CHAPTER 20

FIELD MEASUREMENTS OF WIND WAVE KINEMATICS

by

J.A. Battjes1) and J. van Heteren2)

ABSTRACT

Measurements of surface elevations and three orthogonal subsurface velocity components have been performed from a platform in the southern North Sea, off the Dutch coast, in order to check linear spectral transfer functions relating subsurface velocities to surface elevations, to obtain information about the directional properties of the waves, and to investigate the velocity statistics. The data obtained as far do not cover a sufficiently wide range of conditions to permit general conclusions. Some preliminary results are presented in the paper.

1. INTRODUCTION

Routine instrumental measurements of wind wave parameters are usually restricted to the free surface fluctuation. However, in applications in offshore and coastal engineering one needs information about the internal wave kinematics. Examples are the calculation of the loads on a structure due to a given wave field, or the calculation of densities and fluxes of energy and momentum, which are needed in problems of wave shoaling and refraction, and the prediction of nearshore currents.

Transforming data on surface elevations into velocity data can be accomplished in different ways. These can broadly be classified into two groups, that of a quasi-deterministic non-linear, individual wave analysis, and that of a random linear spectral approach. The principal advantage of the latter is its superiority in dealing with aspects of randomness in time and space (short-crestedness) of the wind waves. Its main drawback is the restriction to linear problems, but there are developments towards the inclusion of some degree of nonlinearity in the formulations (see for example Sharma and Dean, 1979).

1) Professor, Department of Civil Engineering, Delft University of Technology, The Netherlands
2) Staff Member, Hydraulic Division, Deltadepartment, Rijkswaterstaat, The Hague, The Netherlands.
Until a few years ago, there have been rather few published experimental verifications of the linear spectral approach, and those were typically in waves of rather low steepness (Thornton and Krapohl, 1974). It was then decided to initiate a research project in The Netherlands, aimed at such verification, to be based on measurements off an available platform in the southern North Sea. More specifically, the aims were to test the validity of linear spectral transfer functions relating internal velocities to surface elevations, to check the applicability of the model of a Gaussian process to the velocities, and, in addition, to obtain data on the short-crestedness of wind waves.

Due to a variety of causes related to equipment and personnel, progress in this work has been below expectations. Meanwhile, a number of studies by others has been published in which aims were pursued similar to those of the present investigation, and in which significant results were obtained (Cavaleri et al., 1977; Forristall et al., 1978; Guza and Thornton, 1980). It was nevertheless deemed useful to continue the present work, inasmuch as it gives independent additional data, obtained in different geographical conditions, with different sensors, and in part analysed differently. However, due to the delays referred to above, the measurements do not yet cover a wide range of conditions, so that no general conclusions can be drawn. For that reason this paper is kept rather brief. It is planned to give a more extensive presentation elsewhere, after additional data have been obtained and analysed.

The contents of this paper are as follows. Section 2 contains a description of the platform used for the measurements, and of its location. In section 3 the sensors, the data logging system and the transmission are described. The data available at the moment of writing are described in section 4. The methods of estimation of the transfer function, wave direction and short-crestedness are dealt with in section 5. Examples and some results of the estimated transfer functions are presented in section 6, those of the wave directions and short-crestedness in section 7, and those of the statistical properties in section 8. Section 9 gives some preliminary conclusions.

2. LOCATION AND DESCRIPTION OF THE PLATFORM

The measurements were made from a platform (MPN) located in the southern North Sea, about 10 km off the Dutch Coast (see figure 1). The local depth is about 17 m below M.S.L., and the local mean tidal range is approximately 1.80 m.

The platform has been built on a jacket construction, made from tubular elements which are up to 0.80 m in diameter. At three of the four corners of the platform, vertical sensor supports have been mounted (figure 2). These supports are space trusses.
consisting of tubes with diameters up to 0.15 m. They can be lifted out of the water for attachment, inspection and cleaning of sensors.

![Location of the platform (MPN)](image)

Figure 1: Location of the platform (MPN)

3. INSTRUMENTATION

The velocity meter used for the measurements described in this paper is based on the principle of travel time of acoustic pulses. A meter of this type designed for use in tidal rivers has been described by Botma (1978), who also designed the meters used in the present study.

At first the velocity meter consisted of only one pair of transducers/responders. The distance between this pair of transducers/responders was 1.71 m and they were mounted with the axis in the vertical position so that the vertical velocity component (w) could be measured. The calibration factor of this instrument was calculated using a nominal value of the speed of sound in sea water, and an effective travel distance which was 10 cm less than the actual distance between the sensors, to allow for wake effects.

