WIND WAVES TRANSMISSION THROUGH POROUS BREAKWATER

by

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INTRODUCTION

Rubble - mound breakwaters are designed to protect exposed marine areas from excessive wave activity. The resulting interaction of the incident waves with the rubble units is extremely complex due to the variable reflective and frictional properties of the permeable structure. In the past decade considerable effort has been expended to derive rational methods for of such type structure. The theoretioal and experimental investigations have been focused especoialy on the prediction of the reflection and transmission of regular waves incident to breakwater. Sollitt and Cross /1972/ presented the analytical approach to the problem based on the assumption that the original nonlinear govering equation of the wave motion into porous media may be replaced by a linear one so as to give the same average rate of dissipation/Lorentz approximation/. Under the assumption that severe wave conditions for most breakwaters correspond to relatively lond waves, the considerably simple solutions were developed by Kondo and Toma /1972/ and in series of papers by Madsen and co-authors /1974, 1977, 1978/. Madsen's solutions follows rather a physioal than mathematioal rigorous approach to the problem. The momentum equation evaluated him explains the influence of the inertia force assooiated with the unsteady flow arround the solid partioles. Very careful analytical examination of this problem for the long waves past the narrow gaps and holes has been presented by Mei at al. /1974/. The study indicate that apparent mass term can be ignored in most practioal cases.

The number of studies concerning refleotion and transmission of waves by breakwaters conduoted in the field are

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very few. Thornton and Calhoun /1972/ reported the results of the field measurements at the rubble - mound breakwater in Monterey Harbor, California. The incident wave motion was measured at two locations in front of the breakwater in order to resolve the incident and reflected wave components; the transmitted wave was measured at one point behind the breakwater. The incident, reflected and transmitted power spectra were next calculated using the linear wave theory. This model is not truly predictive in that it relies on the knowledge of the wave records at the three locations near the breakwater in any particular case.
For more realistic representation of the interaction between ocean - wind waves and the rubble - mound breakwater, the consideration of the random character of the wave motion is needed.
Thus, the probability theory is the natural frame of reference for the description of such time - varying quantities. For the incident wave the Gaussian model involving superposition of linear waves predicts all the probability properties of the sea surface. Unfortunately, the wave motion into porous media cannot be considered linear.

Massel and Mei /1977/ presented the analytical theory for random waves passing a perforated and porous breakwater. However, these theories are approximate for the shallow coastal zone where the energy of the incident wave is concentrated in the long - wave part of the spectrum. The quadratic damping term is treated by the stochastic equivalent linearization technique. They defined the statistical transmission and reflection coefficients in terms of the standard deviations and the wave spectra.

In this paper an extension of previous work is given. The various aspects of the rectangular porous breakwater - wave interaction are considered under the assumption that the incident wave spectrum is arbitrary. To simplify the analysis the damping terms in the body of breakwater will be replaced by an equivalent linear term. The standard deviations for both wave velocity components are than calculated assuming that they are non - correlated random functions with zero mean values. Results obtained from this analysis are compared with the field measurements of reflection and transmission characteristics of porous breakwater and numerical examples are given.
We consider the motion of an incompressible invicid fluid and Cartesian axes x, z; with z increasing vertically upward. The homogeneous, rectangular breakwater is subjected to normally incident wind - induced waves. The depth of the water is constant.

