CHAPTER 3

MASS TRANSPORT IN PROGRESSIVE WAVES OF PERMANENT TYPE

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INTRODUCTION

Mass transport phenomenon was first recognized by Stokes in 1847 using a Lagrangian description. Later, a basic theory for the mass transport in water waves in viscous fluid and of finite depth was derived by Longuet-Higgins in 1953. Theoretical solutions of mass transport in progressive waves of permanent type are subjected to the definitions of wave celerity in deriving the various finite amplitude wave theories. As it has been generally acknowledged that the Stokes wave theory can not yield a correct prediction of mass transport in the shallow depths, some new theories have been developed. Recently the authors(1974 & 1977) have derived a new finite amplitude wave theory in shallow water for quasi-Stokes and cnoidal waves by the so-called reductive perturbation method, in which the mass transport is formulated both in Lagrangian and Eulerian descriptions.

On the experimental verification, Russell and Osorio(1957) investigated and compared Longuet-Higgins' solution with experimental data of Lagrangian mass transport velocity obtained in a normal closed wave tank of finite length. Since then, many investigations, and nearly all of them, have employed the finite length of wave tank in carrying out their experiments. However, no experiment has yet been attempted at verifying the Stokes drift in progressive waves of permanent type in a wave tank of infinite length. It is not realistic nor economical in constructing such an infinitely long flume to investigate experimentally the mass transport velocity in progressive waves. Instead of using such an ideal wave tank, a new one incorporated with natural water re-circulation was equipped to carry out experiments by the authors(1978). It was confirmed from these experiments that mass transport in progressive waves of permanent type exists in the same direction of wave propagation throughout the depth, and agrees with both the Stokes drift and the authors' new formulations, within the test range of experiments.

THEORETICAL SOLUTIONS OF MASS TRANSPORT

1. Conventional Expressions

The theoretical solutions of mass transport in progressive waves of permanent type can be grouped into two main categories. The first one is derived for the perfect fluid from using a finite amplitude wave theory, whilst the other is resulted from considering viscous effects at the bottom and near the free surface.

In the first category the theoretical expressions of mass transport velocity in Stokes waves have been formulated in Lagrangian description for two different definitions of wave celerity in deriving the wave theory. The first is so-call the Stokes drift in Lagrangian description which is subjected to the definition of wave celerity and then is given for the first and second Stokes definitions respectively as

$$U_M=\frac{H^2\sigma k\cosh 2kz}{8\sinh^2 kh} \qquad \text{...(1)}$$

and

$$U_M=\frac{H^2\sigma k\cosh 2kz}{8\sinh^2 kh}-\frac{H^2\sigma\coth kh}{8h} \qquad \text{.................................(2)}$$

in which U_M is the mass transport velocity formulated in Lagrangian description, H the wave height of progressive waves of permanent, h the water depth at the still water level, $\sigma = 2\pi/T$ the angular frequency of waves, T the wave period, $k = 2\pi/L$ the wave number and L the wave length as shown in Fig. 1. The mass transport velocity in Eulerian description may be given as

$$\bar{u}=\frac{H^2\sigma\coth kh}{8h}\ ; \qquad -\frac{H}{2h}<\frac{z}{h}+1<\frac{H}{2h} \qquad \text{.............(3)}$$

in which u is the mass transport velocity in Eulerian description which is defined to be the horizontal water particle velocity averaged for one wave period. In 1968 Le Méhauté derived the mass transport velocity by means of Laitone's second approximation of cnoidal waves. This seems questionable because the limitation of solution corresponding to solitary wave has a finite value of mass flux.

