CHAPTER 146

WAVE FORCES ON A ROW OF CYLINDRICAL PILES OF LARGE DIAMETER

by

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Abstract

In order to determine wave forces on a row of three cylindrical piles (Figure 2), a numerical computation procedure was applied using a solution of the Helmholz equation, in which the scattered wave field is described as the result of a series of singular sources located along the circumference of the pile [Berkhoff, 1976, reference 1]. Results of the computations were verified by means of model experiments, using both regular and irregular waves.
It is shown that for the two pile geometries, included in the study, strong mutual interference will occur, resulting in transverse forces which are much higher than those found for single piles.

1 Introduction

In order to protect the low-lying hinterland of the lower Rhine delta against flooding, the Dutch have developed the Delta scheme, which involves the closure of a number of estuaries and the construction of three storm-surge barriers, the largest of which is currently being designed. This structure will be located in the Eastern Scheldt and will consist of a group of three sluices, each consisting of a number of gates with a span of approximately 40 m. Gates are suspended between piers founded at the seabed, which locally has a depth of 38 m below sealevel.
For the construction of the piers, cylindrical cofferdams were envisaged, due to time constraints three of these cofferdams had to be used simultaneously. Due to the large dimensions of these structures a strong mutual interference had to be expected which might adversely effect current and wave forces. Therefore, a study was performed to determine wave forces on a row of cylindrical piles. Results of this study are presented in this paper.

2 Theoretical considerations

Wave forces have been the subject of many studies, resulting in a number of theories and computation techniques. Each of these has its own distinct field of application as may be demonstrated with the help of Figure 3.
For objects which are small with respect to the wave length ($D/L < 0.1$) it is assumed that the local flow field is only marginally affected by the presence of the object.
Consequently orbital velocities and accelerations within the fluid are computed for the undisturbed condition using conventional wave theories. Once local accelerations and velocities are known corresponding wave forces may be computed. In this respect two limiting conditions may be considered

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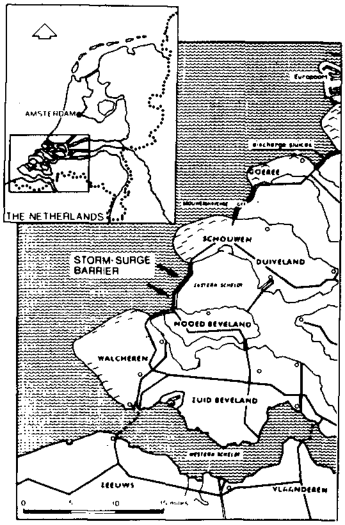
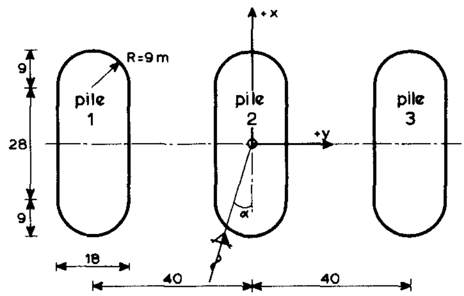
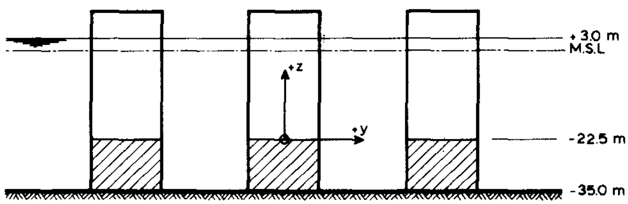