ANALYSIS

Governing Equations

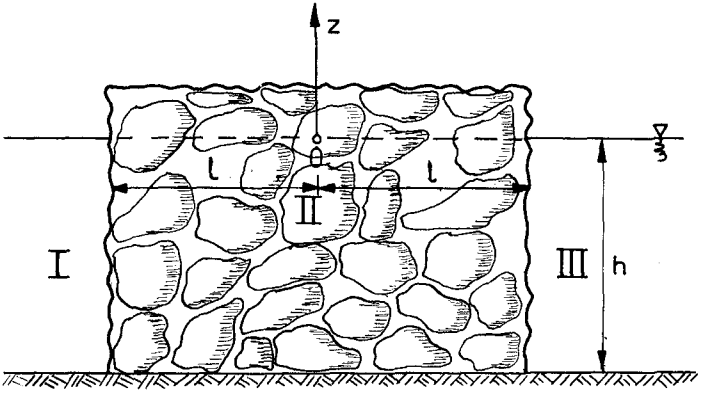

Fig. 1: Definition Sketch

Three different computational regions are identified in Fig. 1.
In the region I, the random incident wave is assumed to be Gaussian with zero mean and is represented by the following Fourier - Stjeltjes integral:

$$\zeta_i = \int_{-\infty}^{\infty} dA(\omega)\; e^{i(kx-\omega t)} \qquad /1/$$

where $\omega^2 = gk\cdot\tanh(kh)$
For a stationary and homogeneous process, the amplitude spectrum dA satisfies:

$$E\left[dA(\omega)\cdot dA^{*}(\omega')\right] = \frac{1}{2}\cdot S(\omega)\cdot\delta(\omega-\omega')\cdot d\omega\, d\omega' \qquad /2/$$

The wave motion in region I consists of an incident and a reflected wave. Thus, the resultant velocity potential takes the form

$$\Phi_1(x,z,t) = \mathrm{Re}\int_{-\infty}^{\infty}\frac{-ig}{\omega}\cdot e^{-i\omega t}\cdot\left\{\left[e^{ik(x+l)} - e^{-ik(x+l)}\right]\cdot\frac{\cosh k(z+h)}{\cosh kh}\right.$$
$$\left. + \sum_{\alpha} M_\alpha(\omega)\cdot e^{\alpha(x+l)}\cdot\frac{\cos\alpha(z+h)}{\cos\alpha h}\right\}\cdot dA(\omega) \qquad /3/$$

in which the wave number α must satisfy the following dispersion relation

$$\frac{\omega^2}{g} + \alpha \cdot tg(\alpha h) = 0 \qquad /4/$$

In the region III, the wave motion is simply transmitted wave. The velocity potential may be expressed as

$$\Phi_3(x,z,t) = \mathrm{Re}\int_{-\infty}^{\infty} \frac{-ig}{\omega} \cdot e^{-i\omega t} \cdot \sum_{\alpha} \left\{ N_\alpha(\omega) \cdot e^{\alpha(l-x)} \cdot \frac{\cos\alpha(z+h)}{\cos\alpha h} \right\} \cdot dA(\omega) \qquad /5/$$

A complete mathematical desoription of flow through a coarse granular material in the region II would be a very diffioult and tedious task. A more resonable approach to the problem is to determine the important physical and hydraulic properties of the media and then evaluate the macrosoopic flow field in terms of these properties. The analysis yields the velocities and pressures which are averaged over the small but finite pore volumes. Thus, for porous media $(-h \le z \le 0, -l \le x \le l)$, we write the equation of motion in the form

$$\frac{\partial \vec{u}_2}{\partial t} = -\frac{1}{\rho} \cdot \nabla(p_2 + \gamma z) \quad + \text{resistance forces} \qquad /6/$$

in which $\vec{u}_2$ - the "seepage velocity", p_2 - water pressure. In order to specify the resistanoe forces, the equation proposed by Forchheimer is adopted in the form

$$\frac{\partial \vec{u}_2}{\partial t} = -\frac{1}{\rho}\nabla(p_2 + \gamma z) - \frac{\nu \cdot n}{K} \cdot \vec{u}_2 - \frac{C_f \cdot n^2}{\sqrt{K}} \cdot |\vec{u}_2| \cdot \vec{u}_2 \qquad /7/$$

where ν - kinematic viscosity of fluid, K - intrinsic permeability, n - porosity, C_f - coeffioient dependent on properties of porous media.
Acoording Arbhabhirama and Dinoy /1973/

