A MEASUREMENT OF SLOPE, CURVATURE, AND DIRECTIONAL SPECTRA OF WIND WAVES IN LAKE MICHIGAN

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1. INTRODUCTION

Wave recording activity among coastal and oceanic engineers has increased rapidly in recent years, and ample wave data covering a large number of gage stations are now available. Directional wave spectra, however, are relatively scarce. The detailed knowledge of directional wave characteristics that correlate with the simultaneously recorded wind field is needed immediately for a proper understanding of the generation, growth, and propagation of a wave field. The data are necessary for engineering design, for verification and improvement of wave prediction models, and for provision of appropriate ground truth for calibrating remote sensing measurements. This paper represents the results of an effort to fulfill this need.

2. MEASUREMENTS

From July to October 1977, the Great Lakes Environmental Research Laboratory (GLERL) installed a light-weight, solar-powered research tower in Lake Michigan to provide a stable platform for supporting wind anemometers, temperature sensors, and an array of four wave staffs to measure meteorological and directional wave variables. The tower was located 2 km offshore from Muskegon, Mich., in 16 m of water (Fig. 1). The wave staffs, 6 m Kelk, Model P116, Zwarts gages (Zwarts, 1974), were placed at the center and vertices of an equilateral triangle of 3.05-m sides. The wave data were recorded digitally at 0.5-s sampling intervals for 30 minutes during each hour. The wind data used in this paper were recorded from an anemometer installed at 10 m above water level. A detailed description of the tower instrumentation data collection and reduction systems are given in Schwab et al. (1980).

3. DATA ANALYSIS

Conventional analysis for calculating the directional wave spectrum with a gage array usually yields poor directional resolution unless a large number of wave gages are used (e.g., Borgman, 1978). In order to

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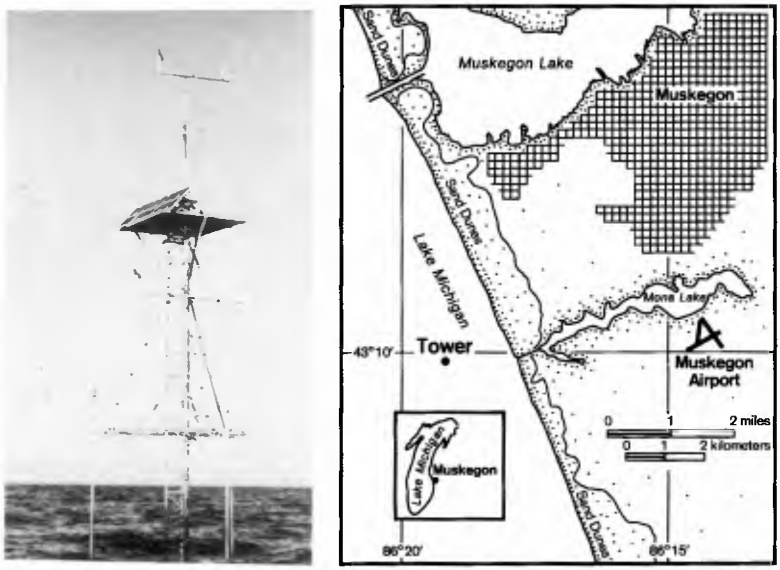

Fig. 1. GLERL research tower and its location.

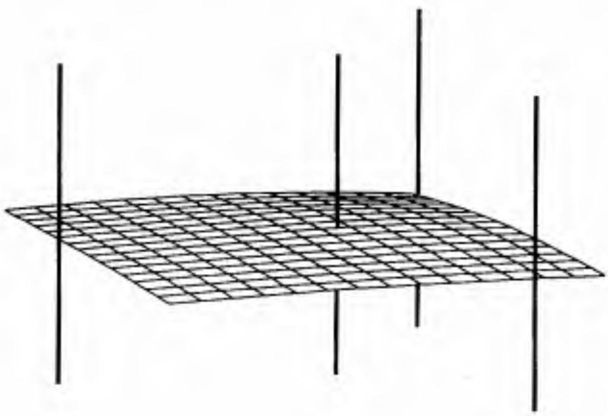

Fig. 2. An example of a fitted wave surface. The wave staffs are 6-m long.

improve the resolution with merely four wave staffs, the present study analyzes the data by fitting a cubic surface through the wave measurements. The fitted surface provides cubic spline approximations to the slopes and curvatures of the waves. Analysis was performed with the six quantities ζ, ζ_x, ζ_y, ζ_{xx}, ζ_{yy}, and ζ_{xy} where ζ is the surface fluctuation at the center wave staff and the subscripts x,y indicate partial differentiations in the east-west and north-south directions, respectively. In addition to directional spectra, which can be calculated readily using these quantities in the method developed by Longuet-Higgins et al. (1963) and extended by Cartwright and Smith (1964), this method also provides an opportunity to examine characteristics and statistical distributions of wave slope and wave curvature. An example of a fitted cubic surface is shown in Fig. 2.