FIG. 1 LOCATION MAP

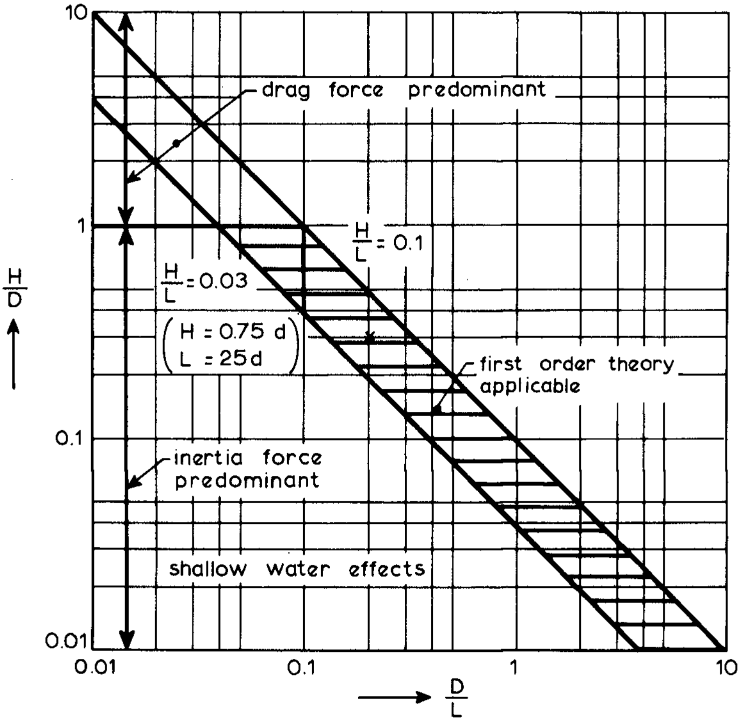


PLAN VIEW OF COFFERDAMS



CROSS SECTION TROUGH y AXIS OF COFFERDAMS

FIG. 2



H = wave height
L = wave length
D = characteristic length of object

FIG. 3 CHARACTERISTIC WAVE PARAMETERS

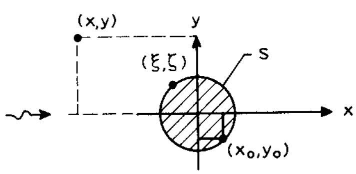


FIG.: 4 NOTATIONS

characterized by large and small values of the Keulegan Carpenter number

$$\frac{U_{max}\,T}{D}\ .$$

which, for deep water, is proportional to the ratio of wave height (H) over pile diameter (D).
For large value of H/D, drag forces are most important whereas for small values of H/D inertia forces predominate. In view of the dimensions of the cofferdam, the latter conditions applies for the present case. However, the ratio of the pile diameter over the wave length is such, that the effect of the structure on the local flow field can no longer be neglected. Resultant forces have to be derived, therefore, from a description of the local wave field which takes into account both the effect of the incident and the scattered waves.

Values of H/D and D/L are such that they fall within the hatched area bounded by the limiting wave steepness in deep water (H/L = 0.1) and the line H/L = 0.03, indicating roughly the limit beyond which shallow water effects strongly manifest themselves. In this area linear wave theory may be applied to describe the wave field. Mathematically the problem is then described as the solution of the Laplace equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad (1)$$

which satisfies the following boundary conditions

free surface: $\frac{\partial^2\phi}{\partial t^2} + g\,\frac{\partial\phi}{\partial z} = 0$ (2)

horizontal bottom $\frac{\partial\phi}{\partial z} = 0$ (3)

surface of the pile $\frac{\partial\phi}{\partial n} = 0$ (4)

Furthermore the well known Sommerfeld condition should be fulfilled, stating that the scattered wave vanishes with increasing distance from the object.

In general, the solution of equation (1) is written as

$$\Phi = \phi_i + \phi_d \qquad (6)$$

where

ϕ_i = potential of incident wave

ϕ_d = potential of scattered wave

The potential ϕ_i is expressed as

$$\phi_i = \frac{H}{2}\frac{g}{\omega}\frac{\cosh k(d+z)\ e^{i(\omega t - mx)}}{\cosh kd} \qquad (7)$$

which satisfies condition 2 and 3, provided the dispersion relationship is valid, stating

$$\omega^2 = gk \tanh kd \qquad (8)$$

The potential ϕ_d may be expressed as

$$\phi_d = \frac{H}{2}\frac{g}{\omega} \cdot \frac{\cosh k(d+z).}{\cosh kd} F(x,y).e^{i\omega t} \qquad (9)$$

which also satisfies condition 2 and 3.
Substitution of (9) in (1) results in the well known Helmholz equation

$$\frac{\partial^2 F(x,y)}{\partial x^2} + \frac{\partial^2 F(x,y)}{\partial y^2} + k^2 F(x,y) = 0 \qquad (1')$$