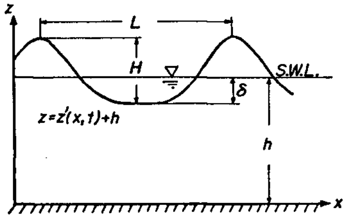


Fig. 1 Co-ordinate system used

On the other hand, Longuet-Higgins' solution represents the second category, and so followed by Huang(1970), Li and Davis(1977) etc. Longuet-Higgins' expression is given as

$$U_M=\frac{H^2\sigma k}{4\sinh^2 kh}\left[3+2\cosh 2kz+3kh\left\{3\left(\frac{z}{h}-1\right)^2+4\left(\frac{z}{h}-1\right)+1\right\}\sinh 2kh+3\left\{\left(\frac{z}{h}-1\right)^2-1\right\}\left(\frac{\sinh 2kh}{2kh}+\frac{3}{2}\right)\right]$$

$$\text{...................(4)}$$

These four kinds of vertitical distribution of mass transport velocity are schematically shown in Fig. 2, these are the Stokes drifts of the first and second definitions of wave celerity, and that of Longuet-Higgins. This figure shows the distinctive nature of mass transport when the Eulerian description is considered. In the figure, $U_M/H^2\sigma k$ is the dimensionless mass transport velocity and z/h the dimensionless vertical level from the bottom. It is clear that as well-known relatively strong forward velocity is expected near both boundaries as derived by Longuet-Higgins when viscosity is taken into account.

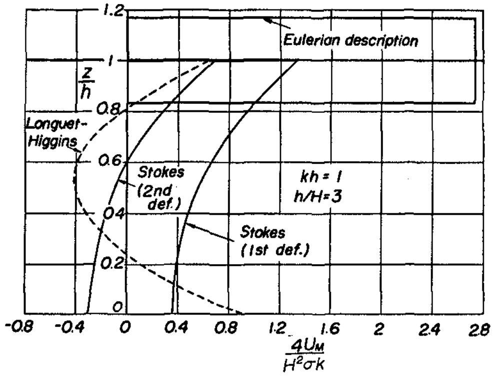


Fig. 2 Vertical distribution of mass transport velocity by conventional expressions

According to the usual expression of mass transport velocity, there is a quite different vertical distribution in Lagrangian and Eulerian descriptions, as a result from the definition of wave celerity in deriving the finite amplitude wave theory. Bagnold's experimental results in 1947 was used to confirm Longuet-Higgins' solution, in which he noted that the mass transport velocities near both boundaries have stronger forward component in the direction of wave propagation, but with reverse flow in the mid-layer between these two viscous boundaries. Ünlüata and Mei(1970) pointed out that Longuet-Higgins' solution corresponed to that would be derived from assuming a constant pressure gradient along the direction of wave propagation thus that no net transport occurs.

2. New Expressions

The new wave theories of quasi-Stokes(1978) and cnoidal waves(1974 & 1977) have recently been derived by the authors from using the so-called reductive perturbation method by Taniuti(1968). From these, the mass transport velocities are then derived.

a) Mass transport in quasi-Stokes waves The mass transport velocity in Lagrangian description is given as

$$U_M/\sqrt{gh} = (\lambda_0^2/16)\{1-(1/2)(2\pi h/L)^2\} + (\lambda_0^2/8)(\sqrt{gh}/c)\,[1+(2/3)(2\pi h/L)^2\{3(z/h-1)^2-1\}] \quad \ldots\ldots (5)$$

And the mass transport velocity in Eulerian description is given as

$$\bar{u}/\sqrt{gh} = (\lambda_0^2/16)\{1-(1/2)(2\pi h/L)^2\} \quad \ldots\ldots (6)$$

in which λ_o and the wave celerity c are given respectively as

$$\frac{H}{h}=\lambda_0+\frac{1}{8\pi^2\varepsilon}\left(\frac{27}{512\pi^2\varepsilon}+\frac{45}{128}\right)\lambda_0^3,\qquad \varepsilon=\left(\frac{h}{L}\right)^2 \quad\cdots\cdots(7)$$

and

$$\frac{c}{\sqrt{gh}}=1-\frac{1}{6}\left(\frac{2\pi h}{L}\right)^2+\frac{19}{360}\left(\frac{2\pi h}{L}\right)^4-\frac{55}{3\,024}\left(\frac{2\pi h}{L}\right)^6+\frac{9\lambda_0}{64}\left(\frac{2\pi h}{L}\right)^{-2} \quad\cdots\cdots(8)$$

b) Mass transport in cnoidal waves The mass transport velocity in Lagrangian description is given as