4. WAVE SLOPES

Perhaps because of a lack of practical measuring instruments, there have been very few field measurements of surface wave slope except for those reported by Cox and Munk (1956) and Longuet-Higgins et al. (1963). A number of laboratory measurements have been conducted by Schooley (1954), Cox (1958), Wu (1971, 1977), and Long and Huang (1976), among others. These previous studies have shown that the statistical distribution of the wave slopes is approximately normal and the mean square surface slope increases as a function of wind speed. The data from this study basically agree with these results.

Fig. 3 presents a typical set of histograms for the statistical distribution of the east-west and north-south components of surface wave slope. They show essentially normal distributions. These histograms represent conditions during a southerly, 5 m s^{-1} wind, and skew is not evident. However, under stronger and growing wind speeds, an appreciable skew toward the windward direction indicated by Cox and Munk (1954) can also be found. The correlation of the mean square slope with wind condition is shown in Fig. 4. The data covered a wide range of wind conditions including both growing and decaying winds. The line drawn through the scattered points is representative of the mean square slope as proportional to the square of 10-m-level wind speed. The points away from the line are those under decaying winds. While the magnitude of the mean square slopes is similar to those oceanic cases calculated by Longuet-Higgins et al. (1963), it is much smaller than laboratory results. This is because laboratory measurements are for short dominant waves that contribute more toward mean square slopes than the long dominant waves found in the field and the instruments used in the field are not capable of measuring short waves.

5. WAVE CURVATURES

The study of wave curvatures is important in understanding the optical properties of the sea surface. Surprisingly few measurements have been made other than those made in the laboratory studies of Wu (1971, 1972, 1977). Theoretical studies (e.g., Kepr, 1969) have shown

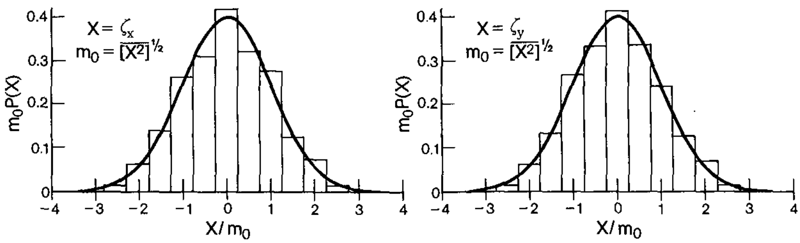

Fig. 3. Typical histograms of the distribution of x- and y-components of surface wave slope.

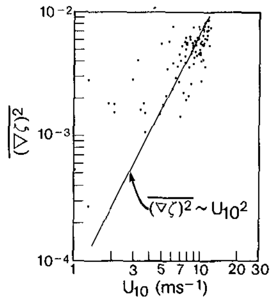

Fig. 4. Mean square surface slopes as a function of wind speed.

that if the surface is represented in the form

$$z = \zeta(x,y) \tag{1}$$

where (x,y) are horizontal coordinates and z is measured vertically upwards, the mean curvature J of the surface is given by

$$J = \frac{(1 + \zeta_y^{\,2})\zeta_{xx} - 2\zeta_x\zeta_y\zeta_{xy} + (1 + \zeta_x^{\,2})\zeta_{yy}}{(1 + \zeta_x^{\,2} + \zeta_y^{\,2})^{3/2}} \doteq \zeta_{xx} + \zeta_{yy} \tag{2}$$

and its total curvature, Ω, is given by

$$\Omega = \frac{\zeta_{xx}\zeta_{yy} - \zeta_{xy}^{\,2}}{(1 + \zeta_x^{\,2} + \zeta_y^{\,2})^2} \doteq \zeta_{xx}\zeta_{yy} - \zeta_{xy}^{\,2} \tag{3}$$