The problem is defined now as a solution of equation (1^1), which satisfies the boundary condition (5) and the Sommerfeld condition.
As early as 1954 McCamy and Fuchs [3] have solved this problem analytically for a single circular pile, by using a series of Bessel functions. Using the same procedure Spring and Monkmeyer [5] derived a solution for an array of two circular piles of arbitrary diameter and solved the problem for an infinite row of equally spaced equal cylinders exposed to perpendicular wave attack. The solution has been extended to waves from arbitrary direction of incidence by Massel [4], who presented an approximate solution. Recently, Chakrabarti [2], using the solution of Spring and Monkmeyer determined a solution for any arbitrary size at arbitrary angles to the incident wave. However, for an array of cylindrical piles of arbitrary cross-section no analytical solutions are available. Recourse had to be taken to a numerical technique, therefore, as described by Berkhoff. [1]
This was done by assuming that the scattered wave field may be considered as a result of a series of singular sources located along the circumference of the pile. Denoting the strength of the source distribution by $\alpha(\xi,\zeta)$ (see Figure 4) and the source potential function by $G(x,y,\xi,\zeta)$ the following expression is obtained for $F(x,y)$

$$F(x,y) = \int_S \alpha(\xi,\zeta) G(x,y,\xi,\zeta)\, ds$$

where
S = circumference of the piles
(ξ,ζ) = coordinate of the source point located at the object

A source potential function which is a solution of the Helmholz equation (1^1) and which satisfies the Sommerfeld condition is found as

$$G(x,y,\xi,\zeta) = \frac{1}{2i} H_o'(k_o r)$$

where

$$r = \sqrt{(x-\xi)^2 + (y-\zeta)^2}$$

and

H_o' = Hankel function of first kind and zero order.

The source strength $G(\xi,\zeta)$ is found from the boundary condition at the pile surface by putting

$$-\frac{\partial \phi_i}{\partial n} = \frac{\partial \phi_d}{\partial n}$$

which results in the integral equation

$$- \left[\frac{\partial e^{-imx}}{\partial n} \right]_{x_o y_o} = \alpha(x_o y_o) + \int_S \alpha(\xi,\zeta) . \frac{\partial}{\partial n} \left[\frac{1}{2i} H_o' \; (k_o r) \right]_{x_o, y_o} ds$$

Where $x_o y_o$ are the coordinates of an arbitrary point at the circumference of the object.
Once the wave potential is known, the pressures may be obtained from the Bernoulli equation

$$p = - \rho \frac{\partial \phi}{\partial t}$$

Integration of the pressures along the circumference of the pile yields the total wave force.

3 Results for circular piles

3.1 Introduction

In order to compare the results of the present theory with previous studies interaction effects were first studied for a row of 3 circular piles with a diameter of 18 metres, located 40 m apart. Results are given in a dimensionless form using the force coefficients

$$C_{F_x} = \frac{F_x}{\rho g \; H \; tgh \; kd . \frac{\pi \; D^2}{4}} \quad \text{and}$$

$$C_{F_y} = \frac{F_y}{\rho g \; H \; tgh \; kd . \frac{\pi \; D^2}{4}}$$

with

$F_{x,y}$ = maximum force in x or y direction respectively

ρ = density of water

g = acceleration of gravity

H = wave height

d = water depth

k = $2\pi/L$ with

L = wave length

D = pile diameter

Resultant forces were first computed by means of the theory described in section 2. In order to verify these results, model tests have been performed subsequently. These tests have been carried out in a model basin with dimensions 11 x 25 m^2, using a linear scale of 1 : 64. Both regular and irregular waves have been applied to determine the transfer function between wave heights and wave forces.
Finally, for a selected number of pile geometries, the effect of the distance between the piles has been studied experimentally.

3.2 Results of computations

Longitudinal and transverse force coefficients C_{F_x} and C_{F_y} are shown on Figures 5 and 6. Forces have been computed for each individual pile, using wave periods between 4 and 12 seconds and directions ranging from 0^o to 30^o. (0^o corresponding to a direction perpendicular to the center-line of the pile array). It appears that results for a single pile are in good agreement with the analytical solution obtained by McCamy and Fuchs. Individual piles of a row of three piles may experience higher forces than single piles, due to mutual interference. These effects are most pronounced for transverse forces and are clearly shown for wave periods of 5 - 6 seconds.

3.3 Experimental verification

Results of computations have been verified experimentally, for the most critical position of the pile as shown on Figure 7. It appears that results for both regular and irregular waves are in good agreement with computed values.
Experiments have been repeated for various distances between the piles. As may be inferred from Figure 8 interference effects are most pronounced for l/D values in the order of 2. For higher values the effect of the mutual interference vanishes. Transverse forces for all tested arrangements were still higher then those for single piles, however.