The wave gauge used for these measurements of the surface elevation, a Baylorgauge, is based on the conductivity of water and it consists of two vertically tensed wires with a length of about 20 m. The distance between the two wires is 22.5 cm. Deviations from linearity and variations in calibration factors were both less than 0.5%.

Both sensors were fixed to the sensor support situated at the
south west side of the platform, with the wave gauge in the centre of the sensor support and the velocity meter at the west side of the sensor support. The horizontal distance between the two sensors was 1.20 m. The elevation of the velocity meter was 5.70 m below M.S.L.

Afterwards this velocity meter has been extended in such a way that three mutually orthogonal velocity components \((u,v,w)\) could be measured. To that end three pairs of transducers/responders were mounted in a glued frame, see figure 3. This frame consists of copper pipes (outer diameter 50.4 mm) to minimize marine fouling. It was mounted outside the sensor support with one of its axes in the vertical position. The orientation of the other axes is arbitrary.

The distance from the vertical axis of the velocity meter to the center of the sensor support has been increased from 1.20 m to 1.75 m so as to decrease the influence of the sensor support and platform piles.

In contrast to the one-dimensional meter, the three-dimensional version compensates for variations in the speed of sound due to variations in temperature and salinity of the sea water. The calculated calibration factor was checked experimentally by oscillating the frame rectilinearly in a laboratory basin. It was found to be correct within 5%, except when the meter was oscillated exactly in line with two sensors measuring one component, in which case the
deviation was up to 10%. This phenomenon is ascribed to the wake development. Since in the field the flow is not rectilinear, this deviation in the calibration is not taken into account in the analysis.

The wave gauge used for the measurements with the extended velocity meter is a stiff step gauge with a length of 15 m. It measures the number of wet electrodes of known mutual distance (0.1 m), so that the calibration is trivial.

The horizontal distance between the two sensors is 1.75 m. The elevation of the velocity meter has been 4.91 m below M.S.L. and 7.94 m below M.S.L.

Figure 3: the three-dimensional velocity meter

Through a data logging system the measured quantities can be scanned at intervals of 0.25 s (per quantity). Because the different channels are scanned successively there is a time interval between two successive channels of 0.25 s divided by the number of channels of the data logging system, in this case 32.

The data are transmitted by an F.M. radio-link to a shore station where they are recorded on magnetic tape. At intervals of 5 min, the significant wave height and the total energy of the frequencies lower than .1 Hz are recorded as well (as part of another project).
4. DATA

The time of recording is normally chosen to be near slack water in order to minimize disturbances due to the wake of the platform and sensor piles.

With the original one-dimensional velocity meter 31 measurements have been made, simultaneous with measurements of the surface elevation, distributed over three days with different climatologic conditions.

With the extended velocity meter two sets of measurements have been performed at the time of writing. First, 26 measurements of the three velocity components only were made, distributed over 6 days. The depth of the centre of the frame was 4.91 m below M.S.L. Later, velocity measurements have been made simultaneous with those of the surface elevation. At the moment of writing 32 such measurements have been made, distributed over 6 days with different climatologic conditions. During the second set of measurements the depth of the sensor was larger, namely 7.94 m below M.S.L.

5. CALCULATIONS

The respective transfer functions are calculated by auto- and cross spectral analyses, which also yield phase and coherence information. Standard procedures as described by Bendat and Piersol (1971) and Jenkins and Watts (1968) have been used based on an F.F.T. algorithm. The length of the analyzed records has been taken 1800 s. For the calculations of the spectra the records are divided in 30 segments, each with duration of 60 s. Because no data window has been used the number of degrees of freedom is 60 and the resolution 0.0167 Hz.

The auto- and cross spectra of the vertical velocity \( w \) and the surface elevation \( \zeta \), written as \( S_{ww} \), \( S_{w\zeta} \) and \( S_{\zeta\zeta} \), have been used to calculate the transfer function

\[
H_{\zeta w} = \frac{|S_{\zeta w}|}{S_{\zeta\zeta}},
\]

the phase function

\[
\phi_{\zeta w} = \frac{\text{arg} \ S_{\zeta w}}{S_{\zeta\zeta}},
\]

and the squared coherence function

\[
\gamma_{\zeta w}^2 = \frac{|S_{\zeta w}|^2}{(S_{\zeta\zeta} S_{ww})}.
\]

The procedure sketched above cannot be used for the relation between the surface elevation and the horizontal velocity \( \hat{u} = (u,v) \), because of the short-crestedness of the waves. Assuming the system to be free of noise, a transfer function can be estimated as

\[
H_{\zeta \rightarrow \hat{u}} = \frac{(S_{uu} + S_{vv})}{S_{\zeta \zeta}}^{1/2}.
\]