The simplified versions in (2) and (3) are obtained by neglecting the squares of the surface slopes since they are much smaller than 1. In general, the statistical distribution of the components ζ_{xx}, ζ_{yy}, and ζ_{xy} are normal, as shown in Fig. 5. Therefore the mean curvature J, being the sum of two normal variates, is also normally distributed (Fig. 6). Longuet-Higgins (1958) derived a strikingly non-normal distribution for the total curvature Ω. This distribution was found to be asymmetrical with positive skewness; its distribution function satisfies a linear differential equation depending on two parameters derived from the moments of the surface energy spectrum. A comparison of Longuet-Higgins' distribution with histograms from the present data is shown in Fig. 6. It is seen that the histogram does exhibit an asymmetrical character with positive skewness. However, its fit with the theoretical distribution is by no means close. Further examination shows that the theoretical distribution has a maximum skewness of 1.15, while the skewness obtained from the data is consistently greater than 1.15 by a factor of 2 or more. One possible explanation is that the statistical characteristics of the variates ζ_{xx}, ζ_{yy}, and ζ_{xy} are assumed to be similar in the theoretical derivations. In the data, however, it has been found that the moments of ζ_{xy} are consistently an order of magitude smaller than those of ζ_{xx} and ζ_{yy}'s.

The variations of curvature with wind have also been examined. Fig. 7 represents mean square values of J and Ω plotted against wind speed. From the data obtained under varied wind fields, the mean square of total curvature appears to be proportional to U_{10}, while the mean square of mean curvature is proportional to the square root of U_{10}.

6. DIRECTIONAL WAVE SPECTRA

Previous studies of directional wave spectra have been concerned mostly with single instances of time. The continuous measurements available from the GLERL research tower provide an opportunity to study directional characteristics of waves episodically. In the present

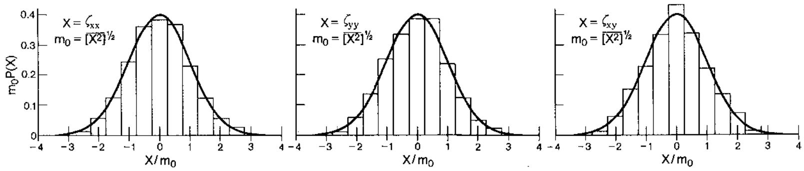

Fig. 5. Typical histograms of the distribution of component curvature.

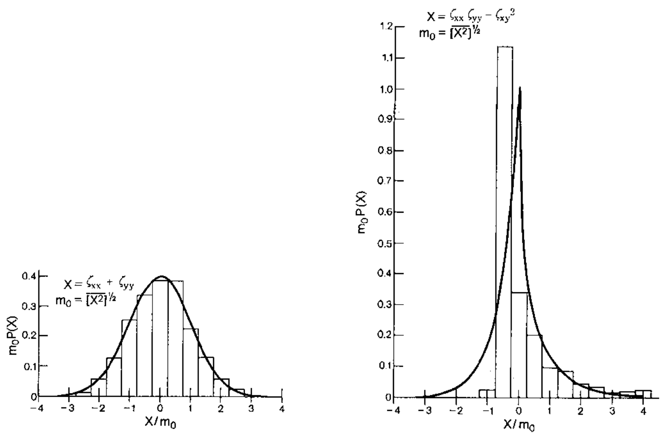

Fig. 6. Typical histograms of the distribution of mean and total curvatures. The non-normal distribution curve represents Longuet-Higgins' theory.

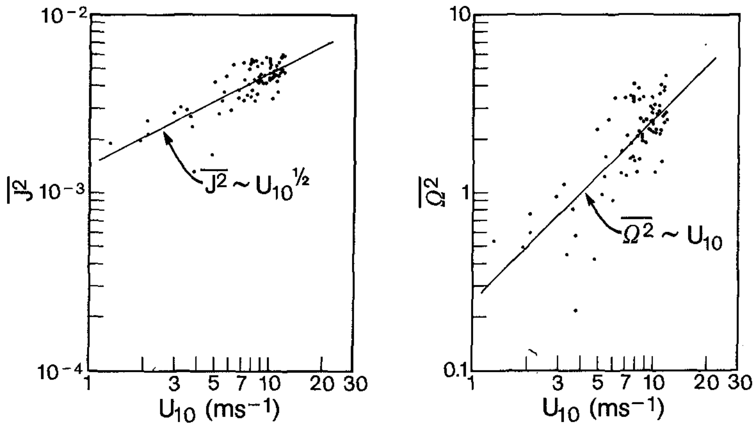

Fig. 7. The mean square values of mean and total curvatures as a function of wind speeds.