4 Results for an array of cofferdams

4.1 Introduction

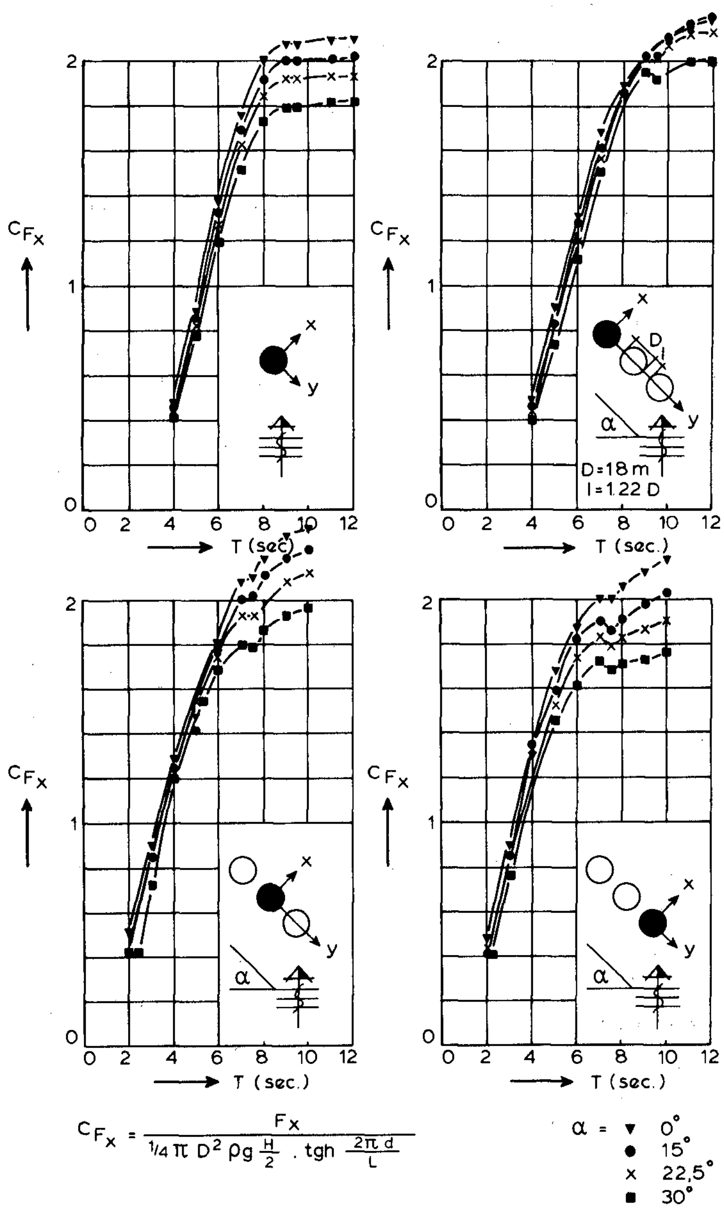
Forces on the array of cofferdams shown on Figure 2, have first been computed. It should be noted that the same dimensionless force coefficients have been used as for circular piles, which explains the high values of these coefficients in the graphs. Subsequently computed values were verified using the same experimental set up as described in section 3.1.

4.2 Results of computations

Longitudinal and transverse force coefficients C_{F_x} and C_{F_y} are shown on Figures 9 and 10. Forces have been computed for each individual pile, using wave periods between 4 and 12 seconds and directions ranging from 0^o to 30^o. It appears that for wave periods between 5...6 seconds, similar interference effects are observed as for circular piles. However, these effects are small compared to another strong interference, observed for a period of 10 seconds, which again is more pronounced for transverse forces. Plots of wave-heights around the piles show a strong amplification of wave-heights between the piles for this wave periods, as a result of interference between scattered and incident waves.