Transfer functions calculated from the measurements as indicated above will in the following be referred to as "measured" transfer functions.
Assuming the water motion and the measurements to be linear and noise-free, the calculated auto- and crossspectra for \((u, v, \xi)\) or \((u, v, w)\) can be used to estimate directional properties of the waves by standard procedures (see Borgman, 1979, for a review). Via the auto spectra and cross spectra the truncated Fourier series and the parameters of the \(\cos^2\)-model of the directional spectrum have been calculated, for different frequencies. From these results, short-crestedness parameters can be calculated, again as a function of frequency.

In addition to the spectrally resolved short-crestedness parameters referred to above, an overall (i.e., non-spectral) measure of the short-crestedness of the waves can be defined. Generally the mean squares and the mean product of \(u\) and \(v\) are non-zero. Let \((u^*, v^*)\) denote the values of \((u, v)\) in a reference frame in which \(u^*\, v^* = 0\). The axes of this reference frame then define the principal directions, and \((u^*_x, v^*_x)\) represent the principal values of the velocity product tensor. An overall measure of the short-crestedness of the waves can then be defined as:

\[
\Gamma = \frac{\text{Var}_x}{\text{Var}_x^2},
\]

such that by definition \(0 \leq \Gamma \leq 1\). Values of this short-crestedness parameter have been calculated from the data.

From the mean squares and the mean product of \(u\) and \(v\), an overall principal wave direction can be obtained, e.g., by using the Mohr circle.

In addition to these calculations the statistical properties of the surface elevation and the velocity components are investigated. The distributions of the instantaneous values are compared with the normal distribution.

6. COMPARISON WITH LINEAR THEORY

6.1. The transfer function of the vertical velocity component and the surface elevation.

As indicated in section 4, the first set of measurements (31) was carried out with a one-dimensional velocity meter, which was so aligned that it measured the vertical velocity component. The results will not be presented here, with one exception. The average of the 31 ratios of the measured transfer function \(H_{uv}\) to the theoretical (linear) one, at the peak frequency of \(S_{\xi\xi}\), was about 0.87. If the effects of possible density variations of the sea water are considered, this ratio is estimated to vary between 0.87 and 0.92, with a standard deviation of 0.04. (Note: in an example given in the Conference preprint of this paper, said ratio was 0.75. However, it later turned out that the calculation procedure com-
At the moment of writing a series of simultaneous measurements have been made of \((\zeta, u, v, w)\), using a wave gauge and the three-dimensional velocity meter. They have not yet been analysed fully. Some results of one measurement will be shown here by way of illustration. The data were recorded on Jan. 16, 1980, from 17 h.10 min. GMT to 17 h.40 min.

In figure 4 the wind- and sea conditions prior to and during the time of recording are shown. Figure 5 shows the surface elevation spectrum \((S_{\zeta\xi})\), and figure 6 the coherence function between surface elevation and vertical velocity. The dashed lines indicate the 90% confidence bands. In the frequency interval which contributes most to \(\zeta^2\), the squared coherence appears to be nearly 1. This indicates that the relation between \(\zeta\) and \(w\) is approximately linear and noise-free in that interval.

In figure 7 the measured transfer function \(H_{\zeta w}\) is compared with the corresponding transfer function based on the linear theory. For the range where the coherence is nearly one the calculated transfer function corresponds well with the theoretical one.

Figure 8 shows the ratio of the measured transfer function to the theoretical (linear) one as a function of frequency. This ratio does not vary significantly from the 100% value. A comparison of the measured velocity spectrum \((S_{uv})\) and the corresponding spectrum calculated from the linear transfer function and the measured elevation spectrum is given in figure 9. As expected from the results contained in the figures 7 and 8, the discrepancies are small in the region where most of the energy is located.
Figure 5: surface elevation spectrum, with peak frequency $f_m$ and area $m_0$.

Figure 6: squared coherence function

Figure 7: measured (—) and theoretical (---) transfer function

Figure 8: measured transfer function as a percentage of the theoretical transfer function.
Figure 9: auto spectrum of the vertical velocity; measured (---); calculated via the linear transfer function and the measured surface elevation spectrum (-----)

The phase difference between surface elevation and vertical velocity is shown in fig. 10. It is seen to vary approximately linearly with frequency; if extrapolated to \( f = 0 \) Hz, the phase difference is 90\(^\circ\), as expected in the linear approximation for all frequencies.