$$\begin{aligned}\frac{U_M}{\sqrt{gh}}=&\frac{\lambda^2}{6\kappa^4}\left\{1+2\left(\frac{\sqrt{gh}}{c}\right)\right\}\left[-\left(\frac{E}{K}\right)\left\{3\left(\frac{E}{K}\right)+2\kappa^2-4\right\}+\kappa^2-1\right]+\frac{2\lambda^3}{5\kappa^6}\left(\frac{\sqrt{gh}}{c}\right)\left(\frac{z}{h}-1\right)^2\left\{2\left(\frac{E}{K}\right)(\kappa^4-\kappa^2+1)-\kappa^4+3\kappa^2-2\right\}\\&+\frac{\lambda^3}{10\kappa^6}\left(\frac{\sqrt{gh}}{c}\right)\left[5\left(\frac{E}{K}\right)^2\left\{9\left(\frac{E}{K}\right)+10\kappa^2-17\right\}+\left(\frac{E}{K}\right)(12\kappa^4-57\kappa^2+47)-6\kappa^4+13\kappa^2-7\right]\\&+\frac{\lambda^3}{15\kappa^6}\left(\frac{\sqrt{gh}}{c}\right)^2\left[15\left(\frac{E}{K}\right)^2\left\{\left(\frac{E}{K}\right)+\kappa^2-2\right\}+\left(\frac{E}{K}\right)(4\kappa^4-19\kappa^2+19)-2(\kappa^4-3\kappa^2+2)\right]\quad\cdots(9)\end{aligned}$$

and that in Eulerian is also given as

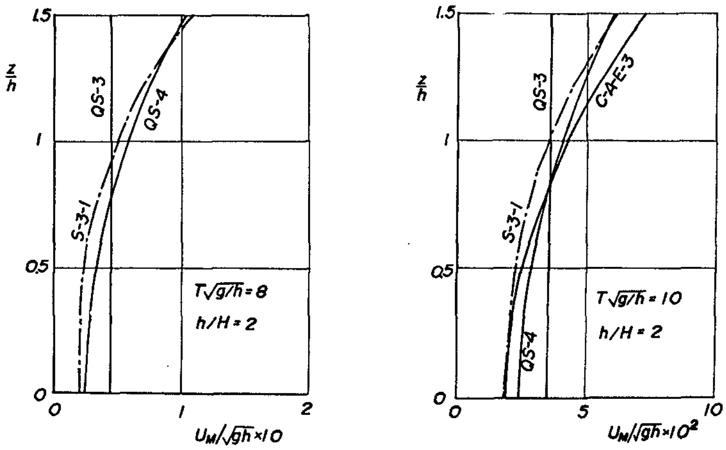
$$\frac{\bar{u}}{\sqrt{gh}}=\frac{\lambda^2}{6\kappa^4}\left[-\left(\frac{E}{K}\right)\left\{3\left(\frac{E}{K}\right)+2\kappa^2-4\right\}+\kappa^2-1\right]+\frac{\lambda^3}{20\kappa^6}\left[35\left(\frac{E}{K}\right)^2\left(\frac{E}{K}+\kappa^2-2\right)+\left(\frac{E}{K}\right)(6\kappa^4-41\kappa^2+41)-3(\kappa^4-3\kappa^2+2)\right] \quad\cdots\cdots(10)$$

in which

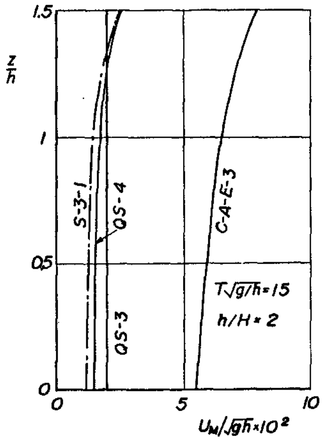
$$\frac{c}{\sqrt{gh}}=1-\frac{\lambda}{2\kappa^2}\left\{3\left(\frac{E}{K}\right)+\kappa^2-2\right\}+\frac{\lambda^2}{40\kappa^4}\left[45\left(\frac{E}{K}\right)\left\{3\left(\frac{E}{K}\right)+2\kappa^2-4\right\}+19\kappa^4-64(\kappa^2-1)\right]\quad\cdots\cdots(11)$$

E and K are the complete elliptic integrals of the first and second kinds, κ their modulus, and λ is approximately given as