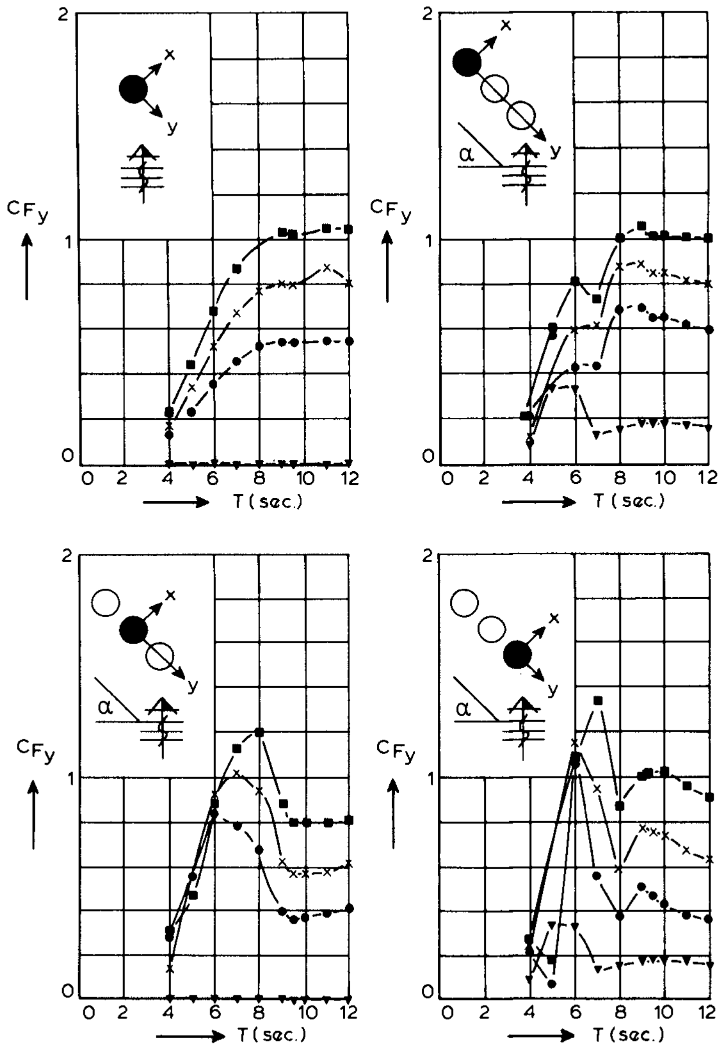
4.3 Experimental verification

Results of computations have been verified experimentally for the most critical position of the pile, as shown on Figure 11. Again computed results agree with experimental data, although transfer functions for C_{F_y} obtained by means of irregular wave show a less distinct amplification for 10 seconds, due to poor resolution of the spectral analysis in this range.



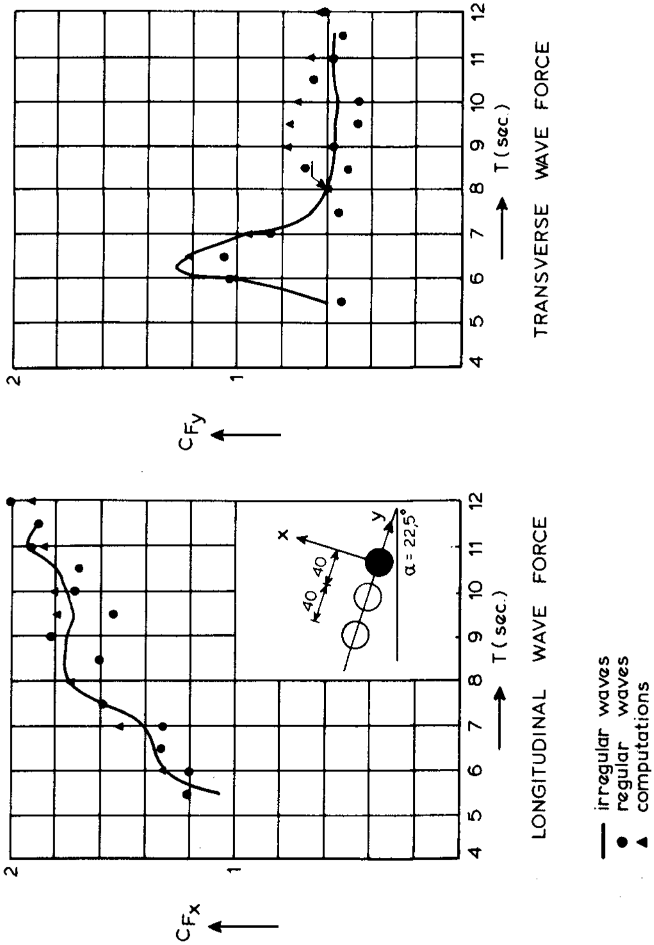
$$C_{F_x} = \frac{F_x}{\frac{1}{4}\pi D^2 \rho g \frac{H}{2} \cdot \operatorname{tgh} \frac{2\pi d}{L}}$$

FIG. 5



$$C_{F_y} = \frac{F_x}{1/4\,\pi\, D^2 \rho g \frac{H}{2} \cdot \text{tgh}\,\frac{2\pi d}{L}}$$

FIG. 6



C_{Fy}
C_{Fx}
T (sec.)
TRANSVERSE WAVE FORCE
LONGITUDINAL WAVE FORCE
x
y
40
40
α = 22,5°
irregular waves
regular waves
computations