The observed phase function indicates a constant (with respect to frequency) time shift (0.83 s in the example of fig. 10) between elevation and vertical velocity, in addition to the predicted 90\(^\circ\) phase difference. This seems very unrealistic physically, and it gave rise to suspicions about the system of instrumentation and data handling. However, search efforts in that direction have so far not given evidence of the source of the "observed" shift. Another contributing factor may be the horizontal separation between the two sensors, but the effect of that would be too small to explain the 0.83 s. At the moment of writing the response times of the instruments are being re-investigated. No further results about phase differences will be reported in the present paper.

In the context of observed phase lags which seem to be unrealistic, in the sense that they cannot be explained by known processes, reference is made to Cavaleri et al (1977), who found good agreement in the observed and predicted phase difference between elevation and vertical velocity, but not for the horizontal velocities. The fact that horizontal and vertical velocities in their results were not in quadrature implies a net vertical transfer of horizontal momentum, which however could not be balanced with other physical processes such as wind stress at the water surface.
6.2. The transfer function of the vertical velocity component and the horizontal velocity component.

Figure 11 shows a comparison of the measured transfer function \( H_{vw} \), defined as \( \left( \frac{S_{vv} + S_{vw}}{S_{ww}} \right)^{1/2} \), to the theoretical one. Since this transfer function is estimated via the auto-spectra, possible noise is not eliminated. In the case of the relation between vertical velocity and surface elevation, the noise level was found to be negligible in the frequency band where most of the energy is concentrated (see figures 6 and 9). Assuming the same for the relation between vertical and horizontal velocities, the transfer function \( H_{vw} \) was calculated in the frequency range in which \( S_{ww} \) is at least 5% of its maximum value. In the lower-frequency part of this range there is a good agreement between the measured and predicted transfer functions, but the measured values show irregularities in the higher-frequency part, which suggests the presence of velocities not related to surface waves.

6.3. The transfer function of the surface elevation and the horizontal velocity component.

A comparison of the measured transfer function \( H_{\xi w} \) to the theoretical one is shown in figure 12, within the range where \( S_{\xi w} \) is at least 5% of its maximum value. In most of this range there is a good agreement between the calculated transfer function and the theoretical one according to the linear theory.

Figure 13 shows three sequences of ratios of measured transfer functions to the linear prediction, at the peak frequency of the auto-spectrum of the input (\( \xi \) in case of \( H_{\xi w} \) and \( \eta \) in case of \( H_{\eta w} \)). The sequences were obtained from a series of 30 min. measurements in one day, with mild and almost-constant wind and sea conditions (see figure 4). The
average value of each of the three ratios is very nearly 100%.
Additional data, including some taken in more severe conditions,
are being analysed so as to obtain a broader base for conclusions.

Figure 11: transfer function of
the vertical velocity
and the horizontal
velocity; measured
(--), theoretical
(- -).

Figure 12: transfer function of
the surface elevation
and the horizontal velo-
city; measured (--);
theoretical (- -).

Figure 13: ratios of
measured to theo-
retical transfer
function values
near the peak of
the auto-spectra,
for some succes-
sive 30 min.
measurements on
Jan. 15 and 16,
1980.
7. WAVE DIRECTION AND SHORT-CRESTEDNESS

Via the auto spectra and the cross spectra of the surface elevation and the two horizontal velocity components two components of the truncated Fourier series per frequency and the parameters of the $\cos^{2\theta}$-model per frequency can be calculated. This can also be done via the auto spectra and cross spectra of the vertical velocity component and the two horizontal velocity components. Because the resolution of this truncated Fourier series is very low ($72^\circ$, Longuet Higgins, 1963) and because the $\cos^{2\theta}$-model only gives one main direction per frequency, interpretation of the results of these calculations is very difficult. Therefore only an indication of the principal wave directions can be obtained. Some typical results will be given in the following, without going into the details.

For the frequencies round 0.10 Hz the calculated main wave directions are found to be in agreement with the direction of swell coming from the northern part of the North Sea if refraction is taken into account (Holthuysen, 1973). For the frequencies round 0.30 Hz the calculated wave direction is about the same as the direction of the wind. Between these frequencies the direction of the waves changes slowly from the direction of the swell near $f = 0.10$ Hz to the wind direction near $f = 0.30$ Hz. It may be that in this range there are two main directions per frequency of which for the lower frequencies the direction of the swell dominates and for the higher frequencies the direction of the sea.