$$C_f = 100\left[d_m\left(\frac{n}{K}\right)^{1/2}\right]^{-3/2} \qquad /8/$$

in which d_m - partiole mean diameter of porous media. Although the damping term in Eq. /7/ is derived from steady state ooncepts it is assumed that it aooounts for the damping due to the instantaneous velocity occuring at all phases of the wave oycle. Thus, the linear term dominates when velooities are low and the turbulent term dominates when velocities are high. The assumptions whioh limit the applioation of the expression /7/ are that oonvective aocelerations be and that the motion be periodio with frequenoy low enough to maintain the validity of the damping term. Thus, Eq./7/ applies when the wave lengths of the partioular spectral components are long with respeot to wave amplitudes and media grain size.
An analytical solution to Eq.7 is possible after its linearization. It is done using a technique that aproximate the turbulent damping oondition inside the porous media. The dissipative nonlinear stress term in Eq. 7 is

replaced by an equivalent stress term linear in $\vec{u}_2$, i.e.

$$\frac{\nu \cdot n}{K} \cdot \vec{u}_2 + \frac{C_f \cdot n^2}{\sqrt{K}} \cdot |\vec{u}_2| \cdot \vec{u}_2 \longrightarrow f_e \cdot \omega_p \cdot n \cdot \vec{u}_2 \qquad /9/$$

in which f_e is a dimensionless friction /damping/ coeffioient and ω_p is the peak frequency. To evaluate f_e in terms of the known damping law in the deterministic case, we ohoose f_e such that the total rate of energy dissipation, integrated over the breakwater oross seotion and averaged over period is unohanged. Alternatively, for random waves one may minimize, the mean square error ϵ^2 when the mean is taken over time in the stoohastic sense as well as space. The relation between f_e and the physical parameters of motion is developed in Appendix A.
As a motion inside the porous media is a result of the harmonio excitation, it may be written as

$$\vec{u}_2(x,z,t),\ p_2(x,z,t) = \left[\tilde{\vec{u}}_2(x,z),\ \tilde{p}_2(x,z)\right] \cdot e^{-i\omega t} \qquad /10/$$

Combining Eqs. 7 and 10 and taking into acoount Eq. 9 and the equation of continuity $\nabla \vec{u}_2 = 0$, gives

$$\frac{\partial \Phi_2}{\partial t} + \frac{1}{\rho}(p_2 + \gamma z) + f_e \cdot \omega_p \cdot n \cdot \Phi_2 = 0 \qquad /11/$$

in whioh Φ_2 - velocity potential in the porous media. Eq. 11 yields the surface displacement in the form

$$\zeta_2 = -\frac{1}{g}\left(\frac{\partial \Phi_2}{\partial t} + f_e \cdot \omega_p \cdot n \cdot \Phi_2\right)_{z=0} \qquad /12/$$

Substituting the above expression into the kinematic free surface condition leads to its final form in the term of potential funotion

$$g \cdot \frac{\partial \Phi_2}{\partial z} - \omega^2\left(1 + i \cdot n \cdot f_e \cdot \frac{\omega_p}{\omega}\right) \cdot \Phi_2 = 0 \qquad /13/$$

At the bottom $(z = -h)$, the foundation is taken to be impervious, i.e.

$$\frac{\partial \Phi_2}{\partial z} = 0 \qquad /14/$$

Finally, in the region II we adopte the potential Φ_2 in the form

$$\Phi_2(x,z,t) = \mathcal{R}e \int_{-\infty}^{\infty} \frac{g \cdot e^{-i\omega t}}{\omega\left(i - n \cdot f_e \cdot \frac{\omega_p}{\omega}\right)} \cdot \sum_{\psi} \left\{\left[P_\psi \cdot e^{-\psi(x+l)} + \right.\right. \qquad /15/$$

$$\left.\left. + Q_\psi\ e^{\psi(x-l)}\right] \cdot \frac{\cos\psi(z+h)}{\cos\psi h}\right\} \cdot dA(\omega)$$

in which ψ - complex wave number being a solution of the following dispersion relation

$$\omega^2\left(1+i\cdot n\cdot f_e\cdot\frac{\omega_p}{\omega}\right)+g\cdot\psi\cdot tg(\psi h)=0 \qquad /16/$$

Eq. 16 has an infinite number of complex roots.