FIG. 7

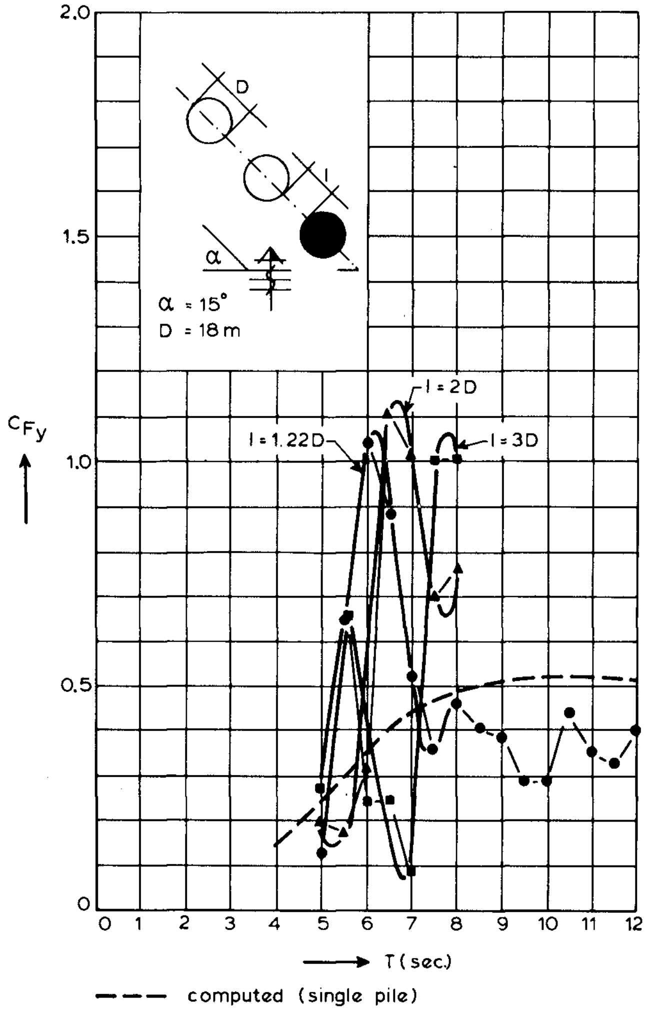

2.0
1.5
1.0
0.5
0
C_{Fy}
D
l
α
α = 15°
D = 18 m
l = 2D
l = 1.22D
l = 3D
0 1 2 3 4 5 6 7 8 9 10 11 12
T (sec.)
computed (single pile)