In figure 14 an example is given of the determination of the overall measure of the main wave direction and of the overall short-crestedness, using the Mohr circle. Measured values of $u^2$, $uv$ and $v^2$ are plotted, from which the principal values $(u^2, v^2)$ and the principal directions $(\theta, \theta + \pi/2)$ can be obtained. The angle $\theta$ is referred to the direction of the $u$-component, from which the angle $\theta'$ is calculated, which is referred to North.

As can be seen from the example, the overall principal direction calculated from $u^2$, $uv$ and $v^2$ ($\theta' = 277^\circ$) has about the same

$\theta = 38^\circ$
$\theta + \pi/2 = 128^\circ = \theta' = 277^\circ$

From $\cos^{2\theta}$-model:
$\theta' = 310^\circ$ to $320^\circ$ for $f = 0.15$ Hz
$\theta' = 270^\circ$ to $280^\circ$ for $f = 0.20$ Hz

(peak frequency of $S_\xi$ is $f_m \approx 0.17$ Hz)

Figure 14: the calculation of the main wave direction and short-crestedness using the Mohr circle.
value as the main directions in the $\cos^2\alpha$-model for frequencies near the spectral peak.

For the example shown the value of the short-crestedness parameter $\Gamma$ is $\frac{\overline{u_x}}{\overline{u_x} + \overline{u_y}} \approx 0.34$. The measurements were made in a mild sea state following about 20 hours of almost constant wind direction, so that this value of the short-crestedness parameter is expected to be typical for wind waves in an ideal, steady generation situation. The value of 0.34 is in accordance with an indirect result by Forristall et al (1978), who found that a value of the ratio $\frac{\overline{u_x}}{\overline{u_x} + \overline{u_y}}$ of about 0.75 gave the best fit of the theoretical distribution of the magnitude of the horizontal velocity to the observations (said ratio is a parameter of this distribution). These measurements were made in a hurricane. Lastly, it is noted that a value $\Gamma = 1/3$ is consistent with a $\cos^2\alpha$-directional spectrum, which has often been used in the past as a rough approximation.

Although this paper does not deal with applications, it is pointed out that the overall short-crestedness is important in the determination of the radiation shear stresses, which are proportional to $(\overline{u_x} - \overline{v_y})$, in a wave field of given total energy, which is proportional to $(\overline{u_x} + \overline{v_y})$ (Battjes, 1972). The radiation shear stress is the driving force for longshore currents.

8. STATISTICAL PROPERTIES

Figure 15 shows a histogram of instantaneous values of a horizontal velocity component and the best fitting Gaussian probability density function. Based on visual observations from results such as shown in figure 15, it appears that for engineering purposes, the three velocity components and the surface elevation are normally distributed. However, it should be borne in mind that the measurements so far have taken place in rather mild to moderate sea conditions.

![Figure 15: example distribution of instantaneous values of a velocity component](image-url)
9. CONCLUSIONS

The measurements available at the time of writing do not cover a sufficiently wide range of conditions to permit very general conclusions. The following tendencies have been observed.

In the first series of measurements, in which a one-dimensional (vertical) velocity meter was used together with a Baylor-type surface wave staff, the measured vertical velocities were found to be on the average about 30% below the predictions using linear theory and measured elevation spectra. No significant deviation has been found in the analysis of the simultaneous measurements of the three mutually orthogonal velocity components and the surface elevation. A reason for these different results has not yet been firmly established.

The observed phase spectra for the relations between elevation and velocity show a linear dependence on frequency, which indicates a constant time shift between the two signals. Although it is believed that this shift is not physically realistic, efforts to locate its origin in the system of instrumentation and data handling have so far not been successful.

The principal wave directions calculated via the auto spectra and cross spectra of the surface elevation and the two horizontal velocity components show for the low frequencies a direction which corresponds with the direction of swell coming from the northern part of the North Sea if refraction is taken into account and for the high frequencies a direction which corresponds with the direction of the local wind. Between these two ranges the calculated main wave direction changes slowly.

From the mean squares and the mean product of the two horizontal velocity components an overall (i.e. non-spectral) short-crestedness parameter and an overall main wave direction have been calculated. It appears that there is a good agreement between the direction of the waves calculated in this way and the directions calculated by conventional spectral methods for the frequencies near the spectral peak.

From the investigation of the statistical properties of the three velocity components it appears that for engineering purposes, the distribution of these components can be accepted as being normally distributed.

ACKNOWLEDGEMENTS

The work described in this paper was carried out as part of the Coastal Research Program (TOW-K) of the Department of Public Works of The Netherlands.
REFERENCES

Borgman, L.E. 1979 - Directional wave spectra from wave sensors, Int Marine Science 8, Plenum Press, N.Y.