Matching Conditions

For the moment we assume that porosity n, permeability K and the linearization coefficient f_e are known. The general solutions developed in the proceeding section then contain 4 unknown functions of frequency: M_α, N_α, P_ψ and Q_ψ. To determine the functions the general solutions for the horizontal mass flux and pressure are matched at $x=\pm l$. This yields 4 equations from which the 4 unknowns may be determined.
It is of interest to note that the matching conditions may be expressed in the form of set of two equations with an infinite number of solutions for coefficients M_α and N_α /Massel, Butowski 1980/.
Solving this set by the standard Galerkin's method we obtain the following expressions for coefficients P_ψ and Q_ψ

$$\begin{aligned} P_\psi + e^{-2\psi l}\cdot Q_\psi &= \sum_\alpha L_{\psi\alpha}\cdot M_\alpha \\ e^{-2\psi l}\cdot P_\psi + Q_\psi &= \sum_\alpha L_{\psi\alpha}\cdot N_\alpha \end{aligned} \qquad /17/$$

in which

$$L_{\psi\alpha}=\frac{1+i\cdot n\cdot f_e\cdot\frac{\omega_p}{\omega}\cdot\cos(\psi h)}{B_\psi^2\cdot\cos(\alpha h)}\cdot\int_{-h}^{0}\cos\psi(z+h)\cdot\cos\alpha(z+h)\,dz \qquad /18/$$

and

$$B_\psi=\left\{\frac{h}{2}\left[\frac{\sin(2\psi h)}{2\psi h}+1\right]\right\}^{1/2} \qquad /19/$$

Transmission and reflection coefficients

Taking the analogy to the deterministic waves we adopted the following expressions for the statistical transmission and reflection coefficients

$$K_r=\frac{\sigma_r}{\sigma_i}=\left[\frac{\int_0^\infty S_r(\omega)\,d\omega}{\int_0^\infty S(\omega)\,d\omega}\right]^{1/2} \qquad /20/$$

$$K_t=\frac{\sigma_t}{\sigma_i}=\left[\frac{\int_0^\infty S_t(\omega)\,d\omega}{\int_0^\infty S(\omega)\,d\omega}\right]^{1/2} \qquad /21/$$

where $S_r(\omega)$ - spectrum of reflected waves, $S_t(\omega)$ - spectrum of transmitted waves, $S(\omega)$ - incident wave spectrum.

The reflected wave spectrum can be expressed as

$$S_r(\omega) = S_{1r}(\omega) = \left[\hat{M}_k(\omega)\right]^2 \cdot S(\omega) \; ; \; \hat{M}_k(\omega) = \left| M_k(\omega) - 1.0 \right|_{\alpha=-ik} \qquad /22/$$

and for the transmitted wave spectrum we have

$$S_t(\omega) = S_3(\omega) = \left[\hat{N}_k(\omega)\right]^2 \cdot S(\omega); \; \hat{N}_k(\omega) = \left| N_\alpha \right|_{\alpha=-ik} \qquad /23/$$

The spectrum $S_1(\omega)$ for the total wave on the incidence side ($x < -l$) is

$$S_1(\omega, x) = \left\{ \left[1 + \hat{M}_k^2(\omega) \right] + 2 \cdot \hat{M}_k(\omega) \cdot \cos\left[2k(x+l) - \varphi_k\right] + \left(M_{\Sigma_1}(\omega, x)\right)^2 + \right.$$

$$+ 2 \cdot M_{\Sigma_1}(\omega, x) \cdot \cos\left[k(x+l) - \varphi_{\Sigma_1}\right] + 2 \cdot \hat{M}_k(\omega) \cdot M_{\Sigma_1}(\omega, x) \cdot$$

$$\left. \cdot \cos\left[k(x+l) - \varphi_k + \varphi_{\Sigma_1}\right] \right\} \cdot S(\omega) \qquad /24/$$

in which

$$M_{\Sigma}(\omega, x) \cdot e^{i\varphi_{\Sigma_1}} = \sum_{\alpha}{}' e^{\alpha(x+l)} \cdot \hat{M}_\alpha(\omega) \cdot e^{i\varphi_\alpha} \qquad /25/$$