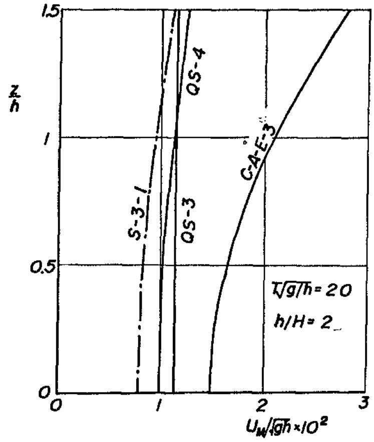
$$\frac{H}{h}=\lambda+\frac{27}{4\,096\pi^4\varepsilon^2}\lambda^3 \quad\cdots\cdots(12)$$



(a) (b)

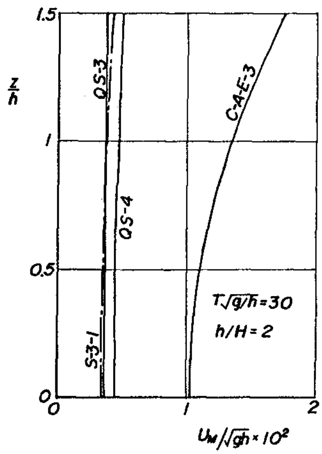


(c)



(d)

It is noted in these expressions that the mass transport exists both in Eulerian and Lagrangian descriptions, and the vertical distribution of Eulerian mass transport is uniform at this order of approximation. Vertical distributions of mass transport velocity in Lagrangian description are shown in Fig. 3. In this figure, the symbol S-3-1 stands for the solution of the usual Stokes wave theory to a third-order approximation and by the first Stokes definition of wave celerity, QS-3 and QS-4 indicate that of the quasi-Stokes wave theory to a third- and fourth-order approximation respectively, whilst C-A-E-3 represents the mass transport velocity predicted by the cnoidal wave theory to a third-order derived by the authors(the letter E for exact). The variation in the dimensionless wave period $T\sqrt{g/h}$ for a constant depth h infers the influence of wave period, or implying the shoaling of a wave with constant wave period T. It is seen that these vertical distributions of mass transport velocity are essentially the same within the range of small dimensionless wave period, and are in the same direction as of wave propagation, but the curve of C-A-E-3 is re-



(e)

Fig. 3 Vertical distributions of mass transport velocity in Lagrangian description markably higher than that of others when a larger dimensionless wave period is considered.

EXPERIMENTAL VERIFICATION

1. A New Wave Flume

In a wave flume test it is highly desired that the uniformity of wave profile, wave celerity and mass transport are achieved to the progressive waves of permanent type produced over a uniform finite depth of water. As the normal wave tank of finite length constrains wave motion by the side walls, sloping end wall, and the wave generator plate, unavoidable forced re-circulation of water mass would eventually be resulted for within the interior of the fluid in the wave tank. This creates the problem of reflecting waves and the secondary flow and so on. This forced recirculation exerts a great influence on the exactness of the wave motion to be measured, especially mass transport and wave celerity.

A new wave flume incorporated with four rectangular orifices at sides as shown in Fig. 4, has been constructed, such that the desired wave conditions in the flume can steadily be maintained. As shown in this figure

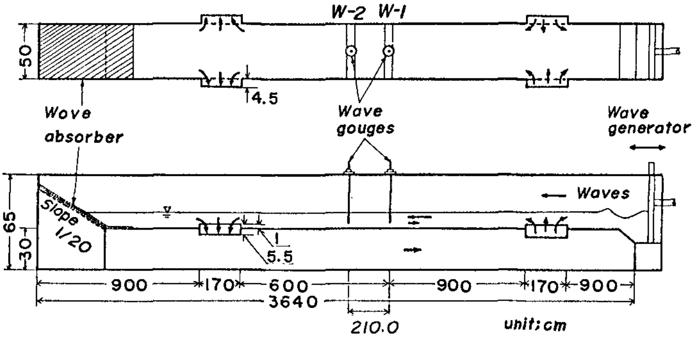


Fig. 4 New wave flume used

the water mass accompanied by the waves can flow into the inlet orifices at sides near the bottom, passing the conduit beneath the wave flume, and finally returns to the upstream part of the flume through the outlet orifices. Punched metal plates were randomly arranged at the end of the flume on a very gentle slope, acting as wave absorber. This enabled to keep the wave reflection coefficient within and below 5% during all experimental operations.