FIG. 8

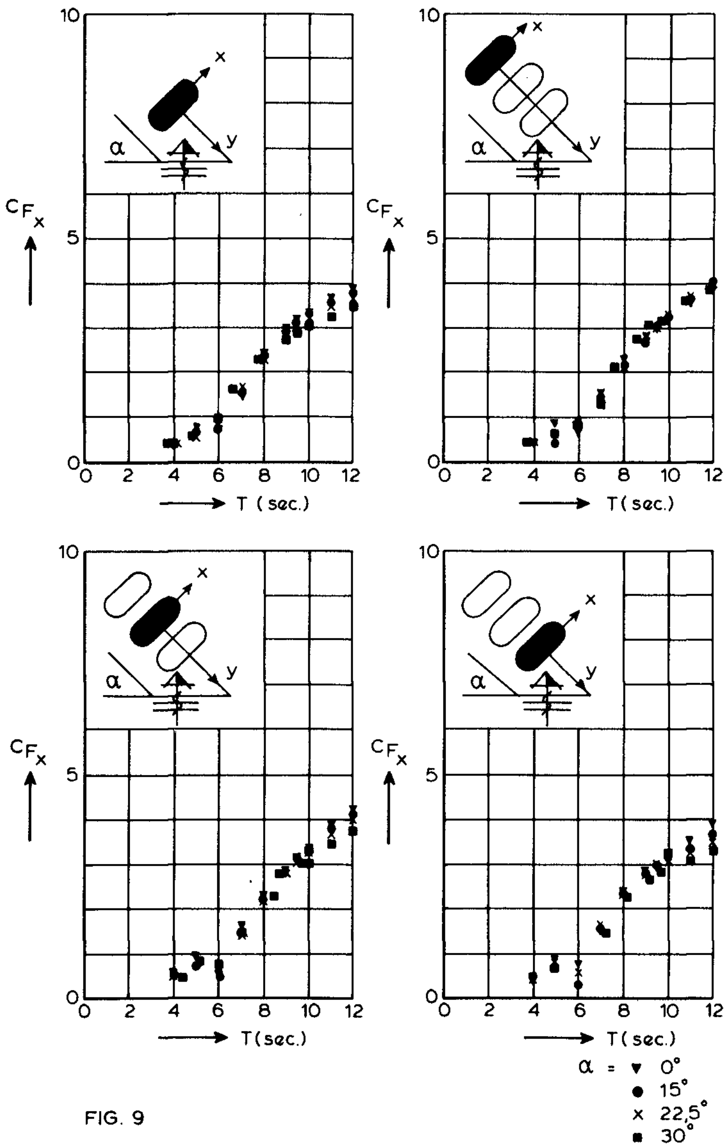


FIG. 9

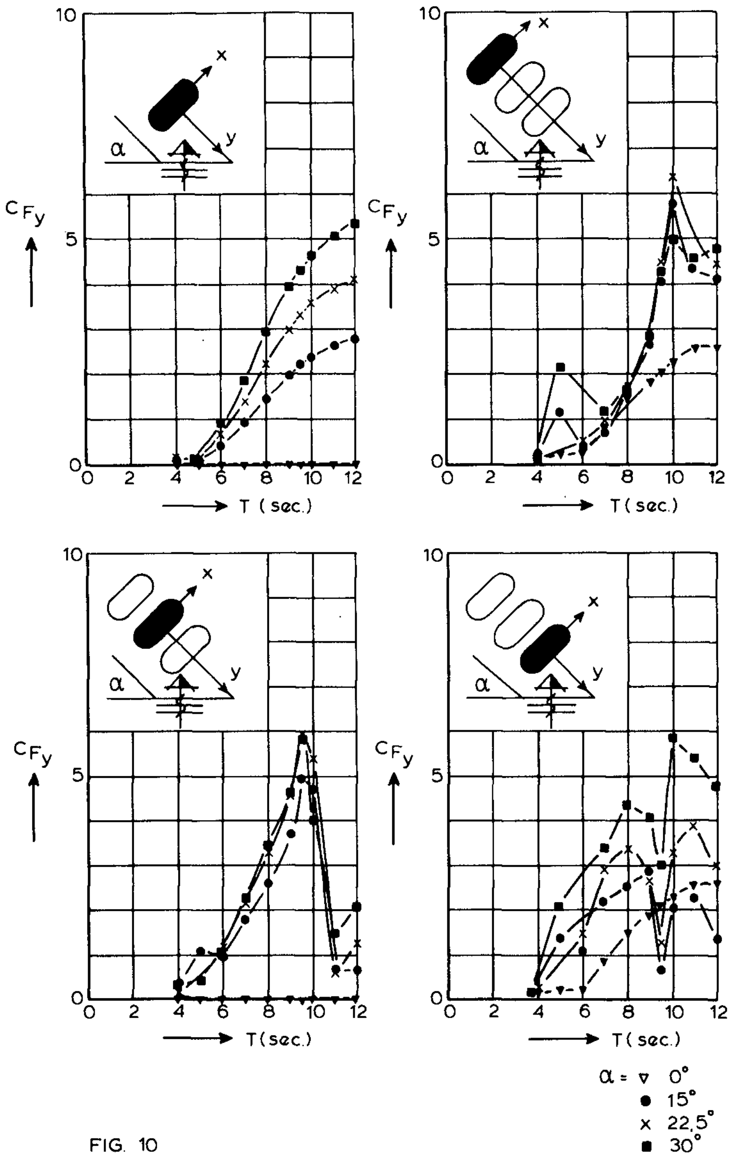
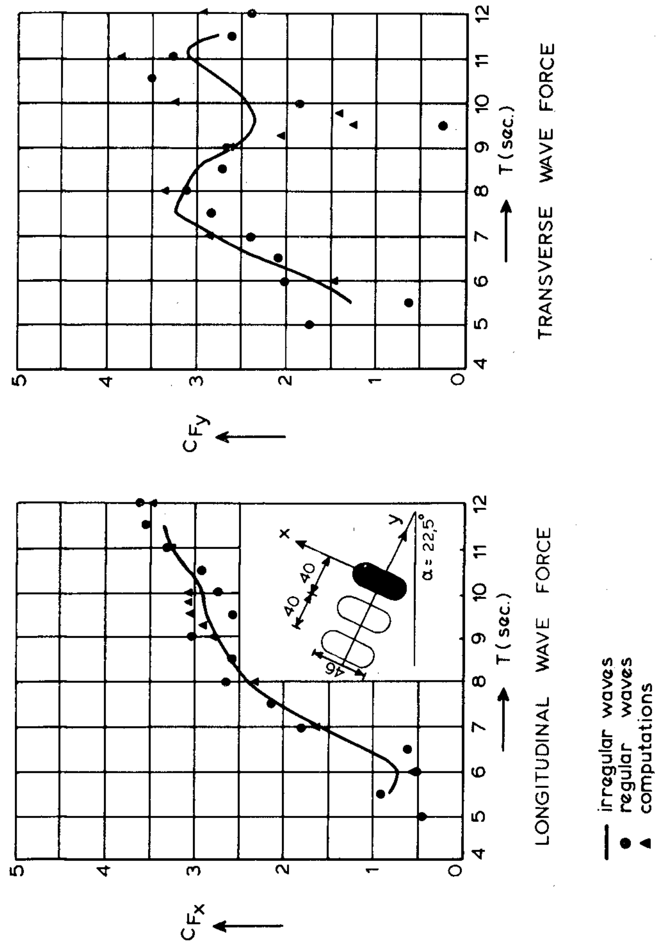