$\hat{M}_\alpha(\omega)$, φ_α - absolute value and phase angle of the complex M_α, respectively. $\sum'$ denotes summation over all the real values of α; i.e. $\alpha = -ik$ is omitted from the sum.

It is of special interest to note that the spectrum $S_1(\omega)$ is also the function of distance from front face of the breakwater.

The transmitted waves at the arbitrary distance from the lee face are described by the spectrum in the form

$$S_3(\omega, x) = \left| \sum_{\alpha} N_\alpha(\omega) \cdot \exp \alpha(l - x) \right|^2 \cdot S(\omega) \qquad /26/$$

When $x \longrightarrow \infty$, we obtain $S_3(\omega, x) \longrightarrow S_t(\omega)$.

It is easy to demonstrate that when $f_e = 0$ /no break - water/, Eqs. 20 and 21 reduce to

$$K_r = 0 \qquad \text{and} \qquad K_t = 0 \qquad /27/$$

NUMERICAL RESULTS

The application of the semi - empirical theory in the previous section is illustrated first for the porous breakwater of thickness $2l = 15$ m. Let the water depth be h = 10 m. Assume that the wind velocity is $U_{10} = 20$ m/sec and the fetch $X = 100$ km. The frequency at the spectral peak is $\omega_p = 0.821$ rd/sec and the mean wave height is $\bar{H} = 2.23$ m. The incident wave energy distribution let to be represented by the JONSWAP formula /Hasselmann et al., 1973/. For a prototype rubble diameter /d = 1 m/,

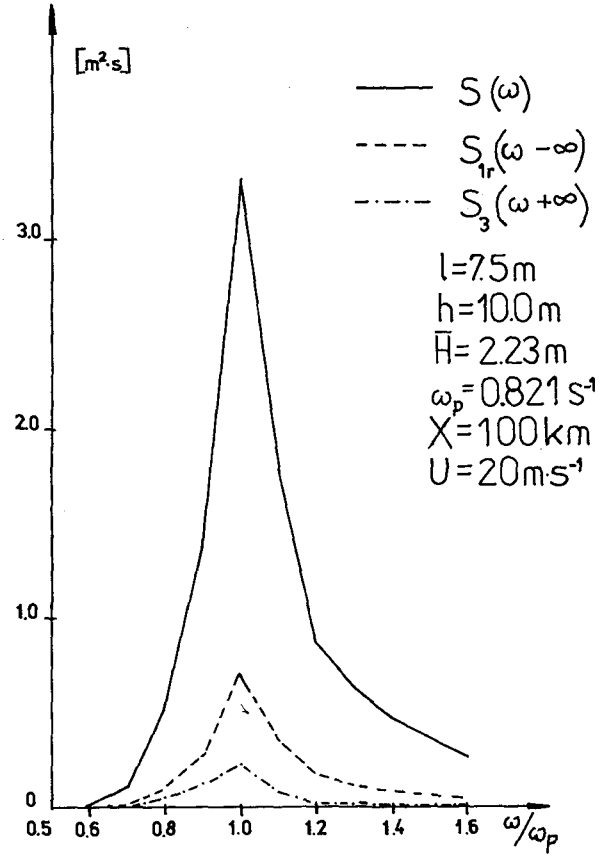

Fig.2 Spectra of the incident $S(\omega)$ and transmitted $S_3(\omega)$ and reflected $S_{1r}(\omega)$ waves.

