### CHAPTER 46

### A NUMERICAL MODEL OF STORM WAVES IN SHALLOW WATER

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### ABSTRACT

For studying storm waves in coastal areas, usual waves theories are no more valid. The presented cnoïdal theory allows the modelling of these problems. Furthermore, thanks to its non-linear properties, it makes possible the simulation of many phenomena usually neglected.

A numerical model using this theory has been developped. it has been tested against analytical results and certain properties of non-linear waves experimentaly observed.

Finaly, the practical problems raised by the utilisation of this model for harbour agitation computations, have been solved.

## 1 INTRODUCTION

Two-dimensional numerical models are now of common use to study tidal propagation and related currents in coastal areas. These models are based upon long waves equations. By assuming that the vertical velocity linearly increases from the bottom to the sea surface, instead of doing the classical hypothesis of hydrostatic pressure distribution, it is possible to simulate non-linear waves in shallow water. This theory (Serre, 1953) belongs to the so called cnoïdal waves theories. This assumption leads to new third-order derivatives in the momentum equations, and it is possible to adapt the tidal models to non-linear waves in shallow water problems by taking in account these new terms.

With such a model, it will be possible to compute non-linear waves of any form over any given bathymetry and current field. As this modelling gives quite correct simulation of breaking waves and associated radiation stresses, it seems that it will possible to compute longshore currents.

The paper presents the modifications made on a tidal numerical model and the problems raised by the new conditions of utilisation.

2 EQUATIONS

By assuming that the vertical velocity linearly increases from the bottom to the sea surface, it is possible to average over the depth the Navier-Stokes equations. So the mass and momentum conservation laws (Serre type equations) can be written as follow :

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$$\frac{\partial h}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0$$
(1)
$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x} \left(\frac{p^2}{h} + \frac{\partial}{\partial y} \left(\frac{pq}{h}\right) + \frac{\partial}{\partial x} \left(\frac{(q+\beta)}{2} + \frac{\alpha}{3}\right) h^2\right)$$

$$= -\left(q + \beta + \frac{\alpha}{2}\right) h \frac{\partial z}{\partial x} - g \frac{p}{c^2} \sqrt{\frac{p^2 + q^2}{h^2}}$$
(2)
$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{pq}{h}\right) + \frac{\partial}{\partial y} \left(\frac{q^2}{h}\right) + \frac{\partial}{\partial y} \left(\frac{(q+\beta)}{2} + \frac{\alpha}{3}\right) h^2\right)$$

 $= - (g + \beta + \frac{\alpha}{2}) h \frac{\partial z}{\partial y} - g \frac{q}{c^2} \sqrt{\frac{p^2 + q^2}{h^2}}$ (3)

where h is the water depth, p and q are x - and y - volume fluxes, z is the bed elevations, g the gravitational acceleration. The new terms  $\alpha$  and  $\beta$  come from the new assumption, and caracterise the vertical accelerations raised by the steepness of the waves and the slope of the bed :

$$\alpha = \frac{d^2h}{dt^2}, \quad \beta = \frac{d^2z}{dt^2} \quad (\text{with } \frac{d}{dt} = \frac{\partial}{\partial t} + \frac{p\partial}{p\partial t} + \frac{q\partial}{q\partial y})$$
(4)

# 3 NUMERICAL MODELS

More than the existence of the new terms  $\alpha$  and  $\beta$  (which are introduced both in the same maner in this theory and then will be computed with the same procedure), the increased non-linearity in waves computation compared to tidal problems, and the reduced number of computed points per wave length required to assure the viability of the system in engineering practice, are the main difficulties to solve.

A finite amplitude permanent (cnoïdal) wave can be in fact regarded as the superposition of several sinusoïdal waves. If the main one is described with a reduced number of points (10 for example), the first harmonic is computed with only  $\mathfrak{H}$  points, and the difference scheme must be able to propagate these coarsly described waves without any relative phase error, which would induce changes in shape, amplified then by the marked non-linearity of the problem. This leads to an extreme sensivity of the solutions of the equations to numerical error influence (Abbott, 1978).

The numerical model developped in the Laboratoire National d'Hydraulique uses finite differences approximations built on a single space grid. The fractionnary steps are used to compute each part of the equations.









# 3.1. One dimensional model :

The advective terms of (2) are computed the first, the propagative and friction terms being solved in a second step. A third-order accuracy scheme based upon characteristic method is used for the advective components of (2); this one has been fitted to total derivatives computations of (4). The elaboration of a high-accuracy difference approximation of propagative components of (1, 2) including  $\alpha$  and B terms, has necessitated the use of an implicit three-stage difference scheme. A Newton discretisation is used to linearize the h cubed term which appears in the pressure gradient of (2). The discretization has been designed to eliminate in the linear case, any damping and phase error for any number of computed points per wave length, at least for a Courant number equal to 1 (Von Newmann stability analysis). These propagative components are solved by a tri-diagonal double sweep algorithm after combination of flux and water depth. In spite of the marked non-linearity of the problem, and as for other reasons the Courant number must be as closer as possible to 1, no iterative procedure is necessary to solve the equations.