2. Experimental Method

Wave profile, wave height, and wave celerity were carefully measured by using two wave gauges of capacitance type, W-1 and W-2 as shown in Fig.

4, which were placed at 2.10 m apart. For measuring the mass transport velocity in Lagrangian description, a visualization method was employed wherein the orbital motions of the neutral buoyant particles, made by mixing xilen and nitrobensen at a ratio of 0.72 to 1 and with colouring, were recorded by a still camera for over 10 wave cycles continuously. An example of the recorded orbital motions is shown in Photo. 1. From

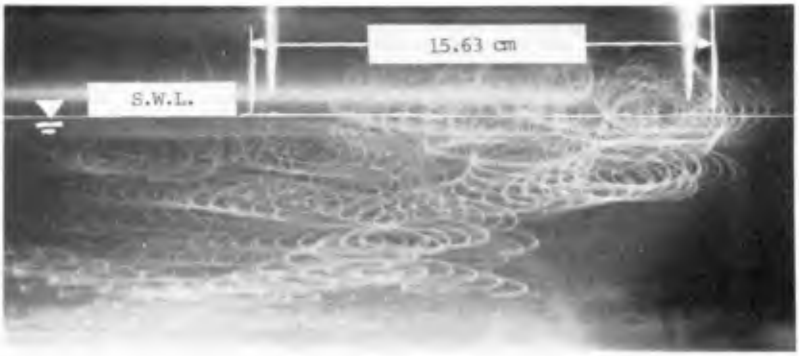


Photo. 1 An example of recorded orbital motions of neutral buoyant particles in the case of $T\sqrt{g/h}$ = 12.7 and h/H= 3.85

this picture, the mass transport velocity can be accessed for particles introduced at various levels throughout the depth.

In addition, a series of the experiment of mass transport in progressive waves of permanent type has been carried out by a normal wave tank to confirm the forced re-circulation of water mass in the tank. It was already published by the authors(1978).

3. Comparison of Data with Theoretical Prediction

The uniformity of wave height and wave celerity produced in the wave flume was very carefully checked as function of time, as given in Fig. 5, in which H/h is the ratio of wave height to water depth and $c/\sqrt{gh}$ the dimensionless wave celerity. The repeatability of mass transport velocity in Lagrangian description was also confirmed, besides the measurement of wave celerity. It was thus concluded that the wave height, wave celerity, and mass transport velocity reached the steady and uniform state after approximately 80 to 100 waves, i.e. reached its permanent type. Effect of viscosity may however appear near the bottom and free surface after a longer wave cycles.

Wave profiles in the steady state are shown in Fig. 6, in which comparison is made with the theoretical curves of quasi-Stokes(QS-4) and cnoidal waves(C-A-E-3), proven good agreement between them. Similarly, due comparisons on wave celerity and mass transport velocity can also be deduced for under the stable wave condition.

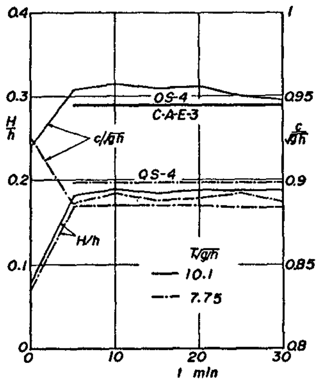


Fig. 5 An example of changes of wave height and wave celerity with time

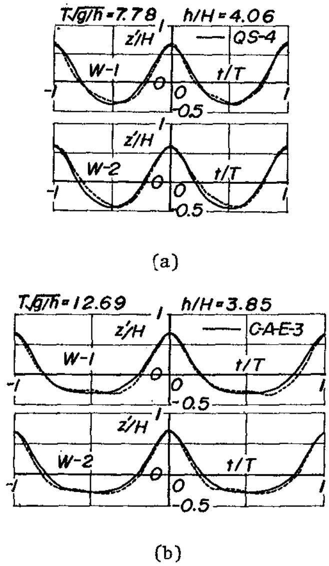


Fig. 6 Comparison of wave profile with theoretical curves

Fig. 7 compares the experimental values of wave celerity with the theoretical solutions for the quasi-Stokes (QS-4) and cnoidal waves (C-A-E-3), for the value of dimensionless wave period $T\sqrt{g/h}$ being 10 and slightly under. Fairly good agreement can again be observed for within the range tested.