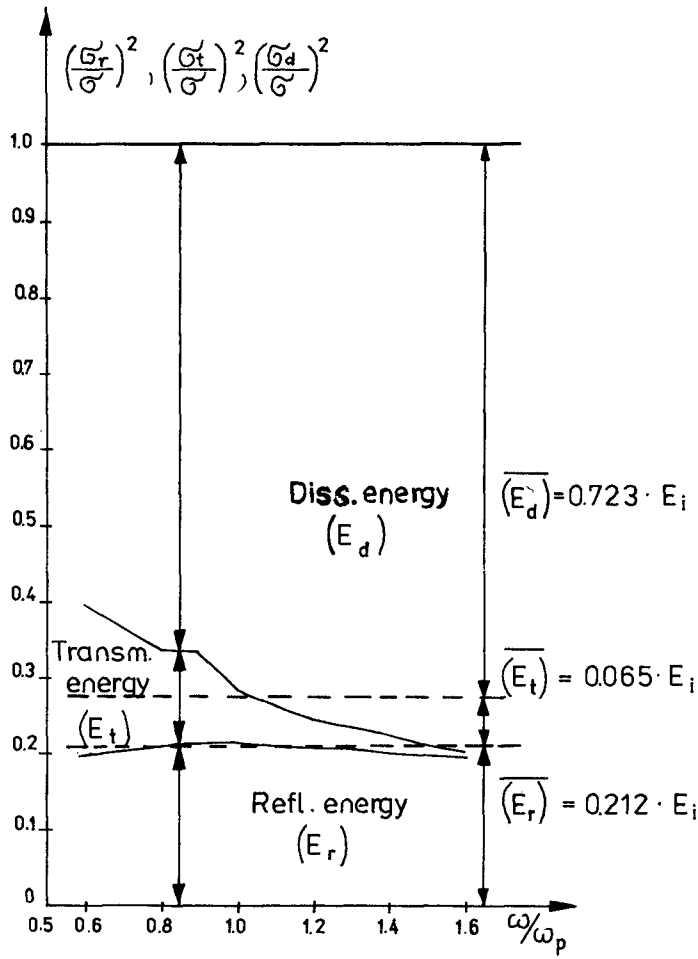

Fig.3 Frequency dependence of the transmission, reflection and dissipation energy.

the permeability and porosity is estimated from the model tests /see Appendix A/ to be

$$K = K_m \left(d/d_m\right)^2 = 5.52 \cdot 10^{-4}\ m^2,\ n = 0.437,\ C_f = 0.228 \qquad /28/$$

From the Appendix A follows that equivalent linear friction ooefficient is $f_e = 4.2654$. Finally be performing the numerioal integration in Eqs. 20 and 21 the statistical transmission and reflection coefficients are found, $K_t = 0.255$ and $K_r = 0.460$. In the Fig.2 the spectra of the incident wave, and the reflected and transmitted wave are shown.
Fig.3 shows the frequency dependence on the respected energies. From this Figure it is evident that almost 73 % of the mean value of inoident energy is dissipated within the body of breakwater. The transmitted energy varies considerably with frequenoy from a maximum at low frequency and decreasing with increasing frequency. The reflection part of energy is almost constant in the band of frequenoy under consideration. Small maximum of reflected energy is observed at the frequency of speotral peak in $S(\omega)$function. Both energies are decreasing for high frequencies.

The measurements and calculations performed by Thornton and Calhoun /1972/ in California represent of some opportiunity to oheck the developed theory against the prototype data. In the fig. 4 the oross seotion of the Monterey Breakwater is shown.