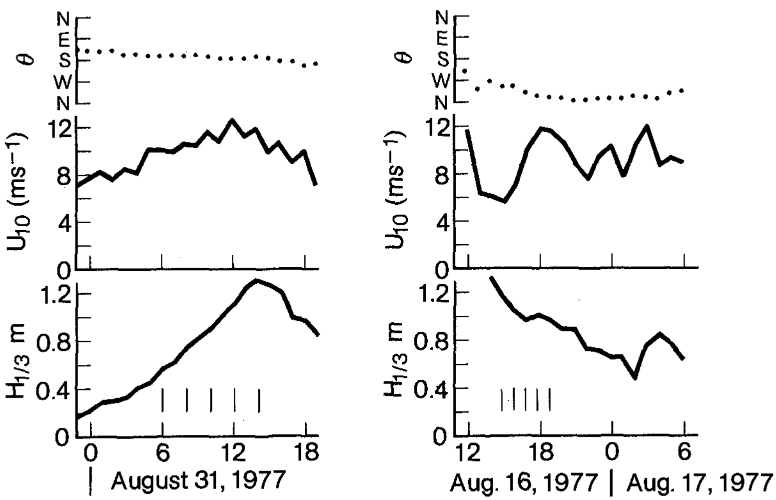

Fig. 8. Two episodes of wind and wind fields for directional spectral study, August 31, 1977, and August 16, 1977.

study two episodes, each with a characteristic wind field, were selected for illustration. Fig. 8 presents the wind and wave fields of the two episodes.

The episode of August 31, 1977, has a constant southerly wind direction with increasing wind speed that generates steadily growing waves. Five sets of single and directional wave spectra at 2-hour intervals between 0600 and 1400 hours, as shown by the tick marks in Fig. 8, are presented in Fig. 9. The directional spectra were plotted on a polar graph with frequency as the radial axis. The contours represent equal spectral energy densities. While contour levels were not indicated, in order to avoid crowding in the figure, it should be noted that the outside contours had the lowest magnitude and remained the same for all the figures. The contour levels increased logarithmically toward the hill, which corresponded to the spectral peak in the single spectrum. It can be seen that as waves grow the contours spread toward lower and higher frequencies and higher spectral density develops around the hill. The directional spectra are unimodal with some skewness toward higher frequency, especially when waves grow higher.

These results also show that the directional spreading tends to be narrow only around the spectral peak and wide for both higher and lower frequencies. This is consistent with the $\cos^{2s}\theta$ model, where a higher s indicates narrower spreading. It has been shown by Mitsuyasu *et al.* (1975) that s is the highest at the spectral peak and decreases toward both higher and lower frequencies.

The episode of August 16, 1977, on the other hand, shows a wind direction changing from south to northwest, with a somewhat fluctuating wind speed. The five directional spectra shown in Fig. 10 were between 1500 and 1900 hours during periods of increasing wind speed, while waves decayed because of the changing wind direction. At 1500 hours, the spectrum was clearly northward owing to previously southerly wind. At 1600 and 1700 hours the waves decayed, the hill remained northward even though the wind direction had changed. There were indications that the contours at both the higher and lower frequency sides tended to spread toward the new wind direction. At 1800 hours an interesting bimodal picture had developed. A new hill showed up in the new south-southwest wind direction, but an old hill remained northward with diminishing magnitude. It was also interesting to note that with the bimodal directional spectrum, there were corresponding two peaks in the single spectrum. Finally, at 1900 hours, the waves conformed to the direction of the prevailing wind. The actual duration to which the wind switched from a steady south to a steady northwest direction lasted over 8 hours. The directional spectra carried the memory of northward direction during most of this time.

Changing wind direction is a familiar and common phenomenon. Hence the response of waves should also be familiar and common. However, the process of direction is also extremely varied; hence a knowledge of these responses is difficult to assess. The results shown in Fig. 10 represent a first look into one of these interesting and intriguing common occurrences.