Similar comparisons on mass transport velocity in Lagrangian description are presented in Fig. 8, in which the theoretical curves were obtained from using the usual Stokes wave theory, the authors' quasi-Stokes and cnoidal wave theories, and that of Longuet-Higgins'. It can be concluded from the figure that the mass transport velocity in progressive waves of permanent type is in the same direction as of wave propagation, and its vertical distribution follows closely the theoretical curves of Stokes waves, particularly to the first Stokes definition of wave celerity, as well as the quasi-Stokes (QS-4) and cnoidal wave (C-A-E-3) theories, within the ranges of $T\sqrt{g/h}$ and H/h presented.

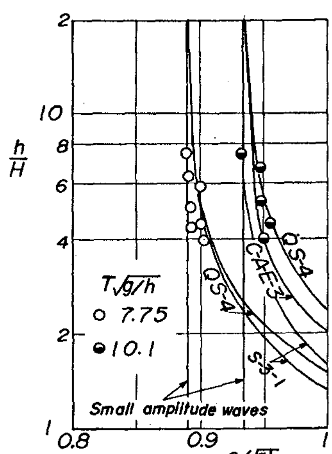


Fig. 7 Comparison of wave celerity with theoretical curves

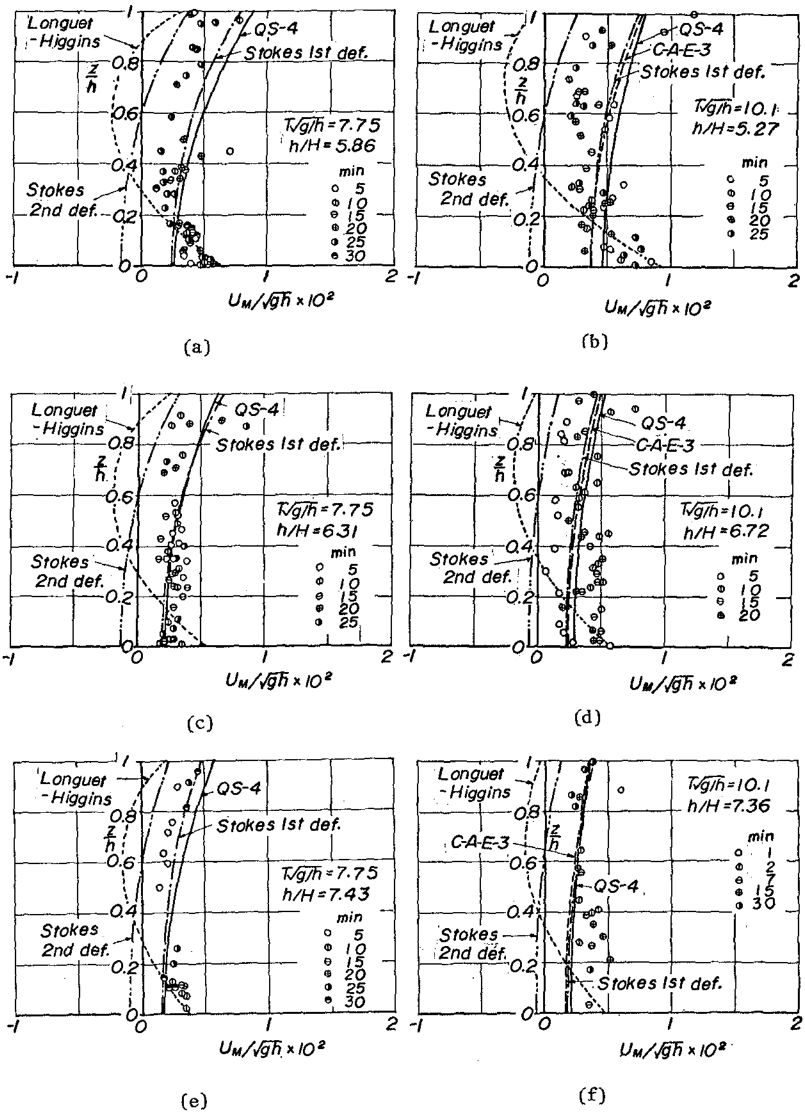
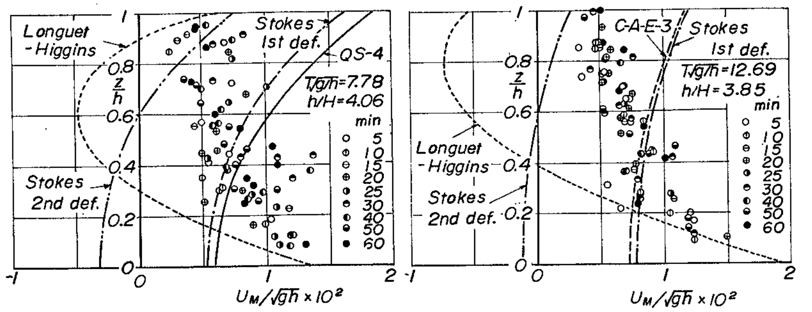