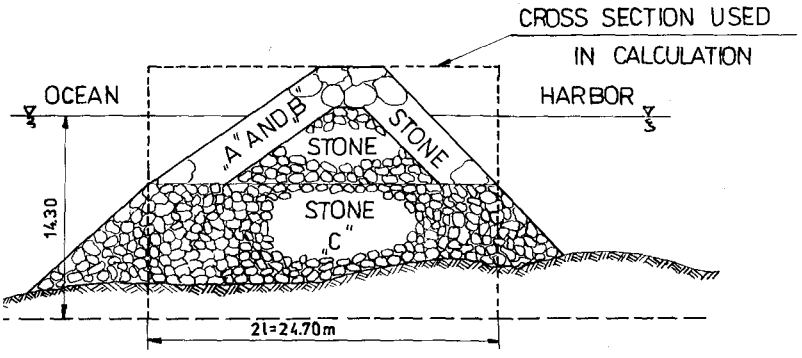

Fig. 4 Monterey Breakwater cross section

As the preceeding analysis is limited to permeable structure of rectangular form only, we adopte the equivalent reotangular cross section to the Monterey Breakwater whioh has approximately the same submerged volume /see Fig.4/. The hydraulic properties of the breakwater /permeability and porosity/ are roughly estimated taking into aocount that the structure is a mound of stones of different sizes and shapes, mainly of "C" type. The averaged total weight of the stone equal to $\sim 10^4$ N with speoific gravity $\gamma \approx 26 \cdot 10^3\ N/m^3$ is used in the calculation. Than, the mean diameter of the stone is approximately $d_m \approx 0.8$ m.
Assuming that the prototype stone angularity and paoking are the same as in the small scale rubble investigated by Sollitt and Cross /1972/ we can calculate the permeability K and the coefficient C_f.
The inoident wave spectrum is specify as in field investigation for December 1, 1970 at 2330 hr /see Tabl.1 in Thornton and Calhoun paper/.
Performing the numerical oalculation of the K_t and K_r coeffioients we obtain:

$$K_r = 0.448,\ K_t = 0.344,\ K_d = \left[1 - \left(K_r^2 + K_t^2\right)\right]^{1/2} = 0.825 \qquad /29/$$

While the field measurement yields

$$K_r = 0.52\ ,\ K_t = 0.11\ ,\ K_d = 0.84 \qquad /30/$$

The above numbers suggest that the transmission rate of energy in the field is higher than that predicted by theory. The reflection rate of energy is almost the same in both methods. The ooeffioient K_d represents the rate of energy which is dissipated in the body of breakwater. A possible explanation for this discrepancy is probably associated with the rectangular shape and value of permeability introduoed in the analysis. Especially, the knowledge of the hydraulic properties of the porous material of Monterey Breakwater is rather poor.
It is worthwhile to note that the experimental results for the reflection coefficient and dissipation rate are in good agreement with the theoretical values.

CONCLUSIONS

The present analysis was undertaken in order to extend a previous simplified theory for the determination of transmission and reflected oharacteristios of porous breakwaters subjeoted to the action of the wind - induced waves. The extension consisted of a more rigovous analysis of wave motion inside the breakwater space

when the incident wave spectrum may be arbitrary. The original nonlinear equation of motion into porous struotu-re was linearized using statistical linearization techniq ue. The computed output consists of the spectral density functions for the reflected and transmitted waves. The statistical transmission and reflection coefficients are introduced in terms of the standard deviations and the wave spectra.

From a comparison with experimental data it appears that the method may be valuable in practioal applications. Probably more good quality experimental data is needed for the deterimation of the applicability range of the method.

APPENDIX A

Equivalent friction coeffiction

Let the error between nonlinear and linear resistanoe forces be

$$\epsilon(x,z,t) = \frac{\nu n}{K}\vec{u}_2 + \frac{C_f \cdot n^2}{\sqrt{K}}|\vec{u}_2|\cdot\vec{u}_2 - f_e \cdot \omega_p \cdot n \cdot \vec{u}_2 \qquad /31/$$

