs nm in analogy to Eq. (2) for the natural set-down, i.e.

$$X^{(2)}(t) = \sum_{n-m=1}^{\infty} \sum_{m=m^*}^{\infty} X^{(2)}_{nm}(t) \tag{21}$$

This signal has to be superposed upon the 1st order control signal $X^{(1)}$ that reproduces the natural short wave train.

EXAMPLES OF APPLICATION

Fig. 7 shows a wave record from Angra, Brazil. By means of pressure cells also the set-down was recorded. Eq. (3) was applied for the computation of the long waves (set-down). It is seen that calculated and measured long waves agree reasonably well. The paddle control signal for the correct reproduction of short and long waves was also calculated, applying Eqs. (21) and (20). The resulting signal is seen in Fig. 8. It is apparant that the correct reproduction of the long waves requires a much larger total stroke of the paddle than is the case for the short waves proper.

Another example of measured and calculated set-down is given in Fig. 9. These waves were recorded at Bintulu, Malaysia.

SHOALING PROPERTIES OF LONG WAVES

Alternatively to the direct compensation of the parasitic waves by a 2nd order control of the wave paddle, the model errors can be reduced by deriving advantage from the shoaling properties of the various long waves. The shoaling of the set-down can be calculated from Eq. (4). For simplicity, however, only the shallow water case shall be considered.

The shoaling of a parasitic wave ξ_p when it própagates from a larger depth h_1 to a smaller depth h_2 can be found by expressing that the energy flux is constant, i.e.

$$\frac{1}{2}\,\rho\, g\, \xi^2_{p,1}\, c_1 = \frac{1}{2}\,\rho\, g\, \xi^2_{p,2}\, c_2 \tag{22}$$

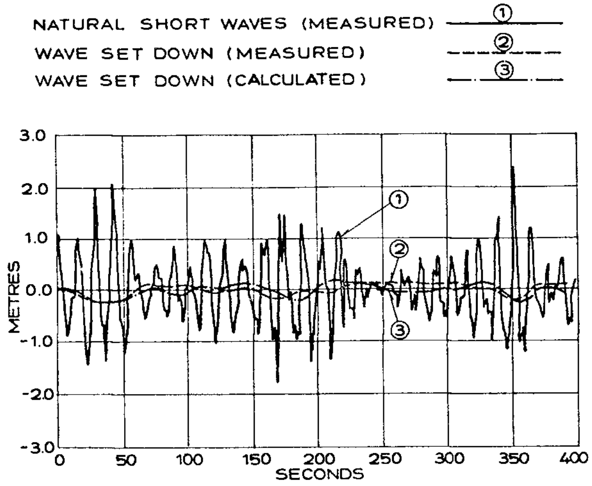


Fig. 7 Wave record from Angra, Brazil.

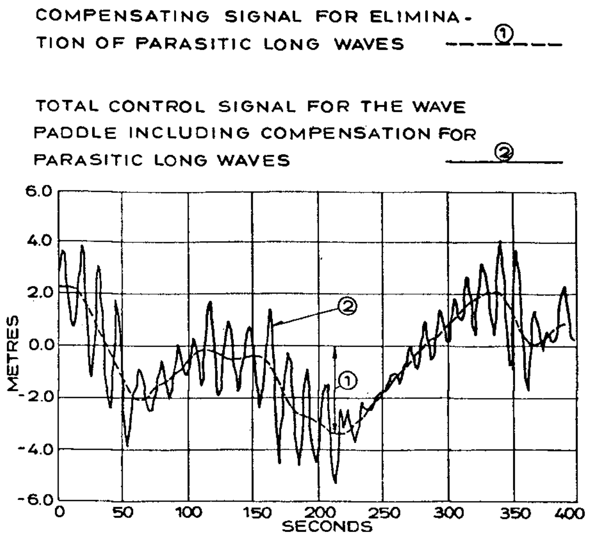


Fig. 8 Paddle control signal (to 2nd order).

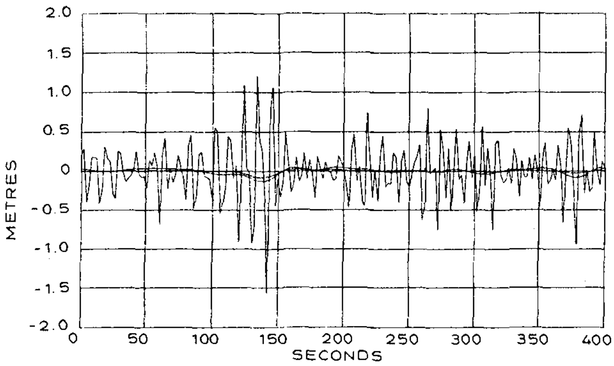


Fig. 9 Wave record from Bintulu, Malaysia.