### 3.2. Two-dimensional model :

The fractionnary steps are also used in the 2.D model. The advective components are solved by the one dimensional characteristic method scheme in the x-direction and then in the y-direction. The  $\alpha$  and  $\beta$  terms are computed with the same procedure (characteristics). To solve the left terms of (1, 2, 3) (propagation), a two-dimensional iterative procedure is used. This one is based on spliting with coordination. Usually, a simple spliting induces differences between the resolutions in the x-and the y-directions. In our model, iterations are used to obtain same water depths in the two directions. The method of coordination used is very fast. It suppresses any polarisation due to the grid (see fig. 8, 10, 11), but the advantages of 1-D-computations are preserved.

## 3.3. Boundary conditions :

In order to simulate entering waves in the model at the seeward boundary, a condition allowing outgoing waves is necessary (see fig. 8, 9). The relation on the outgoing characteristic of the linearised theory has been suited in the following form :

fn - C(h + z) = - 2CWi

(5)

where fn is the outgoing normal flux

- Wi is the sea surface elevation of the incident wave
- C is the celerity which appears in the discrete form of the propagative components.

The same sort of condition has been imposed on inside breakwaters to model partial reflections :









753







d = 8 m H = 2 m DX = 7 m DT = 0,7 s

Fig.7 - PROPAGATION OF A SOLITARY WAVE IN A INCLINED CHANNEL (45°) fn - C  $(1 - \Upsilon)$  (h + z) = 0

(6)

where  $\gamma$  is a coefficient of reflection : (if  $\gamma = 1$  : total reflection, if  $\gamma = 0$  : no reflection).

In the 2.0 model, when there is an inflow a second boundary condition is required (Daubert, 1967). In that case, the normal derivative of the tangential flux is put equal to zero in the advective step. Thanks to the method of coordination used in the propagative step, the condition (6) can also be imposed on boundaries inclined to the grid, although it couples the two components of the volume flux. This coordination nearly suppresses shear effects on inclined boundaries (see fig. 7, 10).

### 4 NUMERICAL RESULTS

The modifications have been tested against analytical results in one and two dimensions for various Courant numbers and numbers of points per wave length. The different tests have shown that, for a Courant number equal to 1, a very good accuracy is obtained with values of the number of points per wave length as low as 20, even with a relative wave height equal to 0.4 (see fig. 1). For much smaller relative wave heights, the non-linearity of the governing equations decreases, and the number of points per wave length required can be reduced to 10. On the contrary, this one must be increased to conserve an acceptable accuracy if the Courant number differs from 1 (see fig. 2, 3).

The influence of the bottom shape and the geometry of the area has also been studied. Figure 4 shows the transformation of a solitary wave propagating over a slope onto a shelf of smaller depth. On the shelf, a desintegration of the initial wave into a train of solitary waves of decreasing amplitude is found. The amplitude of the crests is in good agreement with the results obtained by Madsen (1969) in the same case.

Figure 5 presents the phase shift obtained from non-linear effects after collision between two waves. This shift is in good agreement with the one experimentaly obtained by Maxworthy (1976).

Figure 6 shows the aptitude of the boundary condition (6) to simulate the whole range of reflections on different breakwaters.

The possibility of propagating a wave in a narrow channel inclined in the grid has also been studied. Figure 7 shows such a computation. Thanks to the coordinator, the condition on the normal flux (fn = 0) can be well imposed. The shear effect numericaly induced by the boundary condition nearly disappears in this case.

Figures 8 and 9 show the aptitude of the boundary condition (5) to generate incident waves in the domain (even with an angle of incidence) without presuming anything on outgoing wavés. In these



T = 26,8 s



Fig.8 - GENERATION AND TOTAL REFLECTION OF A WAVE ENTERING THE DOMAIN WITH AN ANGLE (45°)



d = 8 m H = 2 m DX = 5 m DT = 0,5 s

Fig.9 - GENERATION AND PARTIAL REFLECTION OF A WAVE ENTERING THE DOMAIN WITH AN ANGLE (45°)



d = 8 m H = 2 m DX = 5 m DT = 0,5 s

Fig.10-REFLECTION OF A WAVE ON A INCLINED BOUNDARY



d= 8 m , H = 2 m DX = 7 m DT = 0,7s

Fig.11 - DIFFRACTION - REFLEXION OF A SOLITARY WAVE ( BASIN 217 m × 392 m )



Fig. 12 - WAVE FIELD IN THE PORT OF FECAMP AFTER 50 sec. SIMULATION

761

figures two reflection conditions have been imposed on the opposite boundary : total reflection (fig. 8), partial reflection (fig. 9). In the two cases, the reflected wave has to establish itself after a diffraction period, which is best shown in figure 10. This one presents the computation of the reflection on an inclined boundary.

As, any changement in water deoth induces harmonics due to non-linear effects, any variation of the geometry of the area (diffraction) desintegrate the initial wave into a train of waves of decreasing amplitude. This is again shown in figure 11, where a wave entering a closed basin of  $217 \text{ m} \times 392 \text{ m}$  is presented. This example demonstrates that there is no polarisation induced by the grid in the computed wave (coordinator). In this case, the variation of width creates harmonics at the point of diffraction. Then, they propagate into the basin giving circular waves. After reflection on the opposite boundary, this induced a lapping in the basin.

## 5 CONCLUSION

The model, severly tested over the operational range of wave parameters, and over different physical conditions (all the computational tests presented have been done without any bottom friction, so there were no damping decreasing the computed waves) is now applied in coastal engineering practice. Figure 12 presents the first computations done in the port of Fecamp (French port on the English Channel).

#### 6 REFERENCES

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