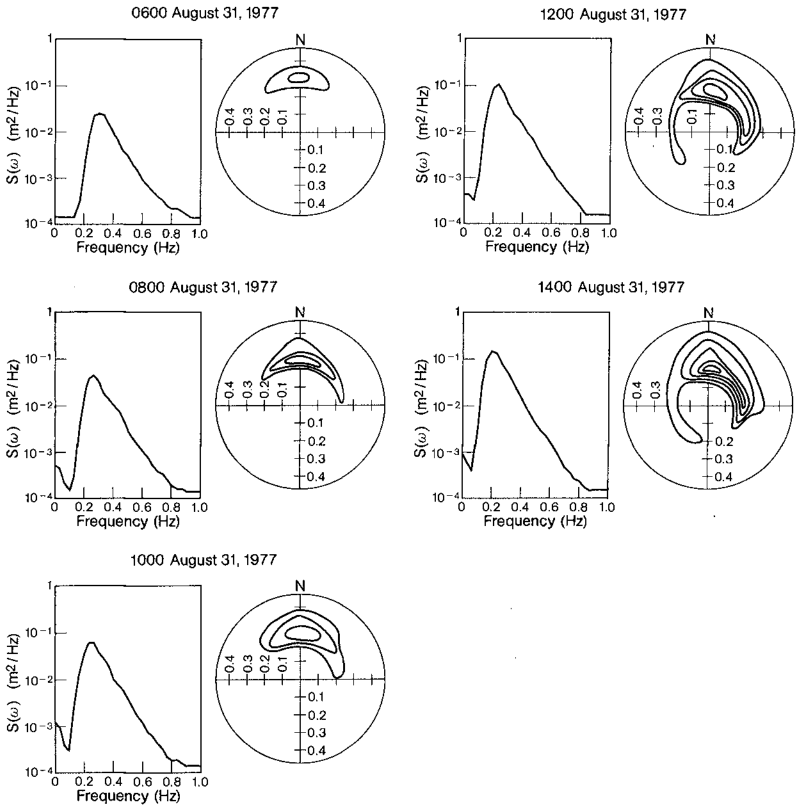

Fig. 9. Single and directional spectra during an episode with constant wind direction, August 31, 1977.

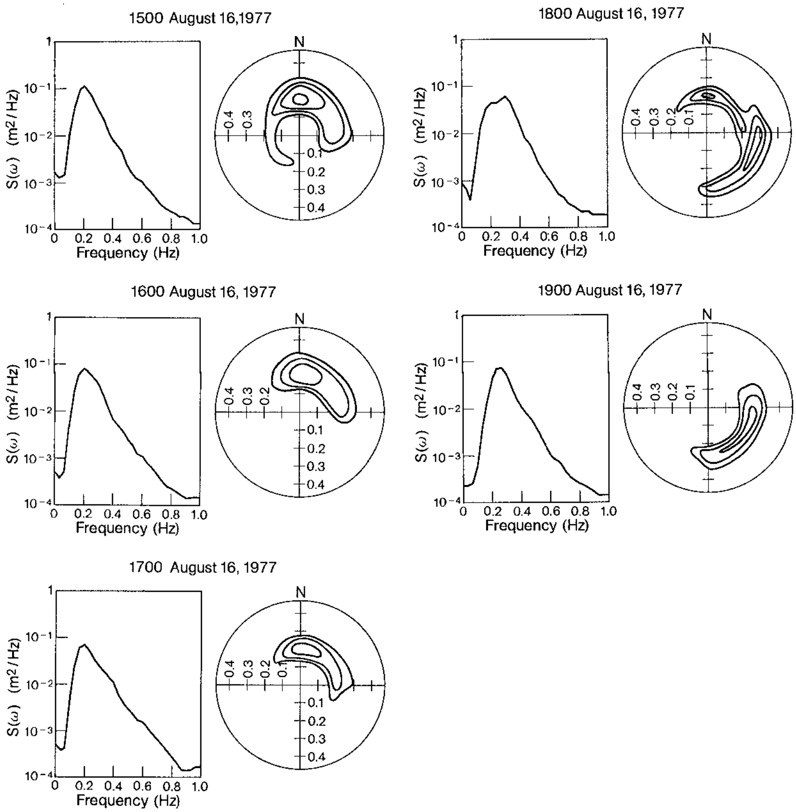

Fig. 10. Single and directional spectra during an episode with changing wind direction, August 16, 1977.

7. CONCLUDING REMARKS

By fitting a cubic surface through the measurements of the four wave staffs, the result of this paper has shown that wave slope, curvature, and directional spectra can be readily calculated with improved resolution over conventional array measurments. Direct measurement of wave slope and curvature in the field has been extremely rare. This study was able to provide these data indirectly and an examination of their statistical characteristics. The results are generally consistent with previous studies. The continuous measurements made from the research tower provided an opportunity for interesting episodic case studies, which is an improvement over previous directional wave analysis that studies mostly single instances in time. This paper represents a first step toward filling the need for definitive directional wave information. Efforts to quantify these results by parameterization, correlation, and modeling are presently under way at GLERL.

8. REFERENCES

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