Fig. 8 Comparisons of mass transport velocity with theoretical curves (1)



(a) (b)

Fig. 9 Comparisons of mass transport velocity with theoretical curves(2)

Notable scattering occurs as wave height increases, i.e. h/H decreases for a constant depth h. However, slightly smaller in magnitude of the experimental data are generally seen as compared with the theoretical curves. This may well be caused from the back drift appeared in the upper part of the flume. This influence can roughly be estimated from assuming a uniform flow of back drift as follows. As the water depth in the upper part of the flume at the orifices was about 10 cm and that in the conduit below was 30 cm, it was therefore estimated that a 25% reduction will be resulted in the mass transport velocity so measured.

For even higher wave height than that presented in Fig. 8, as seen in Fig. 9, the mass transport velocity is still in the same direction as of wave propagation, but the vertical distribution is likely to differ further from the theoretical curves. This might be attributed to the limitation of the inlet and outlet orifices so equipped and even from the undesired reflection.

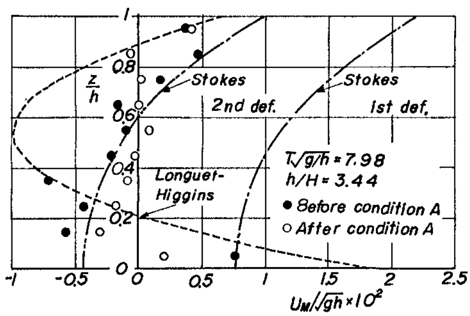


Fig. 10 Comparison of mass transport velocity in normal wave tank with theoretical curves

Finally, an example of the vertical distributions of mass transport velocity in a normal wave tank is shown in Fig. 10 for comparison with Fig. 8. In the figure, the condition A stands for the state at which the steadiness and uniformity of wave profile and wave celerity were made after about 200 to 250 waves. It can be concluded that the vertical distribution of mass transport velocity in a normal wave tank is quite different from those in Fig. 8, and agrees with the theoretical curve of the Stokes waves to the second Stokes definition of wave celerity. It may however be expected that experimental data approach to Longuet-Higgins' curve, as Russell and Osorio(1957) confirmed, when the viscosity effect is considered after very long wave cycles.

CONCLUSIONS

Theoretical formulations of mass transport velocity have been derived for progressive waves of permanent type, both in Lagrangian and Eulerian descriptions, for the quasi-Stokes and cnoidal wave theories by the authors. A new wave flume, equipped with orifices for the natural return of water flow, was constructed for the verification of mass transport velocity, since it is submitted that wave reflection occurs at the end of the normal wave tank which is generally of finite length. It can be concluded that the observed direction of mass transport is the same as that of waves propagating on the water surface, and the vertical distribution agrees with the theoretical curves calculated by the usual Stokes wave theory uder the first Stokes definition of wave celerity, and that of quasi-Stokes and cnoidal wave theories derived by the authors.

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