Upon minimizing the mean square error

$$\frac{1}{2lh}\cdot\int_{-h}^{0}\int_{-l}^{l} E\left[\epsilon^2\right]dx\,dz \qquad /32/$$

one gets

$$f_e = \frac{1}{n\cdot\omega_p}\cdot\left[a + b\cdot\frac{\int_{-l}^{l}\int_{-h}^{0} E\left[|\vec{u}_2|\cdot\vec{u}_2^{\,2}\right]dx\,dz}{\int_{-l}^{l}\int_{-h}^{0} E\left[\vec{u}_2^{\,2}\right]dx\,dz}\right] \qquad /33/$$

in which

$$a = \frac{\nu n}{K}\,, \qquad b = \frac{C_f \cdot n^2}{\sqrt{K}} \qquad /34/$$

The symbol $E[\,]$ represents the statistical average. In order to calculate the statistical moments in Eq. 33 we assume that in the breakwater space the both wave velocity components u_2 and v_2 are the independent random values with gaussian probability density. The mean values are zero and the standard deviations are equal to σ_{u_2} and σ_{v_2}, respectively. Thus, the probability density function for $U = |\vec{u}_2| = (u_2^2 + v_2^2)^{1/2}$ takes the form /Papoulis/

$$p(U) = p\left(|\vec{u}_2|\right) = \frac{U}{\sigma_{u_2}\cdot\sigma_{v_2}}\cdot I_0\left(m_1\cdot U^2\right)\cdot\exp\left(-m_2\cdot U^2\right) \qquad /35/$$

where I_0 - modified Bessel function and

$$m_1 = \frac{1}{4} \cdot \frac{\sigma_{u_2}^2 - \sigma_{v_2}^2}{\sigma_{u_2}^2 \sigma_{v_2}^2}, \quad m_2 = \frac{1}{4} \cdot \frac{\sigma_{u_2}^2 + \sigma_{v_2}^2}{\sigma_{u_2}^2 \cdot \sigma_{v_2}^2} \qquad /36/$$

It can be easy demonstrated that

$$p_2(u_2) = p_2(\vec{u}_2^{\,2}) = \frac{1}{2 \cdot \sigma_{u_2} \cdot \sigma_{v_2}} \cdot I_0(m_1 \cdot u_2) \cdot \exp(-m_2 \cdot u_2) \qquad /37/$$

and

$$p_3(u_3) = p_3\left(\vec{u}_2^{\,2} | \vec{u}_2 |\right) = \frac{1}{3 \cdot \sigma_{u_2} \cdot \sigma_{v_2} \cdot u_3^{1/3}} \cdot I_0(m_1 u_3^{4/3}) \cdot \exp(-m_2 u_3^{2/3}) \qquad /38/$$

By virtue of the definition of the mean of the random value we obtain

$$E[\vec{u}_2^{\,2}] = \sigma_{u_2}^2 + \sigma_{v_2}^2$$

$$E[\vec{u}_2^{\,2} | \vec{u}_2 |] = \frac{12 \cdot \sqrt{\pi} \cdot \sigma_{u_2}^4 \cdot \sigma_{v_2}^4}{(\sigma_{u_2}^2 + \sigma_{v_2}^2)^{5/2}} \cdot \left(\Gamma(5/4) \cdot \Gamma(7/4)\right)^{-1} \cdot \qquad /39/$$

$$\cdot \sum_{n=0}^{\infty} \frac{\Gamma(5/4+n) \cdot \Gamma(7/4+n)}{(n!)^2} \cdot \left(\frac{\sigma_{u_2}^2 - \sigma_{v_2}^2}{\sigma_{u_2}^2 + \sigma_{v_2}^2}\right)^{2n} \qquad /40/$$

Substitiuting Eqs. 39 and 40 into Eq. 33 we are able to calculate the equivalent friction coefficient f_e. To initiate the solution, a value for f_e is assumed / $f_e \approx 1.0$ is suitable/. Then the appropriate potentials are evaluated. The standard deviations of the velocity components are extracted and substitiuted into linearization condition to compute f_e. If the result is different from the assumed value it is necessary to interate in order to obtain next value for f_e.

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