With the phase velocity $c = \sqrt{gh}$ it appears that

$$\xi_{p,2}/\xi_{p,1} = (h_1/h_2)^{1/4} \tag{23}$$

The corresponding shoaling of the wave set-down can be found by applying an equation similar to Eq. (22) in combination with Fig. 4 and is

$$\xi_2/\xi_1 = (h_1/h_2)^{5/2} \tag{24}$$

Thus, the ratio between the wave set-down and the parasitic wave is

$$\xi_2/\xi_{p,2} = (h_1/h_2)^{9/4}\ \xi_1/\xi_{p,1} \tag{25}$$

This implies that the relative effect of parasitic waves can be reduced by generating the waves in somewhat deeper water, letting them propagate into the shallower model area (Fig. 10).

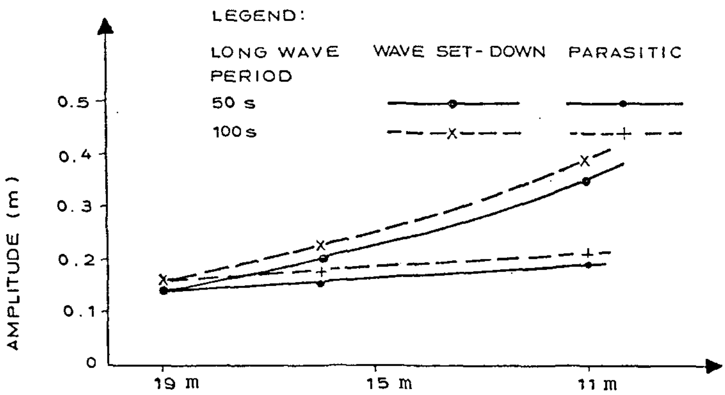


Fig. 10 Shoaling of long waves propagating from 19 m to 11 m depth for short period waves with H_s = 2.4 m and peak period T_p = 13 s.

In this connection, however, it should be mentioned that the method is not perfect. Thus, although the parasitic waves in Fig. 10 are small, they will build up gradually due to reflections from the boundaries of the model basin, because it is virtually impossible to absorb parasitic long waves. Still the method of shoaling is better than doing nothing.

ABSORPTION OF LONG WAVES

It is generally known that long waves constitute a serious problem in physical model tests because they are reflected from the boundaries, thus increasing the long wave disturbances in the basin.

In tests where basically the long waves are generated by the wave groups and appear as wave set-down, it is possible to avoid these boundary effects because the wave set-down disappears during the breaking of the short period waves (Fig. 11). This breaking, however, has to be gentle and take place over a certain distance, i.e. as spilling breakers. If, instead, the waves break abruptly in front of a steep slope, the wave set-down will be reflected as free long waves, which propagate back into the basin as parasitic waves. Therefore the criterion for absorption of wave set-down is simply that the short wave shall be absorbed as spilling breakers.

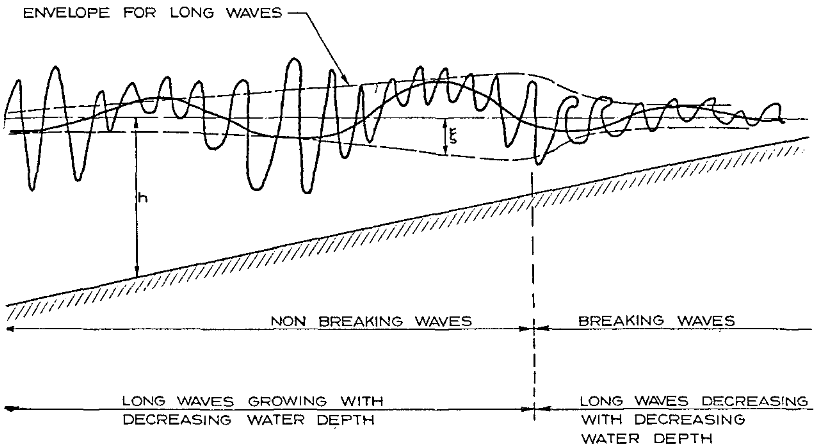


Fig. 11 Wave set-down in breaking waves.

CONCLUSIONS

Long wave phenomena in physical model tests can be correctly reproduced only if the following requirements are met:

(a) The parasitic long waves generated by the reproduction of natural waves must be eliminated by application of the 2nd order control signal defined by Eq. (21) in connection with Eq. (20) or, in the shallow water case, Eq. (12).

(b) Along the boundaries of the model basin waves shall be absorbed by spilling breakers.

As an alternative to the direct compensation of the parasitic waves, the model errors can, to a certain extent, be reduced by deriving advantage from the shoaling properties of the wave set-down and the parasitic waves. This requires:

(c) The wave generator be placed in a depth as large as possible compared with the general depth of the model area.

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