FIG. 10



C_{Fy}
T (sec.)
TRANSVERSE WAVE FORCE
C_{Fx}
T (sec.)
LONGITUDINAL WAVE FORCE
x
y
40
40
46
α = 22,5°
irregular waves
regular waves
computations

FIG. 11

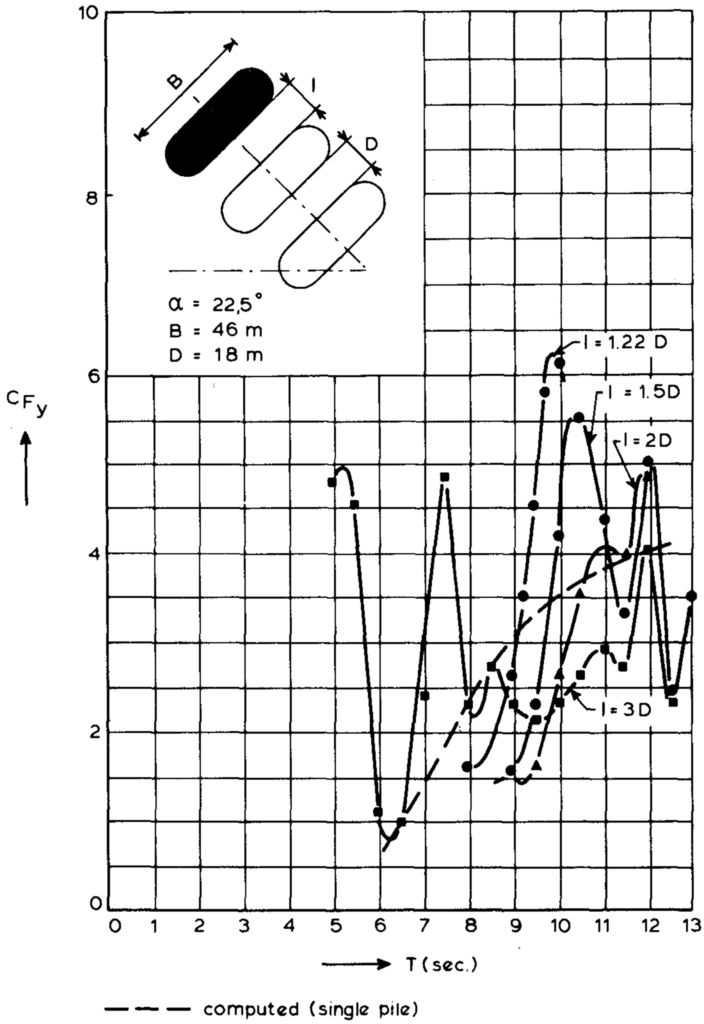

B
l
D
α = 22,5°
B = 46 m
D = 18 m
l = 1.22 D
l = 1.5D
l = 2D
l = 3D
C_{F_y}
T (sec.)
0
2
4
6
8
10
0 1 2 3 4 5 6 7 8 9 10 11 12 13
computed (single pile)

FIG. 12

Transverse forces measured for different distances between the piles (Figure 12), show highest amplification for closely spaced piles (1/D = 1.22), values decrease with increasing distance between the piles. Obviously, critical wave periods causing maximum amplification are also shifted. Maximum forces for all configurations are, however much higher then those found for the individual piles.

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